Prepared for Journal of Engineering in Medicine
Submitted on February 3, 2011
Revision submitted on April 18, 2011

Reconsideration on the Use of Elastic Models to Predict the Instantaneous Load Response of the Knee Joint

L.P. Li *, K.B. Gu

Department of Mechanical and Manufacturing Engineering, University of Calgary,
2500 University Drive, N.W., Calgary, Alberta, Canada T2N 1N4

*Corresponding author:
LePing Li, Ph.D., P.Eng.
Department of Mechanical and Manufacturing Engineering
University of Calgary
2500 University Drive, N.W.
Calgary, Alberta, Canada T2N 1N4

Phone: 1 403 210 7537; Fax: 1 403 282 8406
Email: Leping.Li@ucalgary.ca

ABSTRACT

Fluid pressurization in articular cartilages and menisci plays an important role in the mechanical function of the knee joint. However, the fluid pressure has not been incorporated in the previous finite element modeling of the knee. Elastic models of the knee are widely used instead. It is believed that an elastic model can be used to predict the instantaneous load response of the knee as long as large effective moduli for the cartilaginous tissues are used. In the present study, the instantaneous response of the knee was obtained from a proposed model including fluid pressure and fibril reinforcement in the cartilaginous tissues. The results were then compared with those obtained from an elastic model using the effective modulus method. It was found that the deformations and contact pressures predicted by the two models were substantially different. A unconfined compression of a tissue disk was used to understand the issue. It was clear that a full equivalence between the instantaneous and elastic responses could not be established even for this simple case. A partial equivalence in stress could be conditionally established for a given unconfined compression but not valid for a different magnitude of compression. The instantaneous deformation of the intact tissues in the joint was even more difficult to determine using the effective modulus method. The results thus obtained were further compromised because of the uncertainty over the choice of effective modulus. The tissue nonlinearity was one of the factors that made it difficult to establish the equivalence in stress. The pressurized tissue behaved differently from a solid material when nonlinear fibril reinforcement was presented. The direct prediction of the instantaneous response using the proposed poromechanical model had the advantage to determine the fluid pressure and incompressible deformation.

KEYWORDS: Articular cartilage mechanics; Effective modulus; Fibril-reinforced model; Finite element analysis; Fluid pressure; Knee joint mechanics

1. INTRODUCTION

Finite element models of the knee joints have been developed for various bioengineering applications [1-3]. The analysis is time-consuming because of the complex knee geometry and multiple mechanical contacts involving femur, tibia, patellar, menisci, femoral and tibial cartilages. To simplify the problem, single-phase elastic models of cartilage and meniscus have been widely used in the joint modeling [4-6]. Fibril reinforcement in the articular cartilages and menisci were also considered in a recent model but the fluid pressure in the tissues was still neglected [7]. It is believed that an elastic model can be used to describe the instantaneous load response of the knee or the immediate mechanical behavior of the knee to fast loading, as long as an effective Young's modulus is applied. This effective modulus must be greater than the actual modulus of the tissue matrix, because the fluid trapped in the matrix stiffens the tissue [8]. In addition, an effective Poisson's ratio is set as close to 0.5 as possible to approximate the tissue incompressibility at fast compression [9-10]. This method is referred to as the effective modulus method hereafter.

The equivalence of instantaneous response of fluid saturated materials to the elastic behavior of a single-phase solid material has been long established for 3D problems [11]. For poroelastic beams or columns, only an effective modulus is required for the description of the instantaneous deflections. However, for thin poroelastic plates, both effective modulus and effective Poisson's ratio are required in order to describe the instantaneous mechanical response using elastic solutions [12]. In the mechanics of soft tissues, both the solid and fluid phases are considered incompressible [13,14]. Applying the linear biphasic theory, the effective modulus for the instantaneous compression of a unconfined isotropic disk is $1.5E/(1+\nu)$, where E and ν are the modulus and Poisson's ratio of the tissue matrix [15]. First, we see this effective modulus does not apply to the same tissue disk in confined compression. So it is dependent on boundary conditions. Furthermore, this effective modulus (< 1.5*E*) cannot be applied to the cartilages in the knee joint, for which the effective modulus used was often 10 times greater than a realistic modulus [1,8]. It is not clear how the equivalence can be established in a numerical procedure of a 3D problem, such as for the knee joint, which involves nonlinear and anisotropic properties of multiphasic tissues. The objective of the present study was to determine on what extent an elastic model could describe the instantaneous load response of the knee. Issues on the numerical equivalence of the instantaneous and elastic responses were also investigated.

2. METHODS

Equivalence of Instantaneous Load Response to Elastic Behavior

We first reexamined the concept of equivalence of the instantaneous load response of a hydrated tissue to the elastic behavior of a solid material. The conditions of equivalence can be easily demonstrated using the unconfined compression testing of articular cartilage as an example.

A fibril-reinforced model was chosen for the present study because it highlights the role of collagen fibers in the tissues [16-18]. The model could also describe the strong strain-rate dependent constitutive behavior of articular cartilage that was observed in experiments [19]. In the fibril-reinforced modeling, articular cartilage was considered as a fluid-saturated linear matrix reinforced by a nonlinear collagen network [20]. The linear matrix, referred to as the nonfibrillar matrix, consists of mostly proteoglycans, or the tissue excluding the fluid and collagen fibers. For the case of a tissue disk in unconfined compression, the problem is often simplified as axisymmetric. Using *r* and *z* to refer to the radial and axial directions respectively (*r* is parallel to the articular surface), the stresses in the nonfibrillar matrix can be written in the incremental form of Hooke's law

$$\mathrm{d}\sigma_r^m = \lambda(2\mathrm{d}\varepsilon_r + \mathrm{d}\varepsilon_z) + 2\mu\mathrm{d}\varepsilon_r$$
$$\mathrm{d}\sigma_z^m = \lambda(2\mathrm{d}\varepsilon_r + \mathrm{d}\varepsilon_z) + 2\mu\mathrm{d}\varepsilon_z \qquad (1)$$

where λ and μ are the Lamé constants of the nonfibrillar matrix (denoted by the superscript *m*). The Poisson's ratio $\nu \neq 0.5$ so that the Lamé constants are valid. There is no fibrillar stress in the axial direction, because the fibers are in compression in this direction. The fibrillar stress in the radial direction is

$$\mathrm{d}\sigma_r^f = E_f \mathrm{d}\varepsilon_r \qquad (2)$$

where the modulus of the fibrillar matrix (denoted by the superscript or subscript *f*), E_f, can be a general function of the fibrillar strain, ε_r.

Consider the equilibrium state, or the elastic solution. The fluid pressure vanishes. Using the free boundary condition on the cylindrical surface, the total stress in the radial direction must be zero, i.e.

$$\sigma_r^m = -\sigma_r^f \qquad (3)$$

Substituting Eqs. (2) and (3) into the first equation of (1), the radial strain can be determined by

$$\left[E_f + 2(\lambda + \mu)\right]\mathrm{d}\varepsilon_r + \lambda \mathrm{d}\varepsilon_z = 0 \qquad \text{(Elastic, } \nu \neq 0.5\text{, no fluid pressure)} \qquad (4)$$

if the axial strain is given such as in a relaxation test. Eliminating the radial strain in the second equation of (1) by using (4), the total stress in the axial direction can be obtained by

$$\mathrm{d}\sigma_z = \mathrm{d}\sigma_z^m = \left[\lambda + 2\mu - \frac{2\lambda^2}{E_f + 2(\lambda + \mu)}\right]\mathrm{d}\varepsilon_z \qquad \text{(Elastic, no fluid pressure)} \qquad (5)$$

The radial stress in the matrix is determined by (3) after the relations (2) and (4) are used

$$\mathrm{d}\sigma_r^m = \frac{\lambda E_f}{E_f + 2(\lambda + \mu)}\mathrm{d}\varepsilon_z \qquad \text{(Elastic, no fluid pressure)} \qquad (6)$$

If a compression is applied instantaneously on a homogenous tissue disk, the fluid would be trapped (i.e. no flow) in the tissue at the instant of load application. A uniform fluid pressure is produced instantaneously. Again, the total stress in the radial direction must be zero, i.e.

$$p_f = \sigma_r^m + \sigma_r^f \qquad \text{(Instantaneous response)} \tag{7}$$

The incompressibility of the solid and fluid phases leads to no tissue volume change, or

$$\mathrm{d}\varepsilon_r = -\mathrm{d}\varepsilon_z / 2 \tag{8}$$

Equation (1) is then simplified as

$$\mathrm{d}\sigma_r^m = 2\mu \mathrm{d}\varepsilon_r \ , \quad \mathrm{d}\sigma_z^m = 2\mu \mathrm{d}\varepsilon_z \tag{9}$$

Combining equations (7) to (9) and (2), the total stress in the axial direction can be determined by

$$\mathrm{d}\sigma_z = \mathrm{d}\sigma_z^m - \mathrm{d}p_f = \left(3\mu + \frac{1}{2}E_f\right)\mathrm{d}\varepsilon_z \qquad \text{(Instantaneous response)} \tag{10}$$

If the effective modulus method is used to predict the instantaneous axial stress σ_z, the effective Lamé constants λ' and μ', as defined by the effective modulus and effective Poisson's ratio, must satisfy the following equation

$$\lambda' + 2\mu' - \frac{2\lambda'^2}{E_f + 2(\lambda' + \mu')} \equiv 3\mu + \frac{1}{2}E_f \tag{11}$$

because equations (5) and (10) must be identical. It is observed that the effective material properties must be in general strain-dependent, because E_f is a function of the fibrillar strain. Equation (11) must be satisfied for all strains in a given domain. In order to satisfy the incompressibility requirement, equation (4) must be identical to (8), which requires that

$$E_f + 2\mu' = 0 \tag{12}$$

This requirement can never be satisfied because both properties must be positive (unless $\nu = 0.5$, then equation (8) simply applies). Therefore, one cannot use (4), an elastic formula of compressible material, to calculate the strain of the instantaneous response.

Finite Element Analysis of the Knee Joint

A MRI derived knee model was recently constructed to determine the fluid pressurization in articular cartilages and menisci of the knee [21]. The cartilaginous tissues were modeled using a 3D fibril-reinforced mechanical contact model recently proposed for articular cartilage [22]. The constitutive law of the solid matrix was coded using a FORTRAN subroutine, while the fluid flow was described by Darcy's law formulated in ABAQUS. Site-specific collagen fiber orientations were incorporated per split-line directions [23]. The primary fibers in meniscus were aligned in the circumferential direction, and secondary fibers in the radial direction. Mechanical contact allowing small frictional sliding (coefficient of friction: 0.02) was simulated between the following pairs using ABAQUS: femoral and tibial cartilages, femoral cartilage and meniscus, meniscus and tibial cartilage. The results thus obtained compared reasonably well with data from the literature [21].

In the present study, finite element solution was first obtained using the proposed poromechanical model including fluid pressure (referred to as "full solution"). The SOIL CONSOLIDATION procedure in ABAQUS was used for this purpose. The elastic solution corresponding to the effective modulus method was then obtained using the steady analysis from the SOIL procedure, which ignored the fluid pressurization. The same fiber properties were used for both cases, but the steady analysis was performed using a Poisson's ratio of 0.48 and a large effective modulus for the nonfibrillar matrix of cartilage or meniscus (Table 1). The effective modulus was determined by matching the total forces for the two solutions. This modulus is commonly determined this way in published studies when the force data is available, and finite element analysis is used to obtain contact pressure etc.

Using the proposed model, the instantaneous response was approximated by sealing all tissue surfaces with impermeable boundary conditions and using virtually zero permeability. The permeability was taken to be 10^{-6}mm^4/Ns, which was approximately one thousandth of any realistic permeability for cartilage and meniscus. This small value enhanced the impermeability within the tissues in addition to a rather high ramp compression rate of 100μm/s used in all finite element solutions. These conditions secured negligible fluid flow and incompressibility in the tissues (confirmed by the fluid velocity obtained).

All knee compressions were applied at 100μm/s on the top of the meshed distal femur while the bottom of the meshed tibia was fixed [21]. The femur was also restrained from rigid-body translation in the horizontal plane, but free from any rotations. These boundary conditions excluded significant knee flexion but allowed small sliding between articular surfaces.

3. RESULTS

The full solution associated with fluid pressure was obtained using the proposed model with Young's moduli of 0.26 and 0.50 MPa, respectively, for the nonfibrillar matrices of cartilages and menisci, and a Poisson's ratio of 0.36 for all cartilaginous tissues (Table 1, nonlinear). While all fibrillar properties remained unchanged and the Poisson's ratio for the elastic solution was set to 0.48, the effective modulus of the nonfibrillar matrices was found to be 1.22 MPa, in order to match the total force from the full solution at the compression of 100 μm (star in Figs. 1). Here, for simplicity, the same effective modulus was used for all cartilaginous tissues. However, using this exact effective modulus, the forces from the elastic solutions were, respectively, only 93% and 86% of the full solutions when the compression was increased to 200 and 500 μm (Fig. 1). In fact, the two curves do not match even at small strains, with the full solution more nonlinear.

The maximum contact pressure predicted by the effective modulus method was virtually the same as that from the full solution (Fig. 2; Note: it was true for 100μm compression only, because the equivalence was attempted at this compression). However, the pressure distributions predicted by the two methods were significantly different, with less high contact pressure regions (in red) or larger low contact areas (not shown) when the fluid pressure was present (Fig 2a vs 2b). The fluid pressure also produced a quite different displacement pattern in the lateral condyle (Fig. 3a vs 3b). When the compression was increased to 200μm, the contact pressure patterns were similar to what shown in Fig. 2 for the two cases respectively, but the peak contact pressures were not the same anymore (Fig.2 caption).

When the effective modulus was increased from 1.22 to 12 MPa, a moderate value from literature, the total force would be increased to 6.3 times (Fig. 4), and the peak contact pressure on the femoral cartilage to 7.5 times for the same compression of 100 μm (Fig. 5). The displacement pattern (not shown for the larger modulus) had essentially no similarity to that shown in Fig. 3b.

Two additional cases of the knee joint were further considered in order to determine the factors that influenced the equivalence of the instantaneous and elastic responses. Linear fibrillar property was assumed to run the two cases with and without fluid pressure (last column in Table 1). The effective modulus was adjusted in order to match the forces at 100μm compression for all four cases (Fig. 6). It was seen that the force function from the elastic solution deviated less from the instantaneous response when the material properties were linear (Fig. 6).

Four parallel cases were also considered for the unconfined compression of cartilage disks (Table 2, Fig. 7). The difference between the full and elastic solutions was more evident here, because the tissue disk was fully (uniformly) pressurized, as compared very locally pressurized in the knee joint. The axial stresses in the two linear solutions were identical (independent on compression) as the equivalence could be established for the axial stress by equation (11) when E_f was constant. However, it was not necessary for the effective Poisson's ratio to be close to 0.5. The same force and axial stress would be obtained from the linear elastic solution, if the effective Poisson's ratio were zero and the effective modulus were 7.4868 MPa (instead of 2.5257 MPa when the effective Poisson's ratio was taken to be 0.48, Table 2). These results for unconfined compression disk were obtained from ABAQUS using the same user-defined numerical procedure as that for the knee joint modeling. They are identical to the analytical solutions from the previous section, an indication of good numerical results. It should be noted that the instantaneous response of a homogeneous tissue disk to this uniform compression can be precisely obtained using the finite element method with impermeable boundary condition regardless of the permeability of the tissue and the speed of compression.

4. DISCUSSION

There has been no doubt over the equivalence of the instantaneous load response of soft tissue to an elastic behavior [11-13]. The question is how to establish the equivalence in a finite element analysis, or whether we should just use a poromechanical model to determine the instantaneous response. The present study indicated that a full equivalence could not be always established using constant effective modulus even the Poisson's ratio used was close to 0.5. When only linear constitutive laws were applied, an equivalence in forces and stresses could be established for the case of uniform deformation using an effective modulus (Fig. 7), but could not be established for the case of non-uniform deformation such as in the knee joint (Fig. 6; the difference was small for the linear case though). When any nonlinear constitutive law was involved, the equivalence could not be fully established using the effective modulus method (Figs. 1-3 & 7, equation 11). The effective properties must be strain dependent, although it is not clear whether the equivalence can be expressed with simple equations for general cases. No equivalence will be established in finite element analysis unless numerical procedures are particularly formulated to accommodate the strain dependent equivalence.

Incompressibility must be particularly formulated in order to obtain the instantaneous deformation using an elastic model [7]. The volume strain depends on both the modulus and the Poisson's ratio (Fig. 8). It may not be reliable to simply use a Poisson's ratio close to 0.5 with a generic formulation to approximate the incompressibility of the tissues, because it is asymptotic at 0.5 and numerical singularity will occur if the value is too close to 0.5.

Two factors were found to prevent simple establishment of equivalence between the instantaneous and elastic responses using the effective modulus method: (1) a nonlinear constitutive law obviously made the problem more complicated, as demonstrated with the analytical solution for the case of unconfined compression; (2) A non-uniform deformation produces complex fluid pressure gradients instantaneously. Another factor is the boundary conditions, which is obvious but not investigated in the present study.

There were uncertainties in the results obtained using the effective modulus method, because of the non-uniqueness of the effective modulus. The maximum contact pressure for a given compression (e.g. 100μm) from the elastic solution did not deviate much from the full solution (Fig. 2), when a partial equivalence in force was established for this compression (100μm). However, the pressure distributions were very different even the total forces were equal (Fig. 2). Furthermore, this partial equivalence shown in Fig. 2 would not be possible for other compression magnitudes (e.g. 50 or 200μm) if the same effective modulus were used. A new equivalence must be established for each magnitude of compression with a new effective modulus. For the case of unconfined compression, the effective modulus must be 5.72 MPa to match the force at 200μm compression (not shown), other than 3.14 MPa for the 100μm compression, if the same effective Poisson's ratio 0.48 is used. The deformations from the two solutions were very different, as indicated by the displacement (Fig. 3, 100μm compression), even the partial equivalence was established. In addition, the displacement pattern was also influenced by the magnitude of the modulus: a larger modulus produced more rigid motion for the femoral cartilage (not shown, but more arrows pointed downward compared to Fig. 3b). Therefore, the reliability of the results largely depends on the correct choice of the effective modulus. The importance of proper determination of the effective modulus can also be observed from Figs. 3-4.

It is neither necessary nor sufficient to use an effective Poisson's ratio close to 0.5 to establish the equivalence between the instantaneous and elastic load responses. The equivalence in deformation cannot be established using any Poisson's ratios except for exactly 0.5 (equation 12). The partial equivalence in stress can be established for unconfined compression for a range of effective Poisson's ratio and modulus if the fibrillar modulus is constant (equation 11). However, a unique effective Poisson's ratio may be required for general 2D and 3D problems [24].

Nonlinear fibril reinforcement was considered in the present study. The important influence of fibril reinforcement on the instantaneous response of cartilage has been recognized in experimental studies [25], and demonstrated in modeling [20]. The fibers were recently incorporated into whole knee joint modeling without considering the fluid pressure, but an incompressible material model was necessarily used [7]. In the present study, the uncertainty in the effective modulus was not a direct consequence of the nonlinear fiber properties. In fact, the same fiber properties were used in the comparison of the instantaneous and elastic load responses. The difference was produced by the fluid pressurization in the tissues. The pressurized tissue behaved very differently from a solid material when the fibril reinforcement was nonlinear [20].

The major limitation of this study was the use of small deformation theory, which may have caused certain numerical error for the large compression shown in Fig. 1. However, most of the results presented were for 100μm compression, which produced strains well below 2.5% for most contact regions (if 50μm compression went to the femoral cartilage, and the rest to tibial cartilage/menisci). The conclusions from these results should be qualitatively correct, and supported by the analytical solutions in the methods section. The solutions obtained from large deformation theory would be more nonlinear, resulting in more difficulties to establish numerically the equivalence between the instantaneous and elastic responses. Furthermore, if realistic large compression were considered, the difference between the full and elastic solutions would be much more significant than what is observed in Fig. 6, because the nonlinearity would be more evident when the tissues are more compressed. This prediction can be deduced from the case of unconfined compression shown in Fig. 7, which was subjected to 10% uniform compressive strain. It is important to point out that the difference in the contact pressure distributions from the two solutions was significant even the compressive strain was small and the total forces were equal (Fig. 2).

In conclusion, the effective modulus method with an elastic model cannot be used to describe the incompressibility of the cartilaginous tissues at instantaneous knee compression. The *deformation* thus obtained is greatly compromised even the effective Poisson's ratio is very close to 0.5. Some equivalence in *stress* between the instantaneous and elastic responses might be established for a given knee compression if the effective modulus and Poisson's ratio were determined for the same compression. The stresses may only be predicted for the compression at which the equivalence has been established. Different effective properties must be used for a different compression. The effective modulus method is more suitable for linear than nonlinear materials, only if the deformation is not a concern. The proposed poromechanical model accounting for the fluid pressure is more time-consuming than the elastic model, but has the potential to provide further or more precise information. In addition, only real material properties are needed in the poromechanical model, which eliminates the uncertainties in the determination of compression-dependent effective properties encountered when an elastic model is used. Finally, the findings presented here might serve as a guide to interpret the elastic solutions should the effective modulus method be used to seek preliminary results.

ACKNOWLEDGEMENTS

This study was partially supported by Natural Sciences and Engineering Research Council of Canada.

REFERENCES

[1] Bendjaballah MZ, Shirazi-Adl A, Zukor DJ. Biomechancs of the human knee joint in compression: reconstruction, mesh generation and finite element analysis. *The Knee* 1995, **2**, 69-79.

[2] Li G, Gil J, Kanamori A, Woo SL. A validated three-dimensional computational model of a human knee joint. *J Biomech Eng* 1999, **121**, 657-662.

[3] Peña E, Calvo B, Martínez MA, Palanca D, Doblaré M. Finite element analysis of the effect of meniscal tears and meniscectomies on human knee biomechanics. *Clinical Biomech* 2005, **20**, 498-507.

[4] Haut Donahue TL, Hull ML, Rashid MM, Jacobs CR. A finite element model of the human knee joint for the study of tibio-femoral contact. *J Biomech Eng* 2002, **124**, 273-280.

[5] Penrose JM, Holt GM, Beaugonin M, Hose DR. Development of an accurate three-dimensional finite element knee model. Comput Methods Biomech Biomed Eng 2002, **5**, 291-300.

[6] Ramaniraka NA, Terrier A, Theumann N, Siegrist O. Effects of the posterior cruciate ligament reconstruction on the biomechanics of the knee joint: a finite element analysis. *Clinical Biomech* 2005, **20**, 434-442.

[7] Shirazi R, Shirazi-Adl A, Hurtig M. Role of cartilage collagen fibrils networks in knee joint biomechanics under compression. J Biomechanics 2008, **41**, 3340-3348.

[8] Yao J, Funkenbusch PD, Snibbe J, Maloney M, Lerner AL. Sensitivities of medial meniscal motion and deformation to material properties of articular cartilage, meniscus and meniscal attachments using design of experiments methods. *J Biomech Eng* 2006, **128**, 399-408.

[9] Besier TF, Gold GE, Delp SL, Fredericson M, Beaupré GS. The influence of femoral internal and external rotation on cartilage stresses within the patellofemoral joint. *J Orthop Res* 2008, **26**, 1627-1635.

[10] Yang N, Nayeb-Hashemi H, Canavan PK. The combined effect of frontal plane tibiofemoral knee angle and meniscectomy on the cartilage contact stresses and strains. *Ann Biomed Eng* 2009, **37**, 2360-2372.

[11] Biot MA. General theory of three-dimensional consolidation. *J Applied Physics* 1941, **12**, 155-164.

[12] Cederbaum G, Li LP, Schulgasser K. Poroelastic structures. Elsevier Science Ltd., Amsterdam, July 2000.

[13] Ateshian GA, Ellis BJ, Weiss JA. Equivalence between short-time biphasic and incompressible elastic material responses. *J Biomech Eng* 2007, **129**, 405-412.

[14] Bachrach NM, Mow VC, Guilak F. Incompressibility of the solid matrix of articular cartilage under high hydrostatic pressures. *J Biomech* 1998, **31**, 445-451.

[15] Armstrong CG, Lai WM, Mow VC. An analysis of the unconfined compression of articular cartilage. *J Biomech Eng* 1984, **106**, 165-173.

[16] Korhonen RK, Laasanen MS, Töyräs J, Lappalainen R, Helminen HJ, Jurvelin JS. Fibril reinforced poroelastic model predicts specifically mechanical behavior of normal, proteoglycan depleted and collagen degraded articular cartilage. *J Biomechanics* 2003, **36**, 1373-1379.

[17] Taylor ZA, Miller K. Constitutive modeling of cartilaginous tissues: a review. *J Applied Biomechanics* 2006, **22**, 212-229.

[18] Wilson W, van Donkelaar CC, van Rietbergen B, Ito K, Huiskes R. Stresses in the local collagen network of articular cartilage: a poroviscoelastic fibril-reinforced finite element study. *J Biomechanics* 2004, **37**, 357-366. Erratum: **38**, 2138-2140.

[19] Oloyede A, Flachsmann R, Broom ND. The dramatic influence of loading velocity on the compressive response of articular cartilage. *Connective Tissue Research* 1992, **27**, 211-244.

[20] Li LP, Buschmann MD, Shirazi-Adl A. Strain-rate dependent stiffness of articular cartilage in unconfined compression. *J Biomech Eng* 2003, **125**, 161-168.

[21] Gu KB, Li LP. A human knee joint model considering fluid pressure and fiber orientation in cartilages and menisci. *Med Eng Phys* 2011, **33**(4), 497-503.

[22] Li LP, Cheung JTM, Herzog W. Three-dimensional fibril-reinforced finite element model of articular cartilage. *Medical & Biological Engineering & Computing* 2009, **47**, 607-615.

[23] Below S, Arnoczky SP, Dodds J, Kooima C, Walter N. The split-line pattern of the distal femur: a consideration in the orientation of autologous cartilage grafts. *J Arthroscopic and Related Surgery* 2002, **18**, 613-617.

[24] Li LP, Cederbaum G, Schulgasser K. Theory of poroelastic plates with in-plane diffusion. *Int J Solids Structures* 1997, **34**, 4515-4530.

[25] Mizrahi J, Maroudas A, Lanir Y, Ziv I, Webber TJ. The “instantaneous” deformation of cartilage: effects of collagen fiber orientation and osmotic stress. *Biorheology* 1986, **23**, 311-330.

Table 1 Material properties used in all finite element analyses of the knee joint. The collagen network was considered as nonlinear in the proposed poromechanical model, where the modulus was a function of the tensile strain ε. The compressive stiffness of the fibers was neglected. Linear fibrillar property, or constant fibrillar modulus, was used for comparison.

Tissue	Fibrillar moduli, Nonlinear (MPa)		Nonfibrillar matrix		Fibrillar moduli, Linear (MPa)	
	Primary fiber direction	Normal directions	E (MPa)	ν	Primary fiber direction	Normal directions
Femoral cartilage	$3+1600\varepsilon$	$0.9+480\varepsilon$	0.26	0.36	10	3
Tibial cartilage	$2+1000\varepsilon$	$2+1000\varepsilon$	0.26	0.36	5	5
Menisci	28	5	0.5	0.36	28	5
Bone	$E_b = 5000$, $\nu_b = 0.3$					
Effective moduli for cartilages & menisci ($\nu = 0.48$):			Nonlinear 1.22,		Linear 1.25	

Table 2 The four cases of finite element analysis on articular cartilage in unconfined compression. The poromechanical model was used to consider the fluid pressure ("Full solution" and "Linear trapped"). The effective modulus method was used when no fluid pressure was considered (Nonlinear "Elastic solution" and "Linear elastic"). The Lamé constants for the nonfibrillar matrix were $\mu = E / 2(1+\nu)$ and $\lambda = E\nu / (1+\nu)(1-2\nu)$. The radius and thickness of the disk were, respectively, 3 and 2 mm.

Assumed nonlinear fibrillar matrix $E_f = (2+1000\varepsilon_f)$ MPa	Assumed linear fibrillar matrix $E_f = 14.4$ MPa
"Full solution" Fluid saturated matrix $E = 0.26$ MPa, $\nu = 0.36$	"Linear trapped" Fluid saturated matrix $E = 0.26$ MPa, $\nu = 0.36$
"Elastic solution" (Nonlinear) Effective solid matrix $E' = 3.14$ MPa, $\nu' = 0.48$	"Linear elastic" Effective solid matrix $E' = 2.5257$ MPa, $\nu' = 0.48$

FIGURE CAPTIONS

Fig. 1 Total reaction force in the knee joint as a function of the compression magnitude. The full solution was obtained using the proposed model considering the fluid pressure in cartilages and menisci. The elastic solution ignored fluid pressurization and was obtained using an effective modulus that enabled a match of the forces at 100 μm compression as marked by the star.

Fig. 2 Contact pressure on the articular surface of the femoral cartilage at 100μm knee compression. (a) full solution with fluid pressure considered; (b) elastic solution using a large effective modulus. The broken lines indicate the position of menisci (inferior view: medial meniscus on the right-hand side). The peak contact pressures at 200μm compression were (not shown) 0.321 and 0.296 MPa for cases (a) and (b) respectively. Note that the contours do not actually show the boundaries of the contact areas.

Fig. 3 Surface displacement vectors of the femoral cartilage at 100μm knee compression. (a) full solution (8−225μm); (b) elastic solution (35−142μm). The broken lines indicate the position of menisci (inferior view: medial meniscus on the right-hand side).

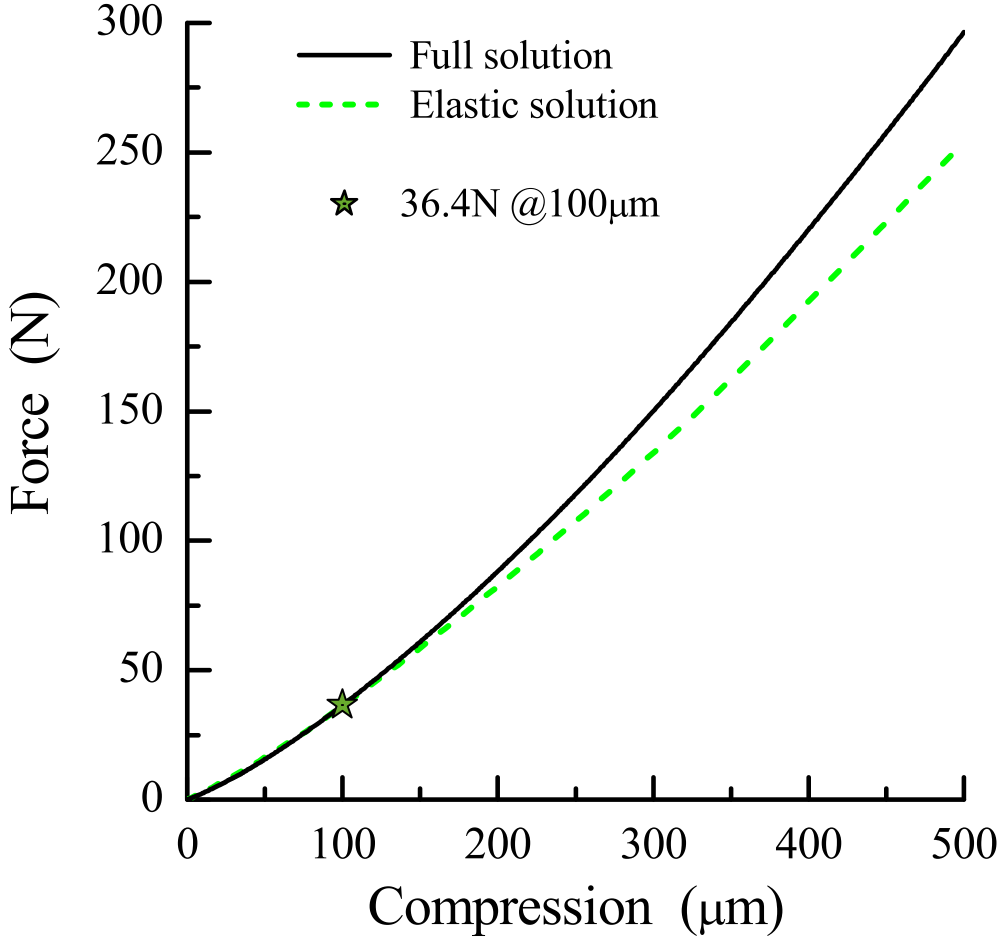
Fig. 4 Total reaction force in the knee joint as a function of Young's modulus of the nonfibrillar matrix, as determined by the effective modulus method. The effective Poisson's ratio was 0.48.

Fig. 5 Maximum contact pressure on the femoral cartilage as a function of Young's modulus of the nonfibrillar matrix, as determined by the effective modulus method. The effective Poisson's ratio was 0.48.

Fig. 6 Total reaction force in the knee joint as a function of the compression magnitude. The "Full solution" and "Elastic solution" were part of the results shown in Fig. 1, assuming nonlinear fibrillar property. Two additional solutions were obtained assuming linear fibrillar property: "Linear trapped" was the solution considering fluid pressure, while "Linear elastic" was an elastic solution. Material properties (Table 1) were chosen so that the forces for all cases were the same at 100μm compression as marked by the star.

Fig. 7 Total reaction forces for the cartilage disk in unconfined compression. The four cases were parallel to those in Fig. 6 for the knee joint, and are illustrated in Table 2.

Fig. 8 Volume strain as a function of the Poisson's ratio when the Young's modulus was given. This was determined for a cartilage disk in unconfined compression when fluid pressure was not considered.



Full solution
Elastic solution
36.4N @100μm
Force (N)
Compression (μm)
0
50
100
150
200
250
300
0
100
200
300
400
500

Figure 1

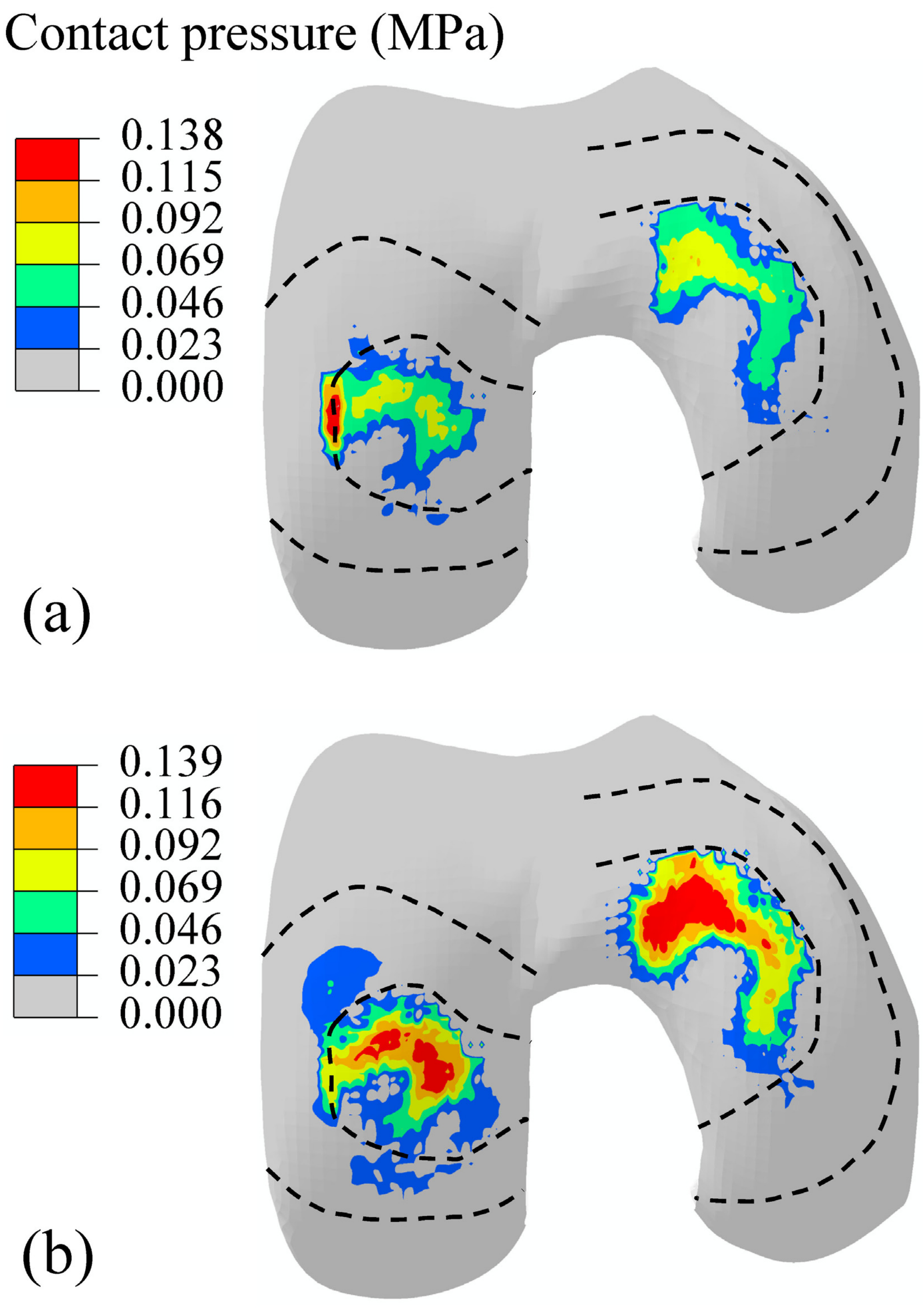
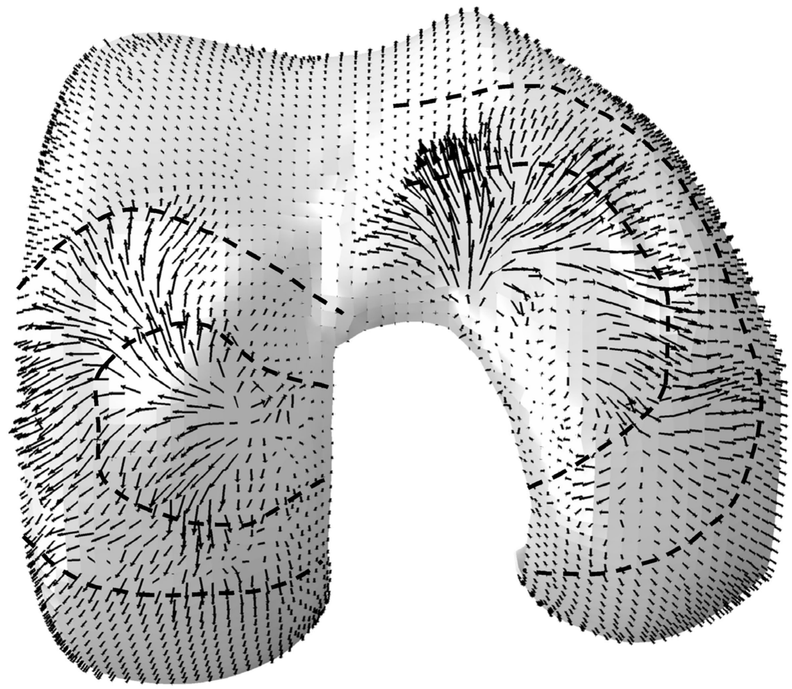
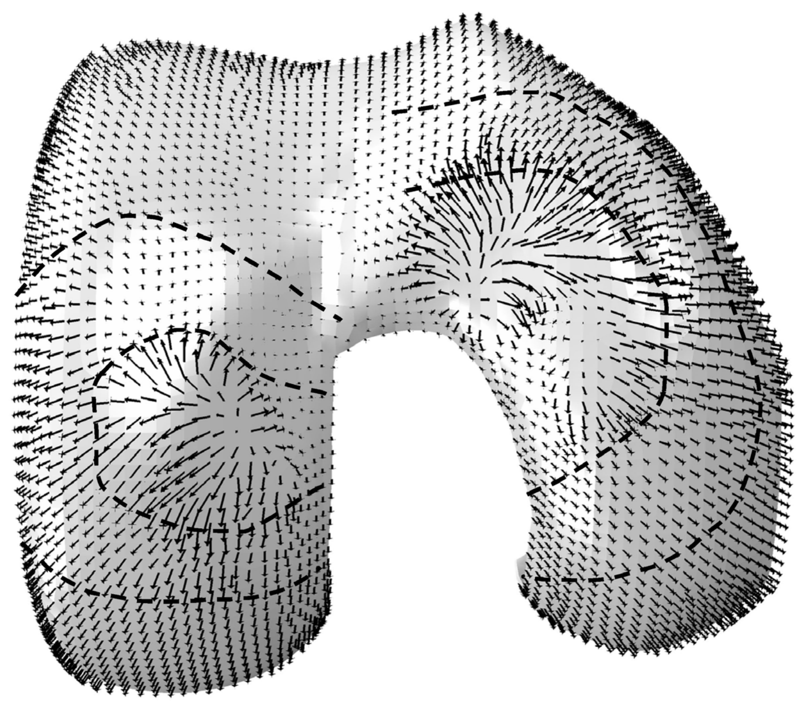

Contact pressure (MPa)
0.138
0.115
0.092
0.069
0.046
0.023
0.000
(a)
0.139
0.116
0.092
0.069
0.046
0.023
0.000
(b)

Figure 2



(a)



(b)

Figure 3

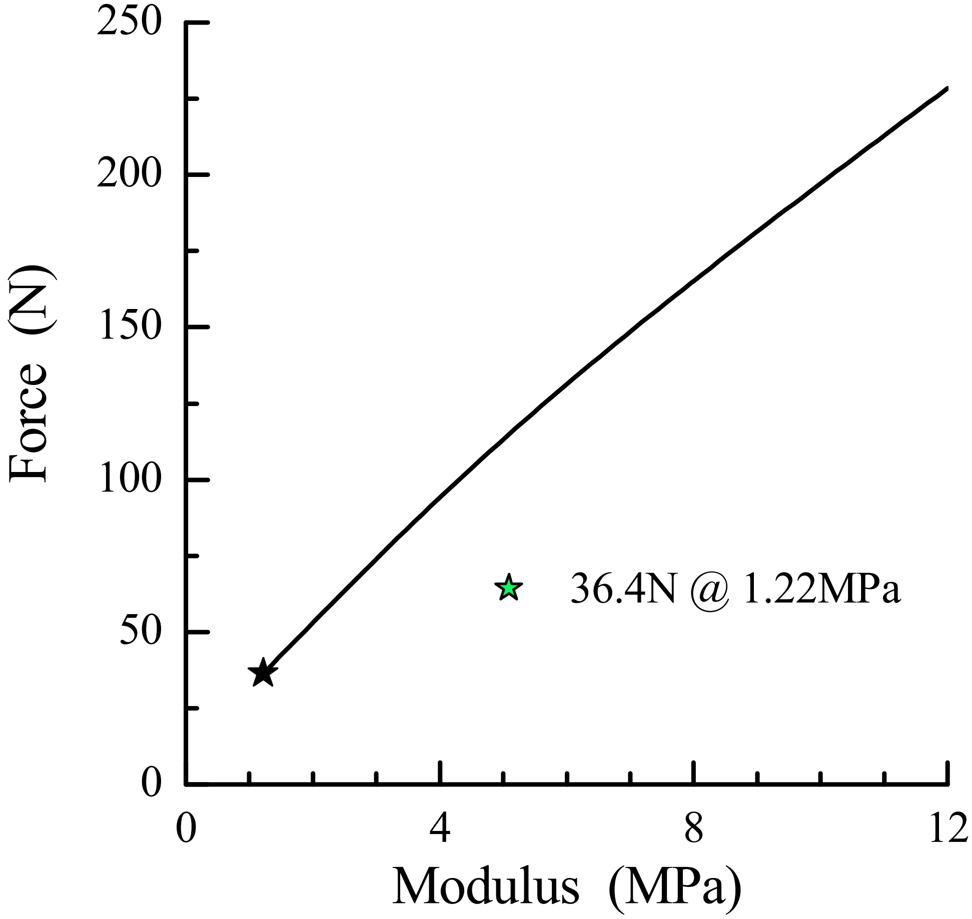
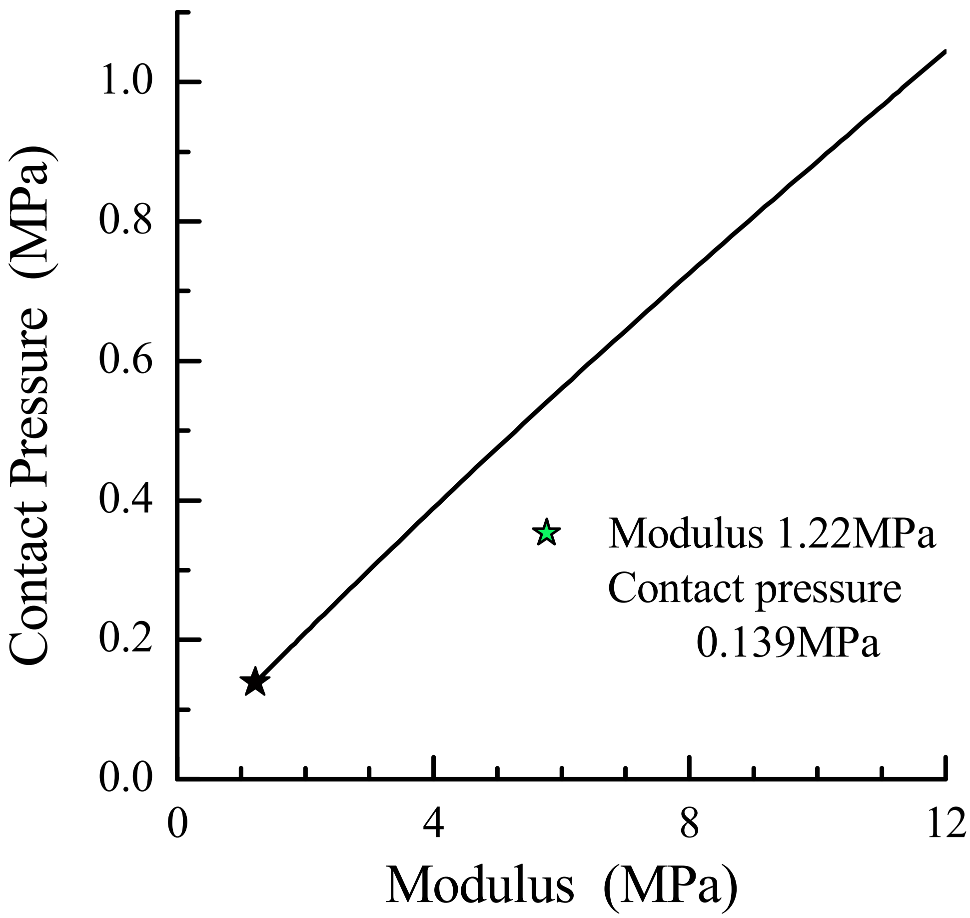


Figure 4



Contact Pressure (MPa)
1.0
0.8
0.6
0.4
0.2
0.0
0
4
8
12
Modulus (MPa)
Modulus 1.22MPa
Contact pressure
0.139MPa

Figure 5

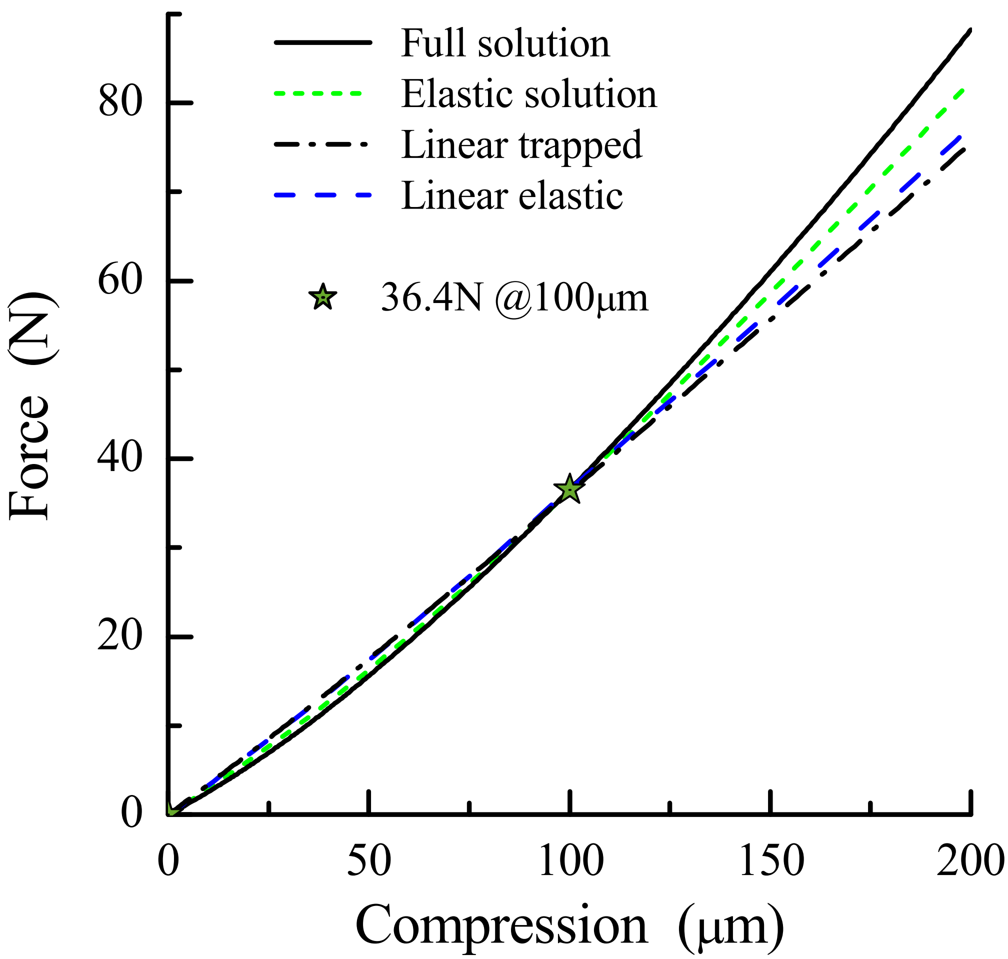

Full solution
Elastic solution
Linear trapped
Linear elastic
36.4N @100µm
Force (N)
Compression (µm)
0
20
40
60
80
0
50
100
150
200

Figure 6

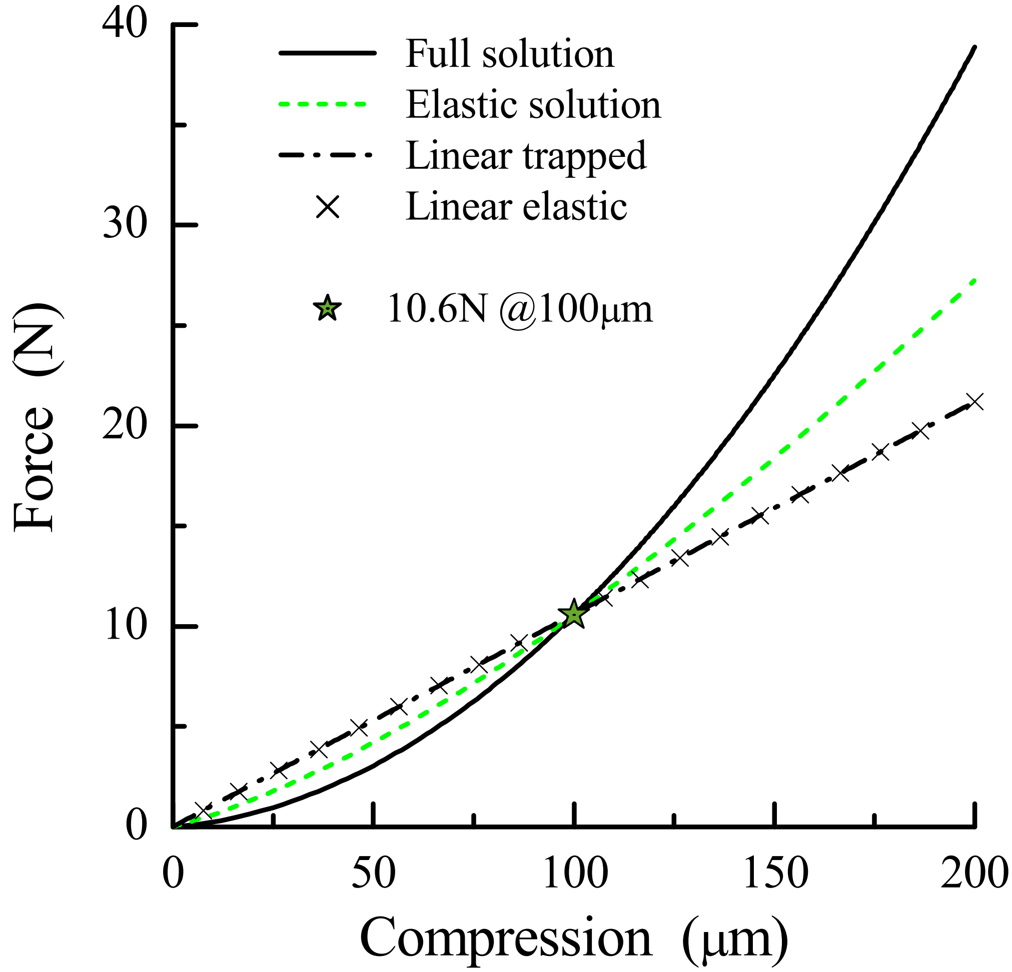
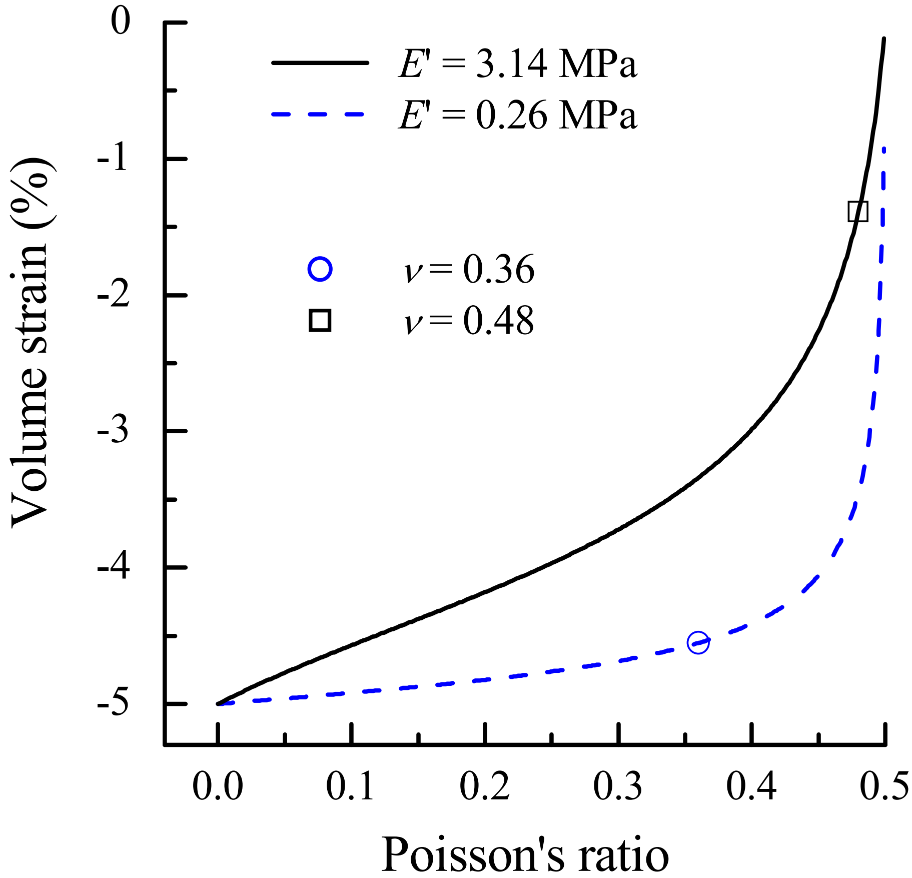

Full solution
Elastic solution
Linear trapped
Linear elastic
10.6N @100µm
Force (N)
Compression (µm)
0
10
20
30
40
0
50
100
150
200

Figure 7



Volume strain (%)
Poisson's ratio
E' = 3.14 MPa
E' = 0.26 MPa
ν = 0.36
ν = 0.48
0
-1
-2
-3
-4
-5
0.0
0.1
0.2
0.3
0.4
0.5

Figure 8