From last time...

1) The slot machine in the picture has 3 "states," cherries, plums, and lemons.

a) List all the *microstates* of the machine.

b) How many microstates belong to the *macrostate*, "all the same"?

c) How many microstates belong to the *macrostate*, "mixture"?

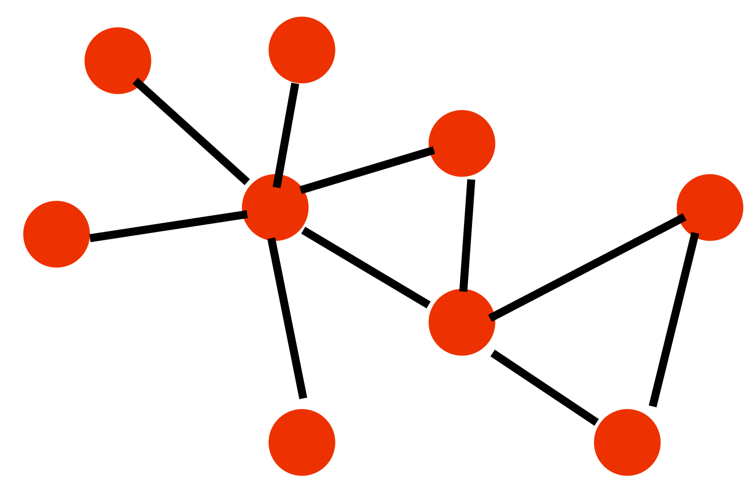
d) What is the likelihood of achieving each macrostate?

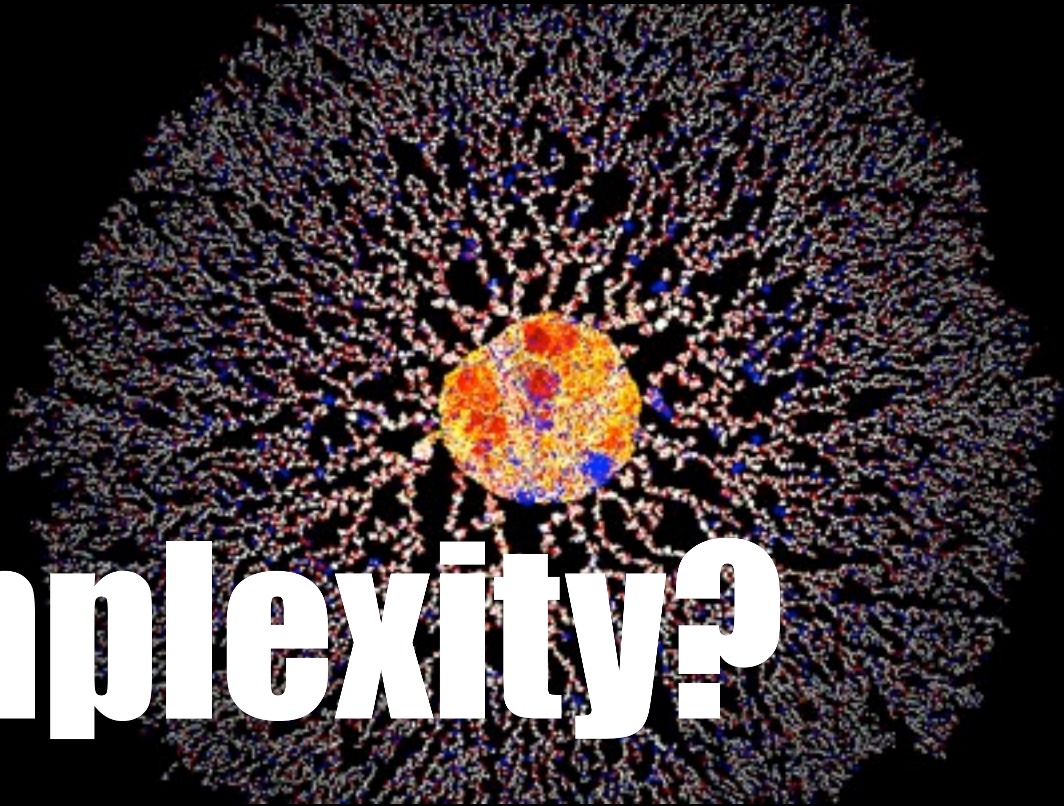
e) Which macrostate is the most likely? What does this tell you about its entropy? Is this macrostate an *ordered* or *disordered* macrostate? Why (try to use *information* in your argument)?

2. Use the network at right to answer the following:

a) What is the *degree* of each *node*?

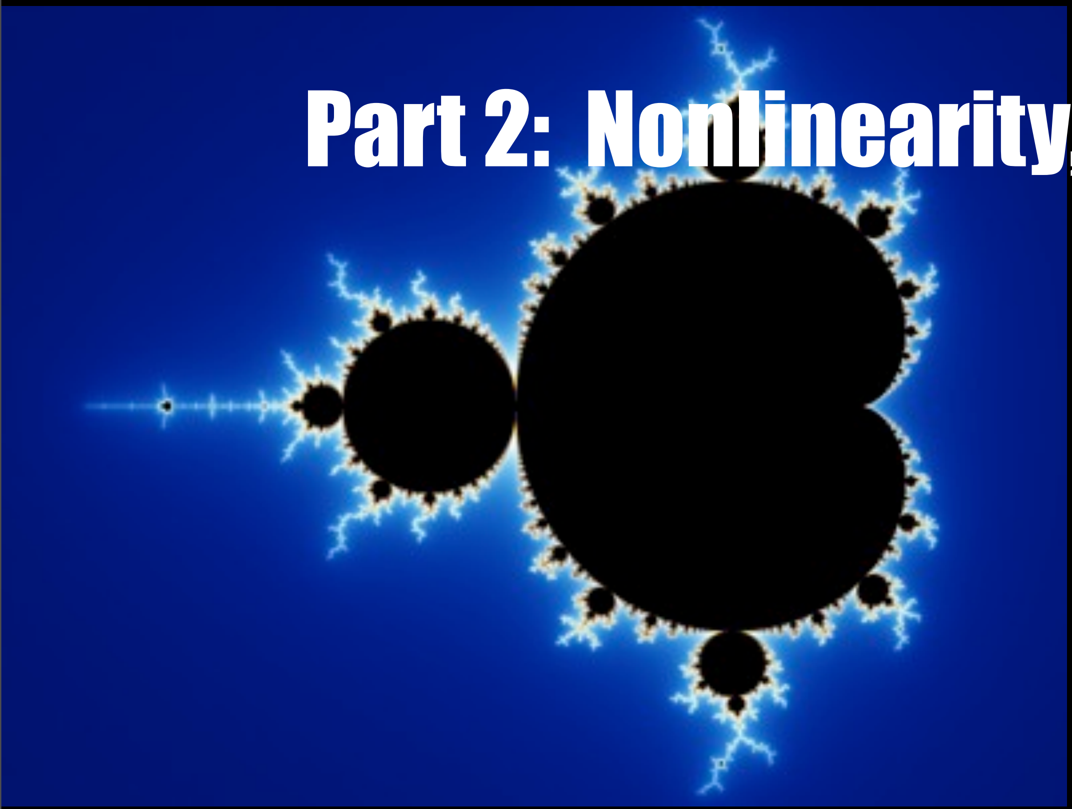
b) What is the *average clustering coefficient*?
(Hint: C=(3xNumber of Triangles)/(Number of Triads)



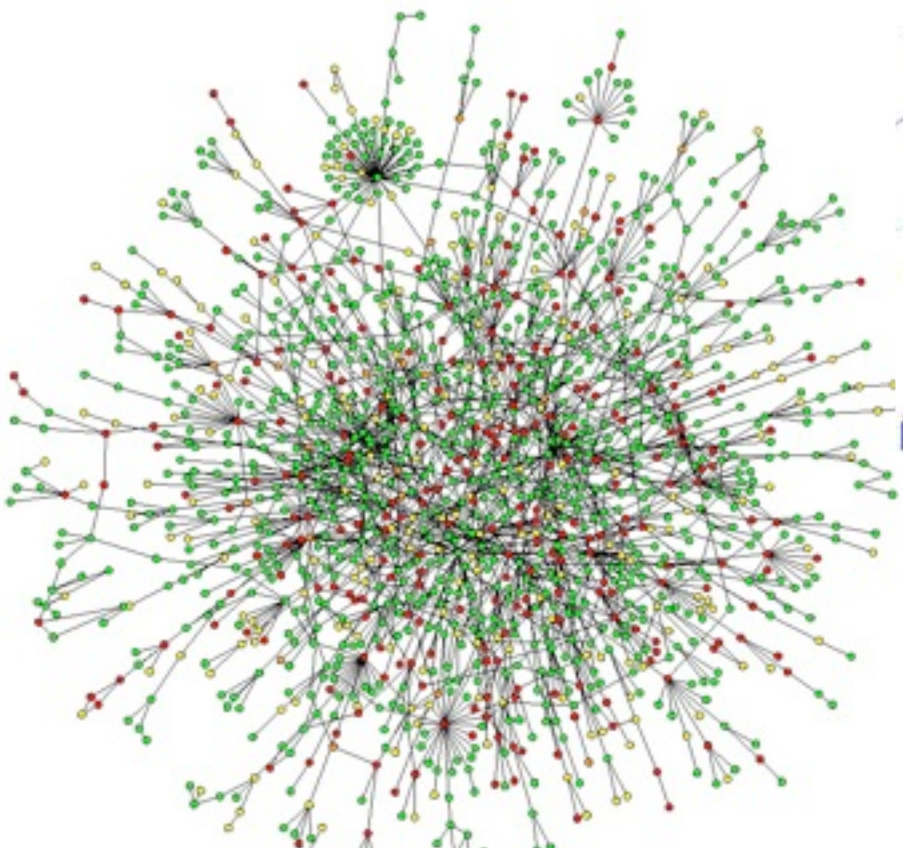


What is Complexity?

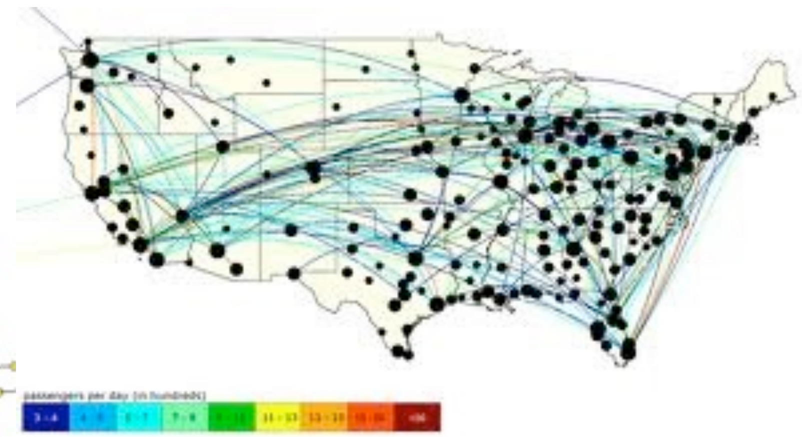
Part 2: Nonlinearity, Chaos and Emergence; Scaling and Fractality



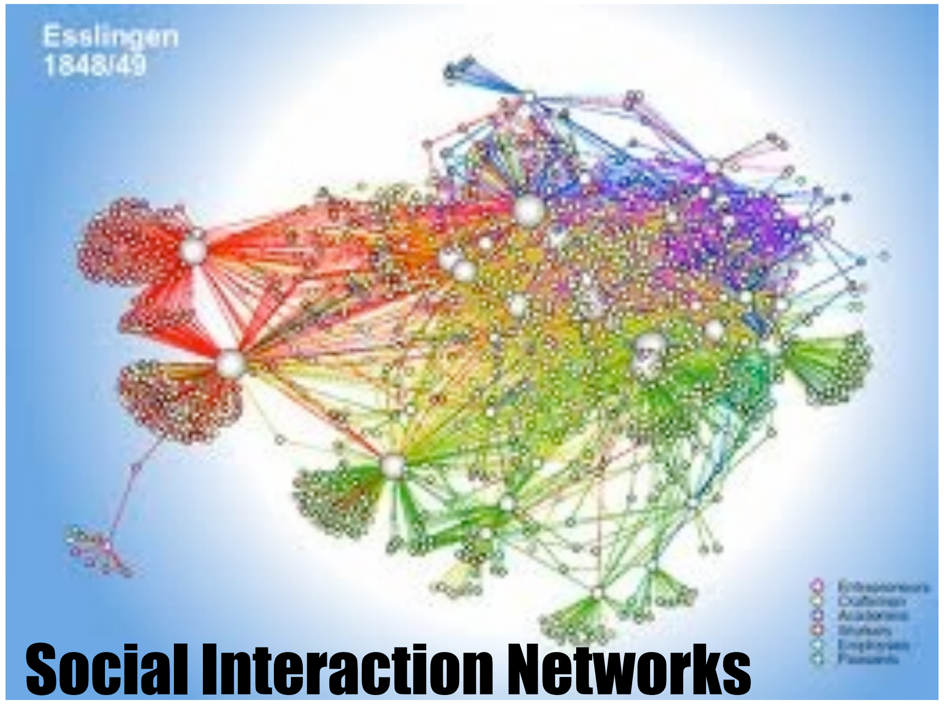
Systems as diverse as...



Protein-protein Interactions



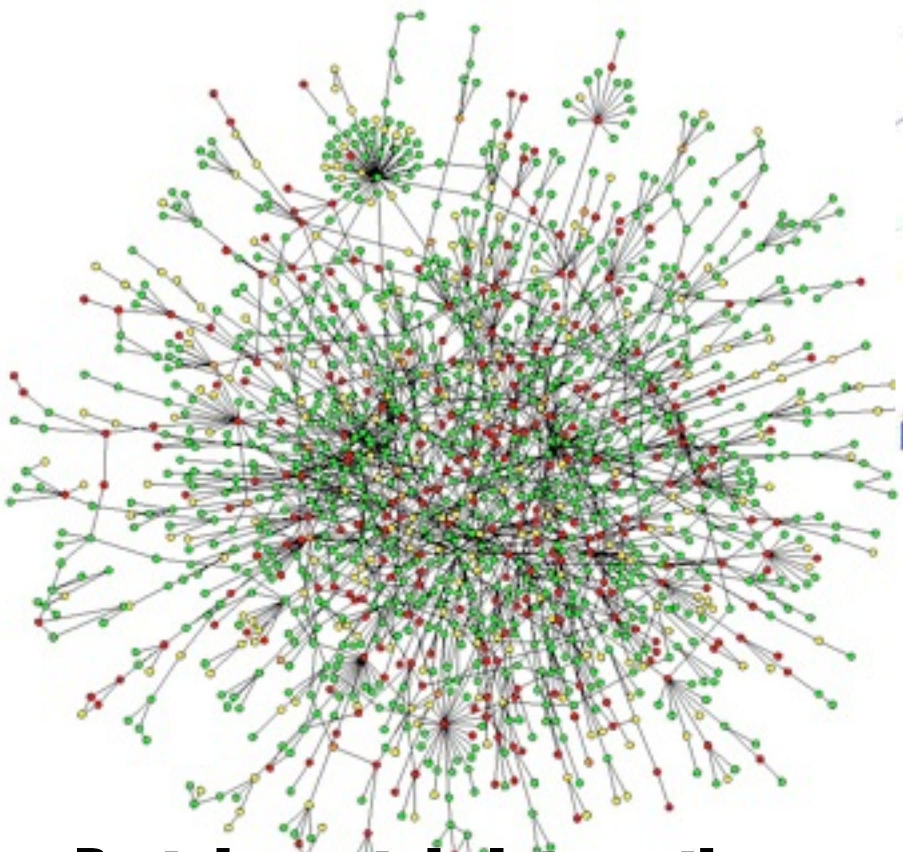
Air Traffic Networks



Social Interaction Networks

Contain similar information on a global scale because of...

Systems as diverse as...



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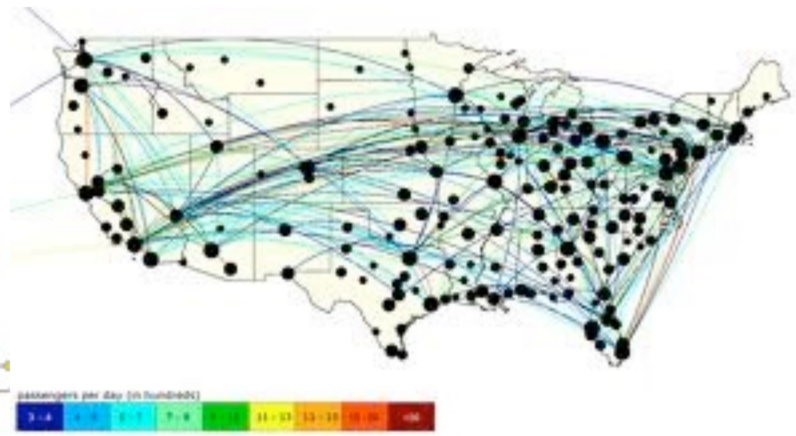

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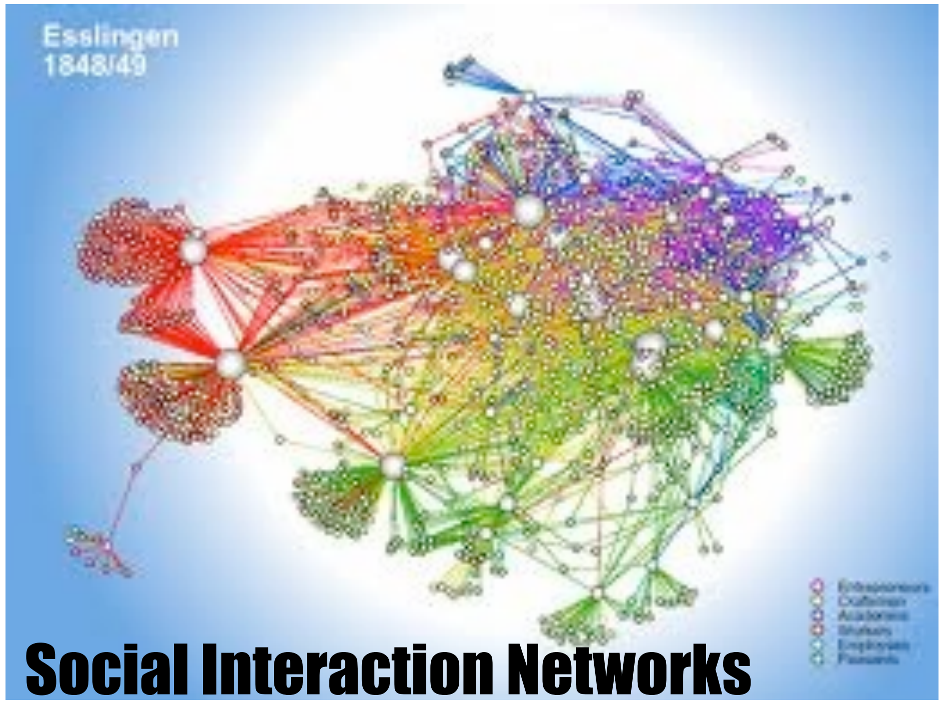

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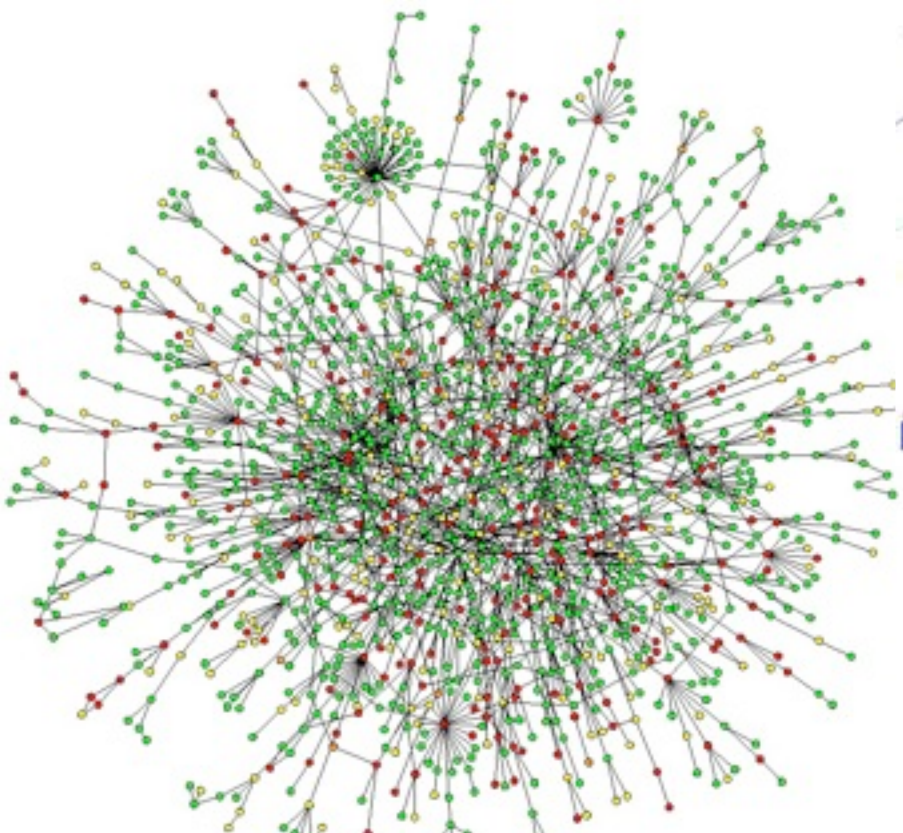

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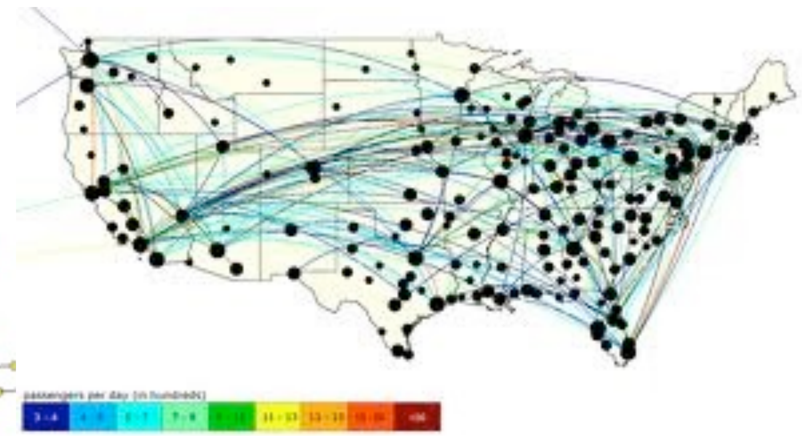

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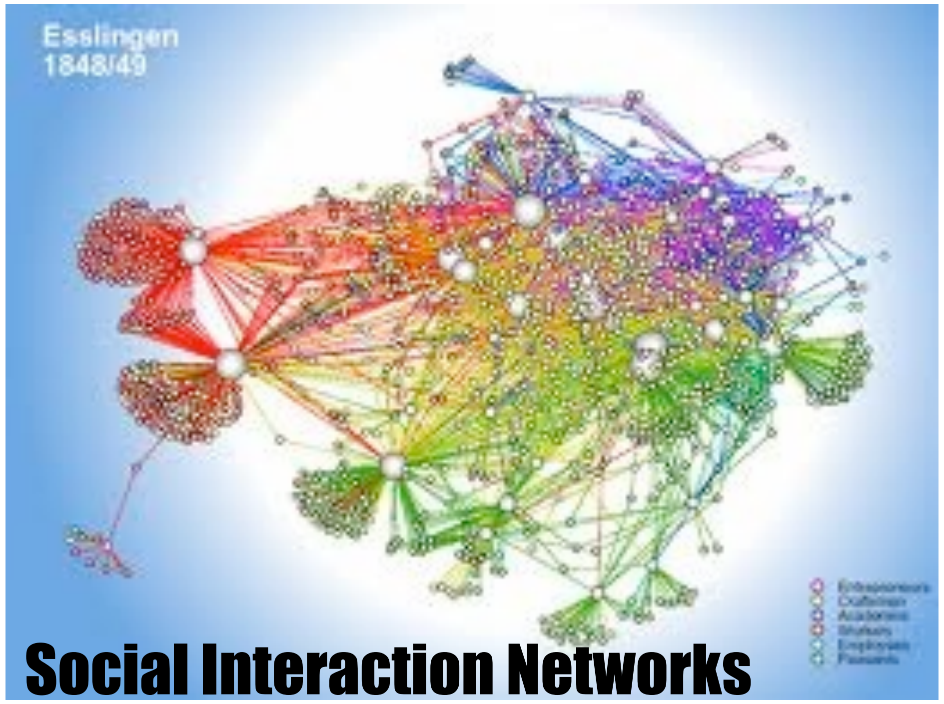

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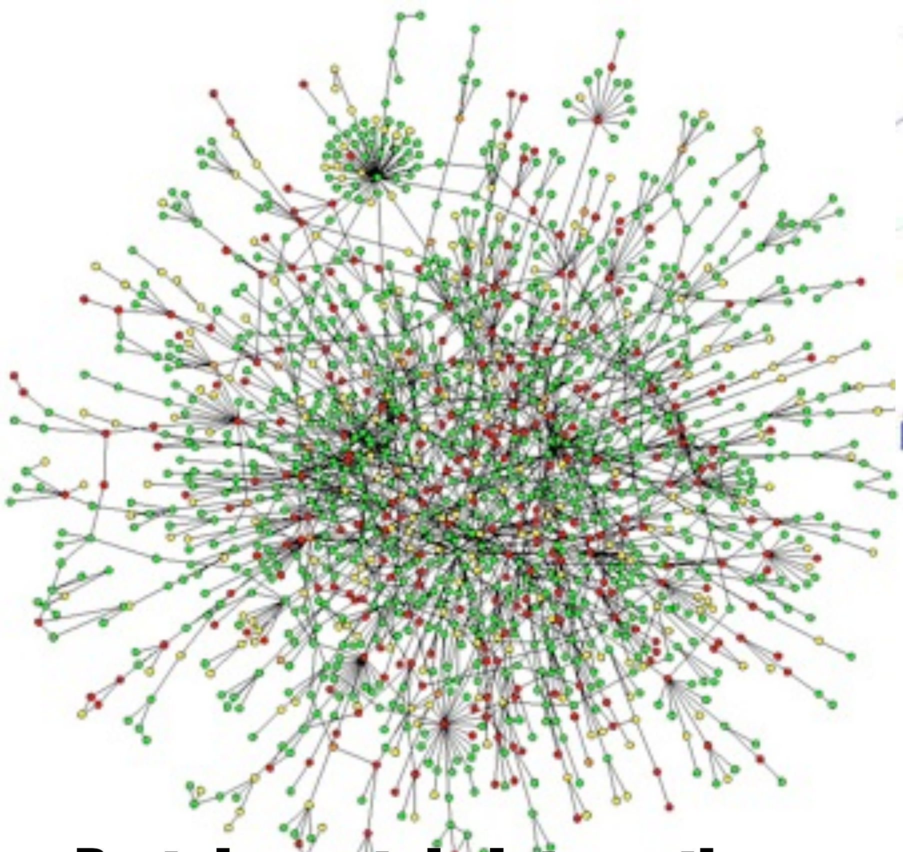

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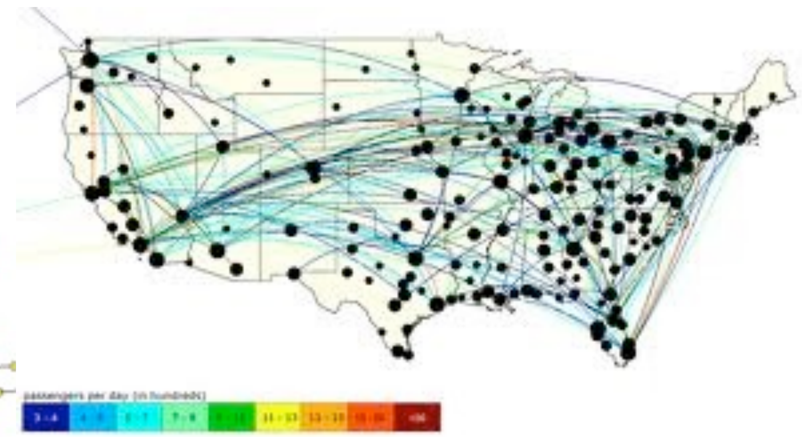

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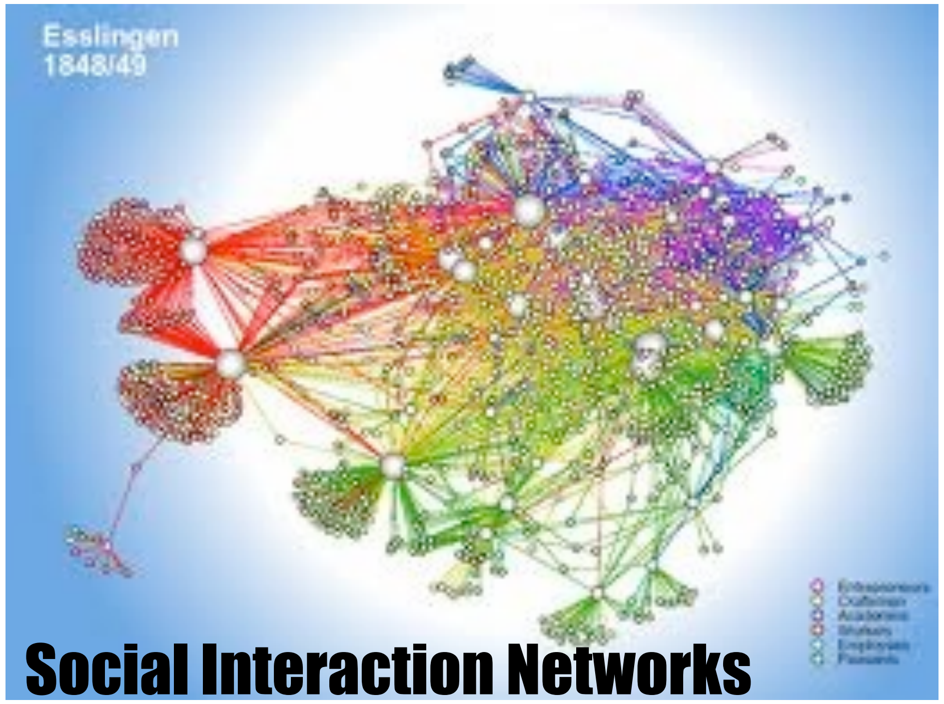

Social Interaction Networks

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Self-Organizing Behavior

Shared emergent properties!

Where do we use Complex Networks?

- Modeling network formation and structure (e.g. How do large social networks, like Facebook form?)

- Exploring disease/information spread in populations

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Statics

↓

Dynamics

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Descriptions of systems that show complex CHANGING behaviour that emerges from the collective actions of many interacting components

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- ***How*** **can systems change?**
- ***What types of behaviours*** **are possible?**
- ***What predictions*** **about those behaviours can be made?**

A brief history of dynamics...

Isaac Newton

- **Invents Calculus (math of motion and change)**
- **Laws of Motion**
- ***Kinematics* (how) vs. *Dynamics* (why)**

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Library of Congress

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- **3-body problem**
- **“Sensitive dependence on initial conditions”**

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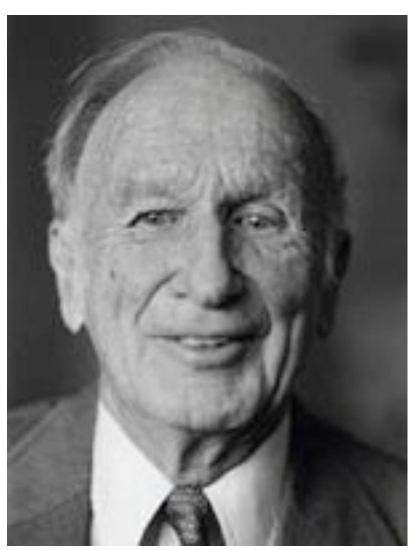
Edward Lorenz (and many others)



- "Butterfly Effect"
- Chaos

But *why* do some systems have this "sensitivity to initial conditions"?!

Linear Systems

- Understand parts individually
- *Add* behaviours back together to understand whole system

Nonlinear Systems

- Individual parts *might* be understood
- Behaviour of whole is NOT simple sum of parts

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Nonlinearity

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A linear system

Population of breeding frogs. Each year, all frogs pair up to reproduce and each set of parents has exactly 4 offspring. The parents then die.

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•
•
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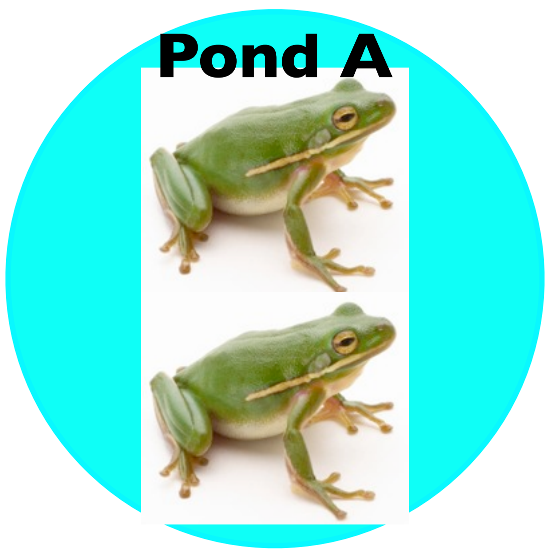

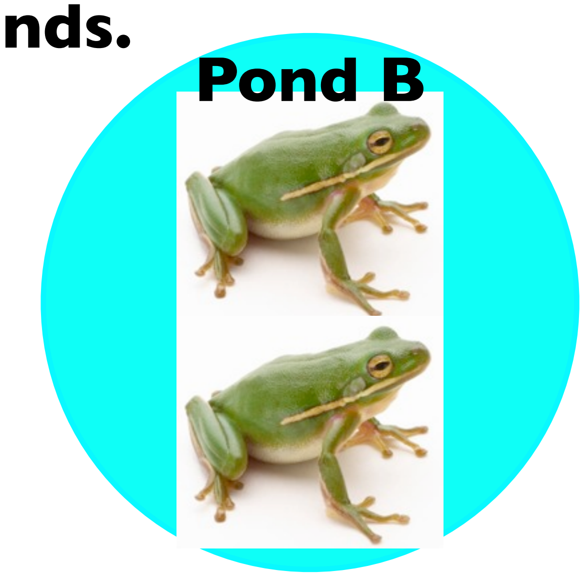

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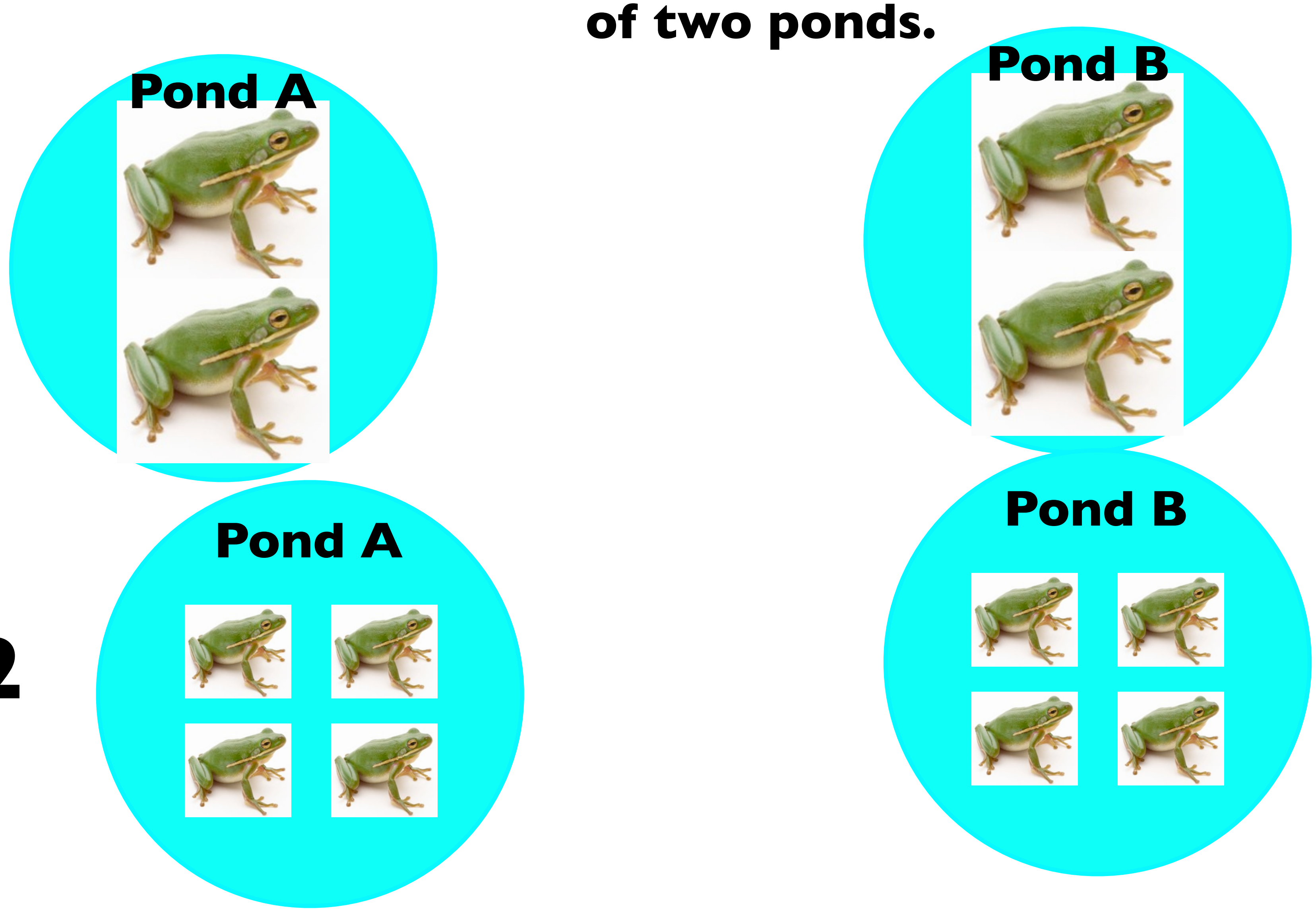

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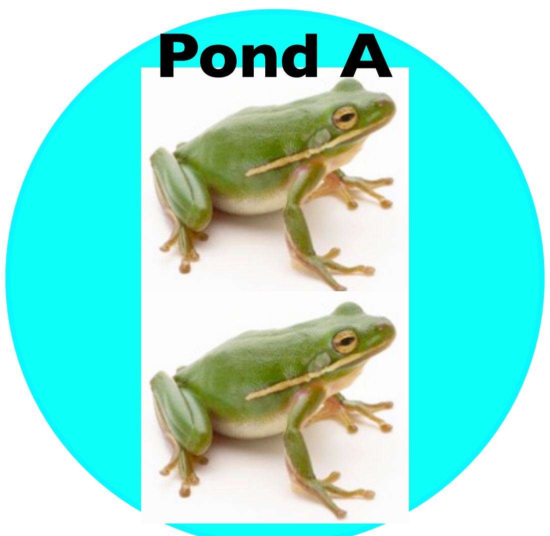
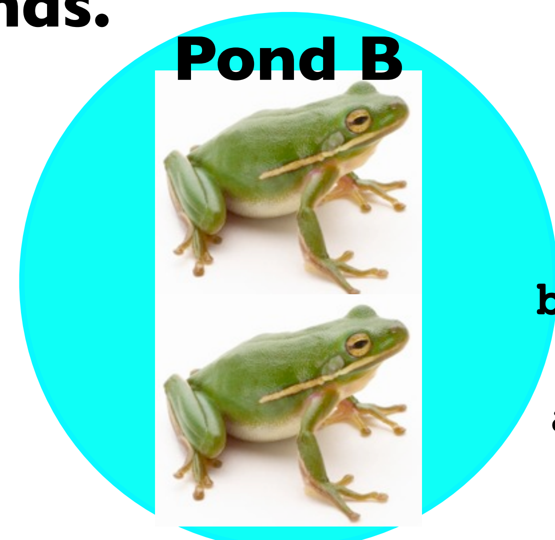
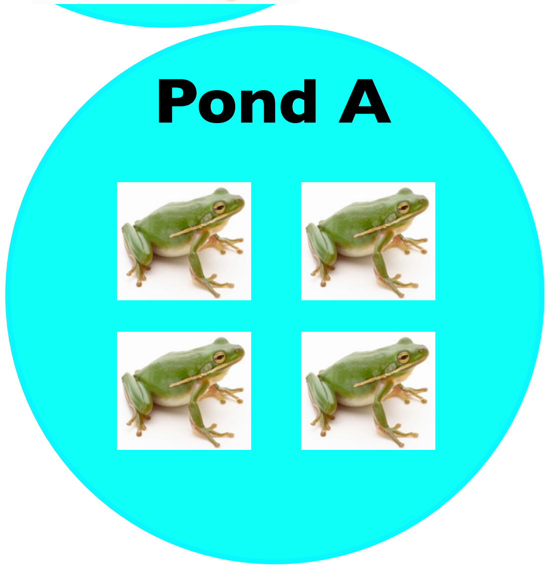
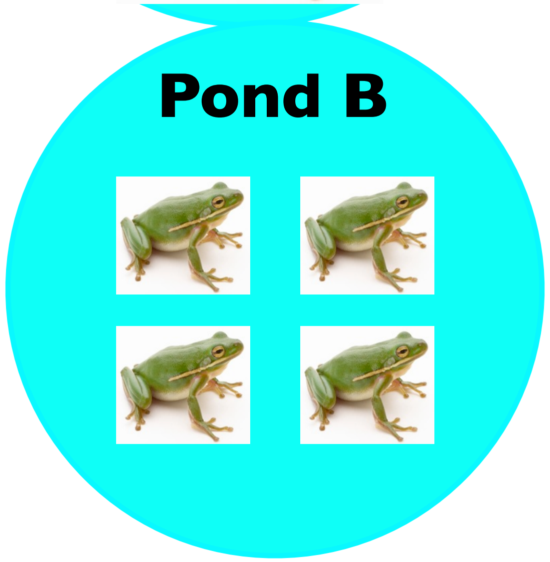




The "sum of frogs" remains the same because the reproductions can be "broken" apart and added back together.

Year 2





A nonlinear system: The Logistic Map

$$\boxed{x_{t+1} = Rx_t(1-x_t)}$$

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Combo of birth/death rate

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Size of next generation's population

Size of current generation's population

What happens if we change the value of R (and x_0)?

The Logistic Map: Experiment 1

$$\boxed{x_{t+1} = Rx_t(1-x_t)}$$

Start with $R=2$ and $x_0=.5$

–Make a plot of $x(t)$ vs. t

Repeat for $R=2$ and $x_0=.2$

–Make a plot of $x(t)$ vs. t

Repeat for $R=2$ and $x_0=.99$

–Make a plot of $x(t)$ vs. t

What do you notice?

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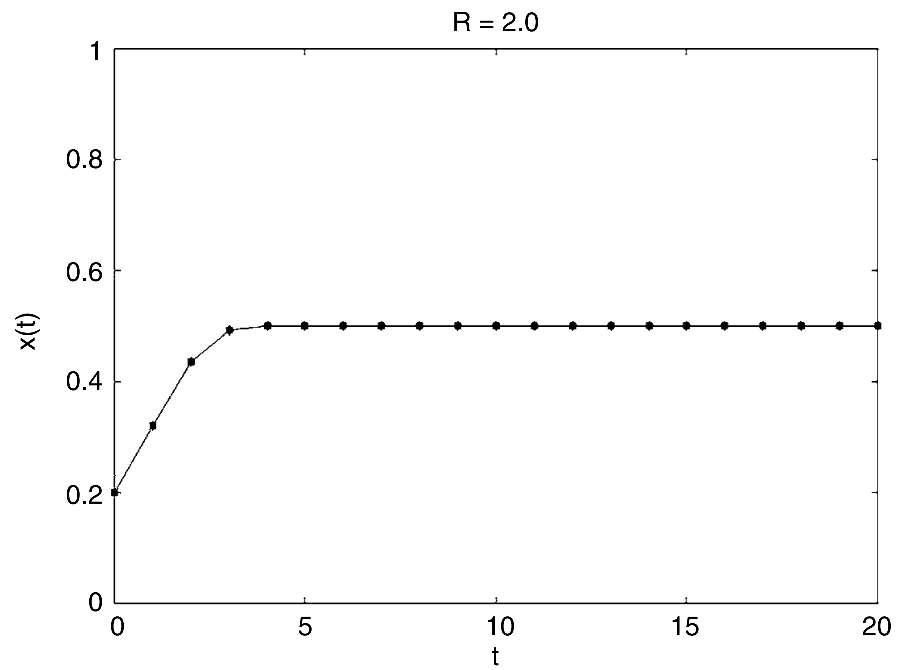

$x(t)=.5$ is called a *fixed point*

Time to get there depends on x_0

Value is determined by R

What do you notice?

The Logistic Map: Experiment 2

$$\boxed{x_{t+1} = Rx_t(1-x_t)}$$

Start with $R=3.1$ **and** $x_0=.5$

–Make a plot of $x(t)$ *vs.* t

Repeat for $R=3.1$ **and** $x_0=.2$

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What do you notice?

Is there a fixed point?

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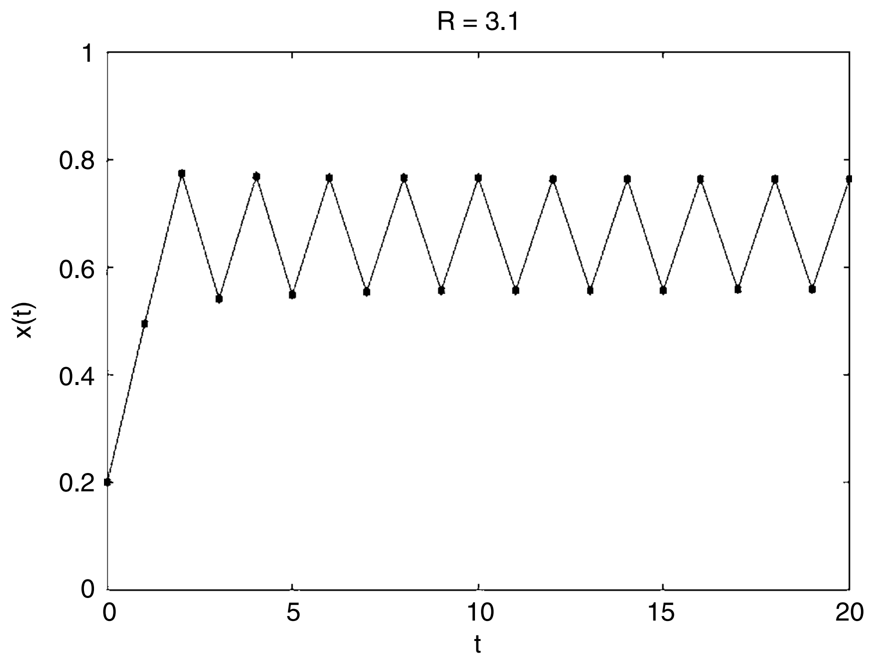

What do you notice?

Is there a fixed point?

This is an *attractor* (the f.p. was, too)

- **In this case, it is a *period-2* attractor because it oscillates between 2 values**
- **Time to "settle into" attractor depends on x_0**
- **Values and period of attractor depend on R**

The Logistic Map: Experiment 3

$$\boxed{x_{t+1} = Rx_t(1-x_t)}$$

Start with $R=3.49$ and $x_0=.5$

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Repeat for $R=3.49$ and $x_0=.2$

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What do you notice?

How many fixed points are there?

What is the period?

The Logistic Map: Experiment 3

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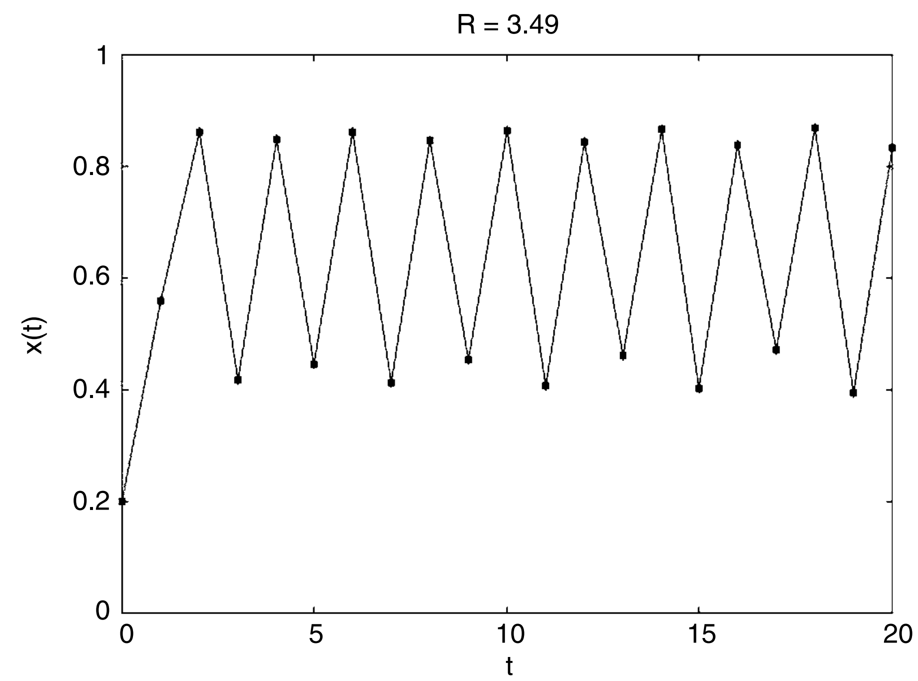


Period-4 attractor

The map has undergone a *period doubling*

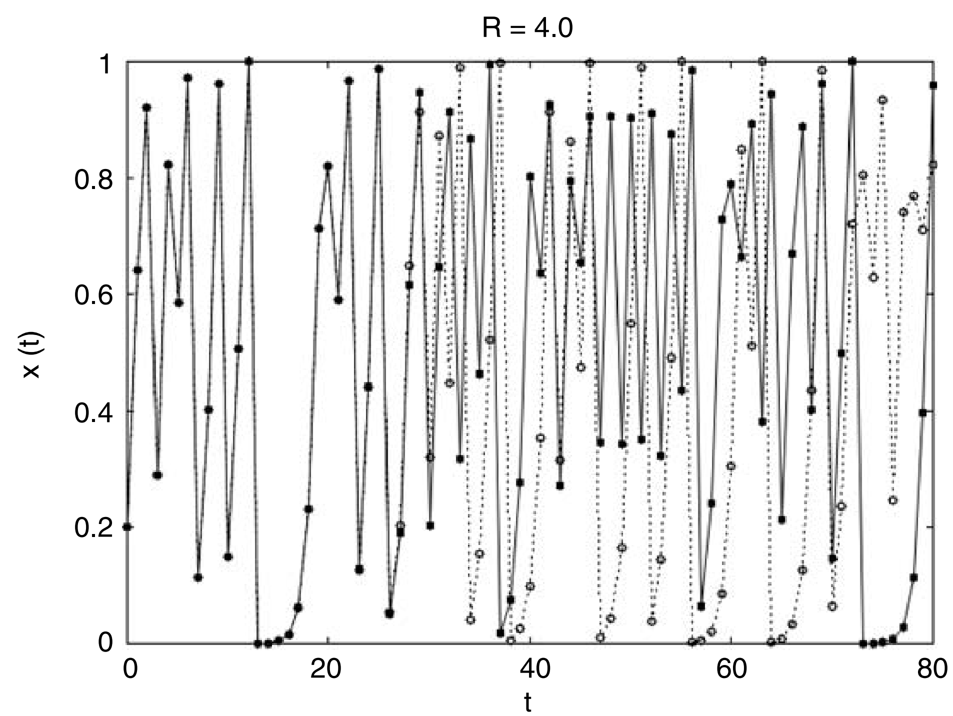
What do you notice?

How many fixed points are there?

What is the period?

The path to chaos...

For smaller and smaller changes in *R*, period doublings keep happening until...



R=3.569946 (or thereabouts!)

Period becomes *infinite*-- i.e. CHAOS

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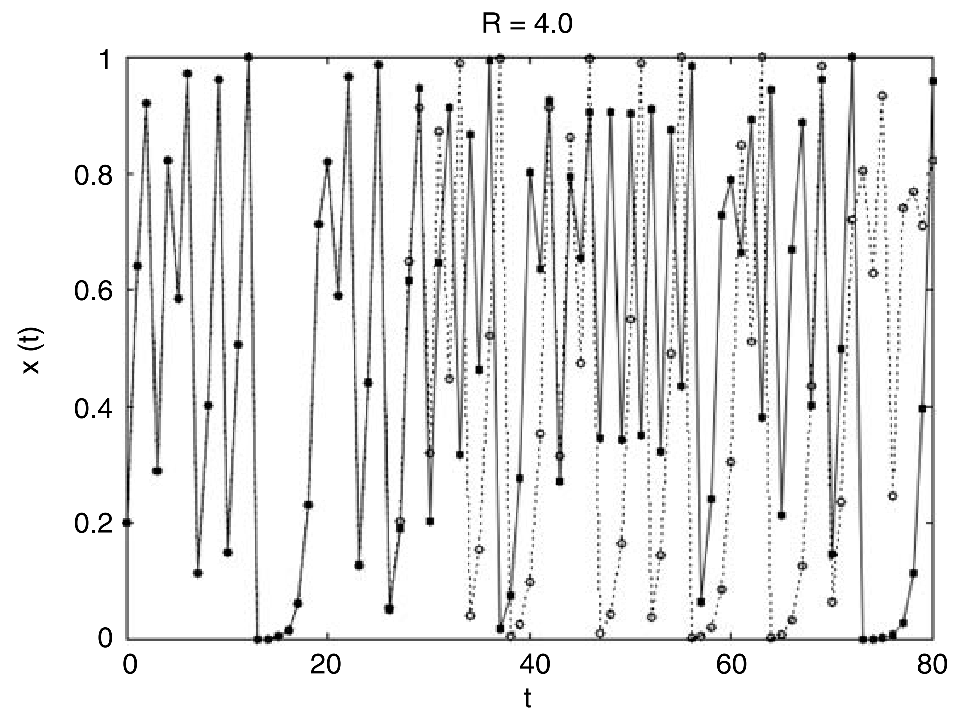

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Implications:

- **Before infinite period, behaviours of logistic map are roughly predictable; after they are NOT**
- **For any *R* in chaotic region, any two values of x_0 *arbitrarily close* will yield trajectories that DIVERGE!**

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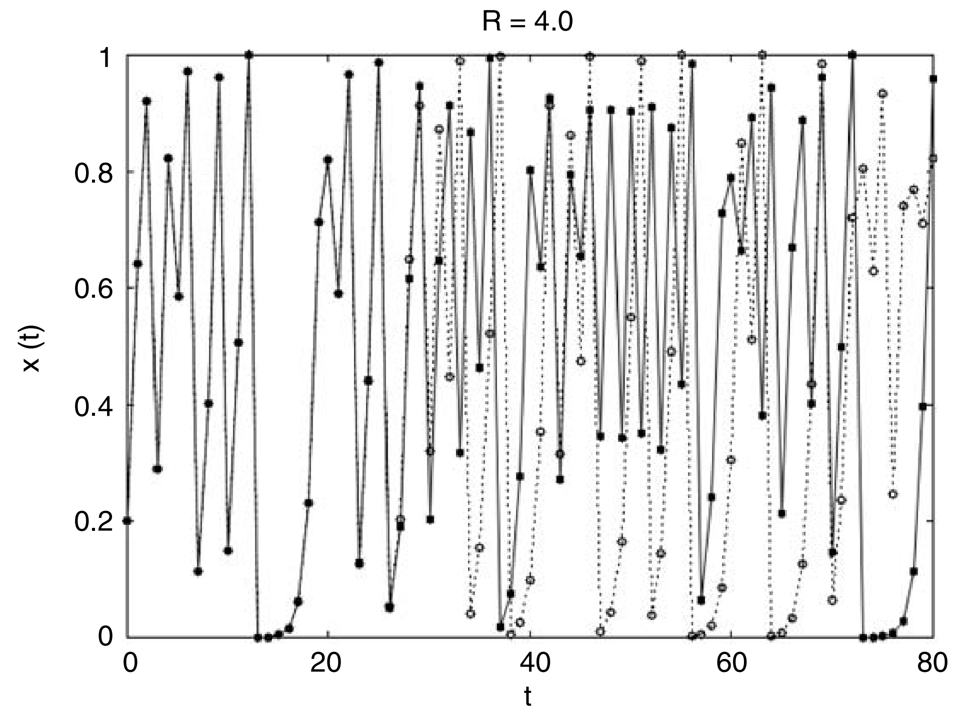

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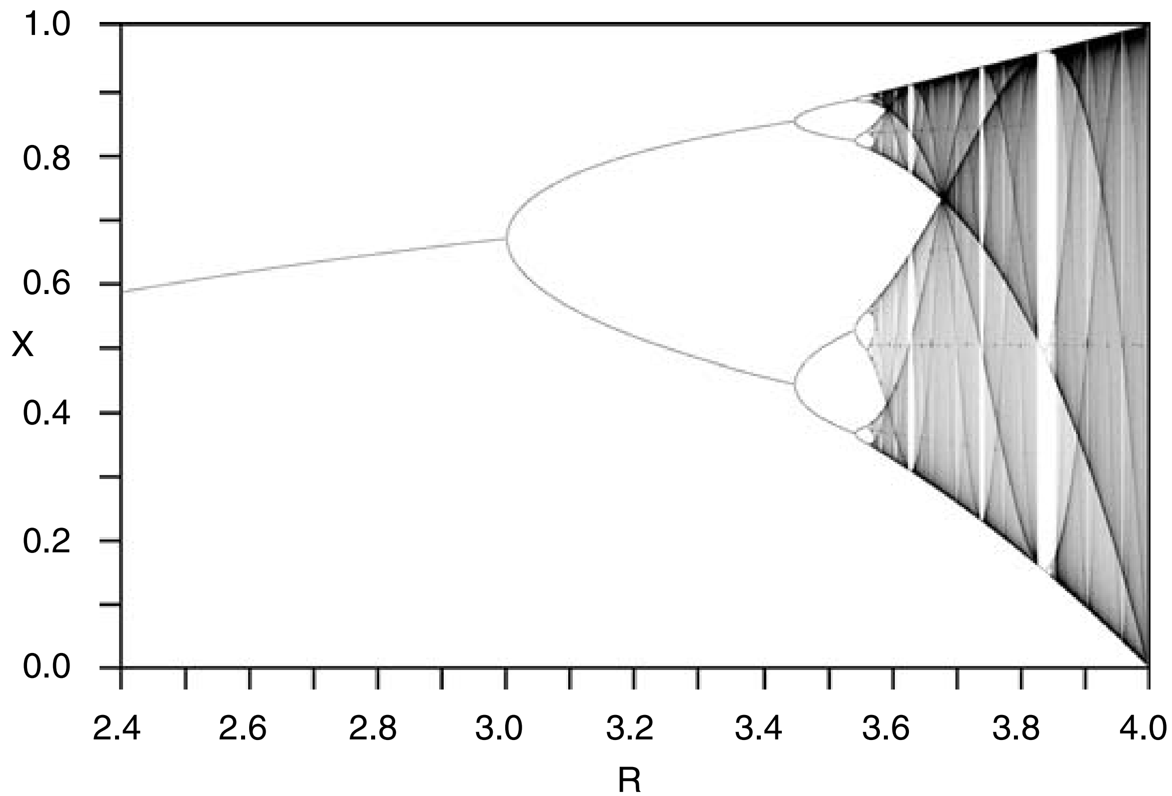
Apparent randomness can arise from VERY simple deterministic systems!

Should we abandon all hope?!

Surprisingly, there is substantial *mathematical* order in chaos!



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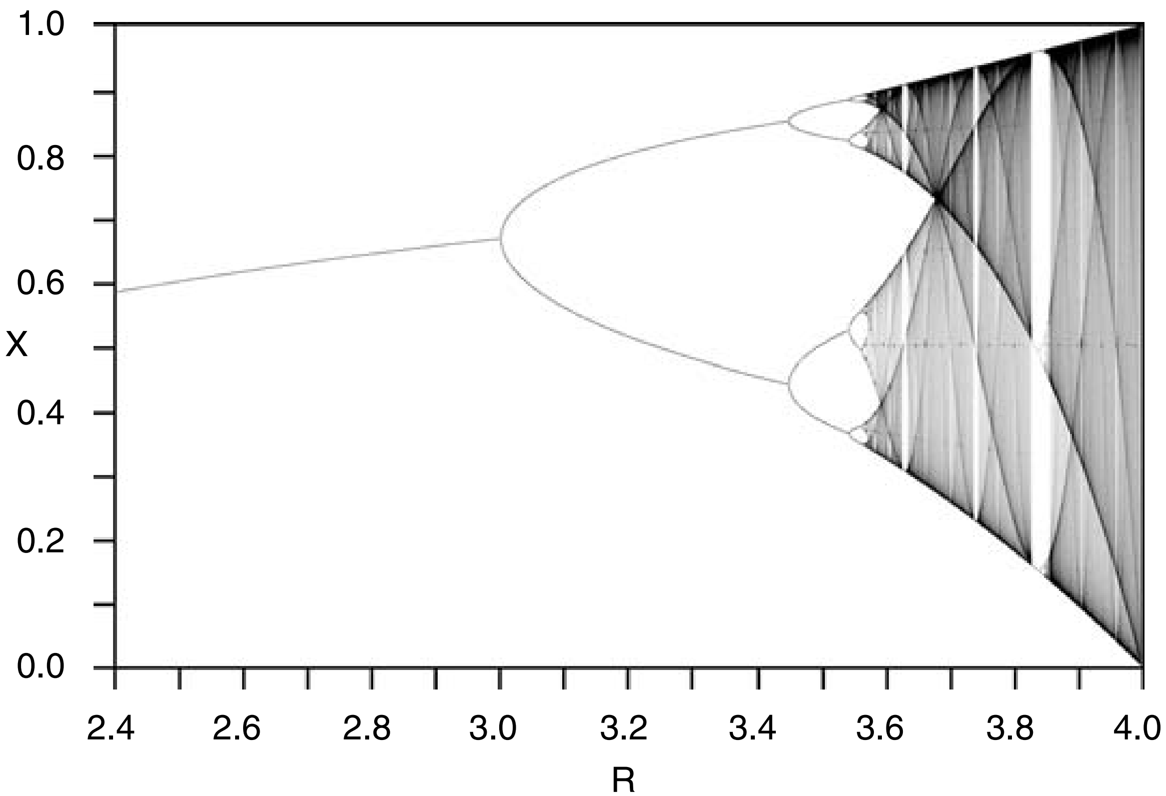
Universal Features



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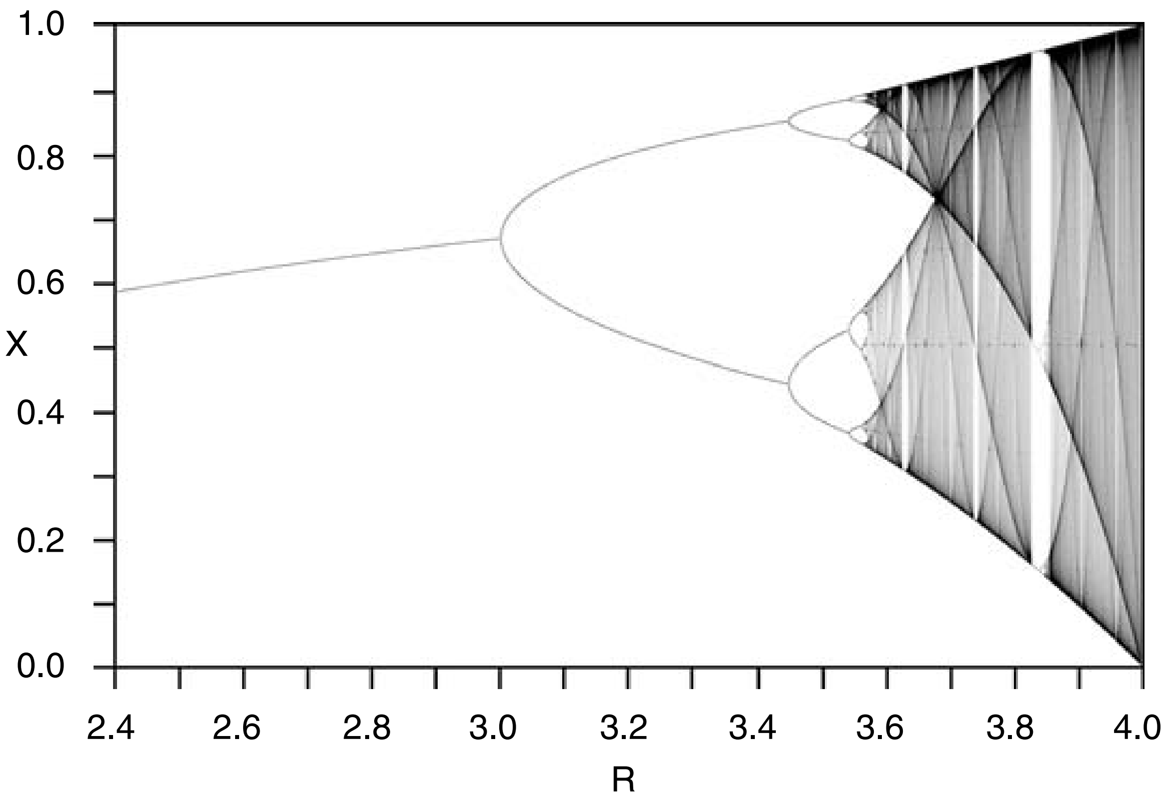

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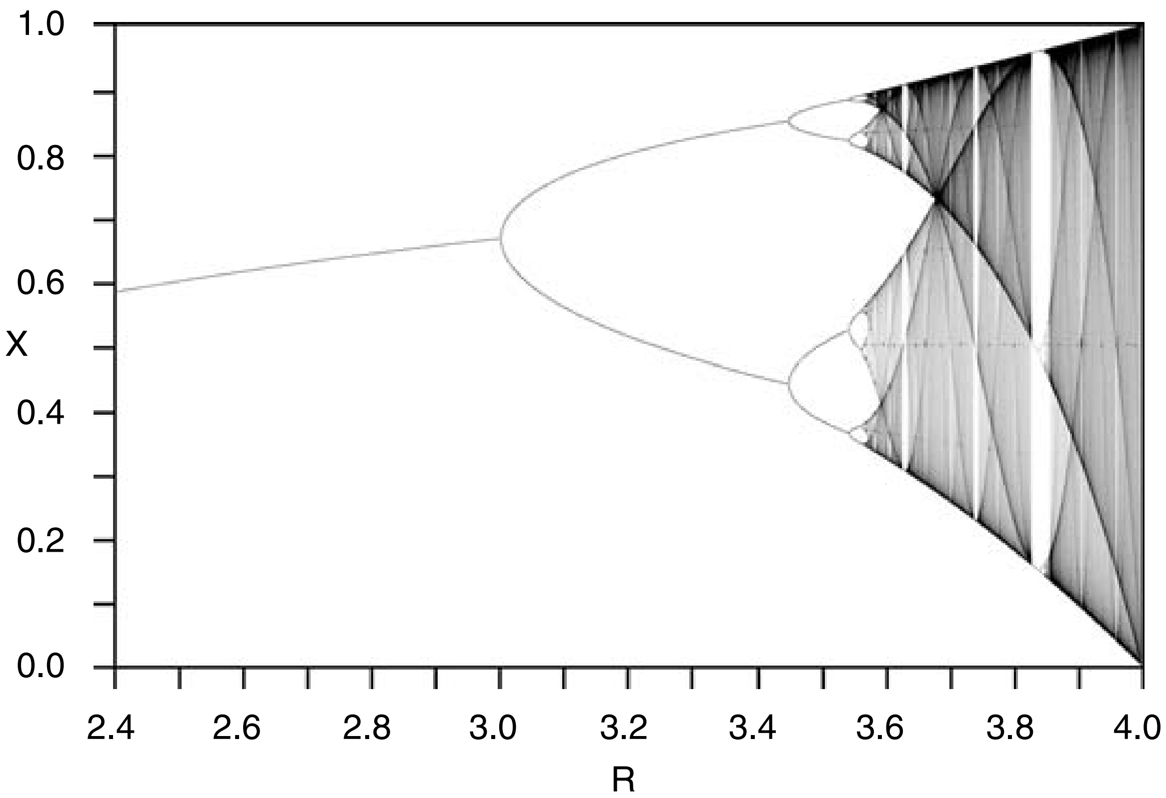

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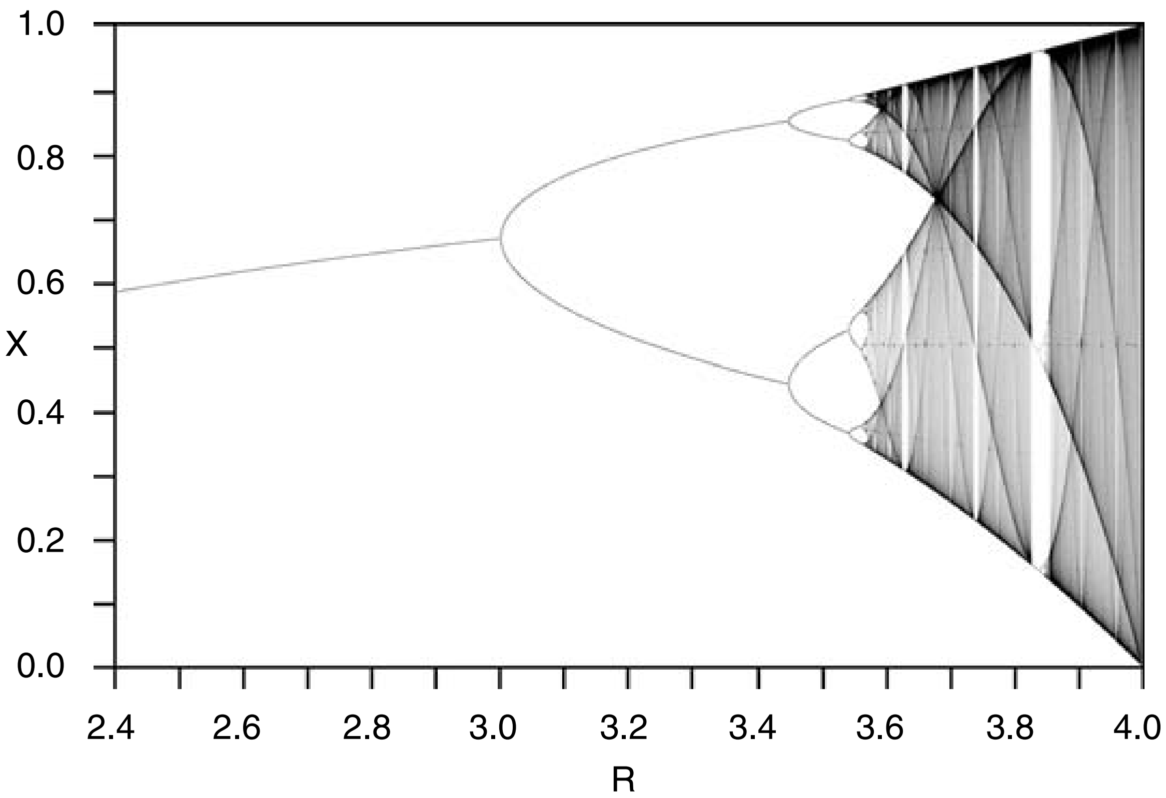

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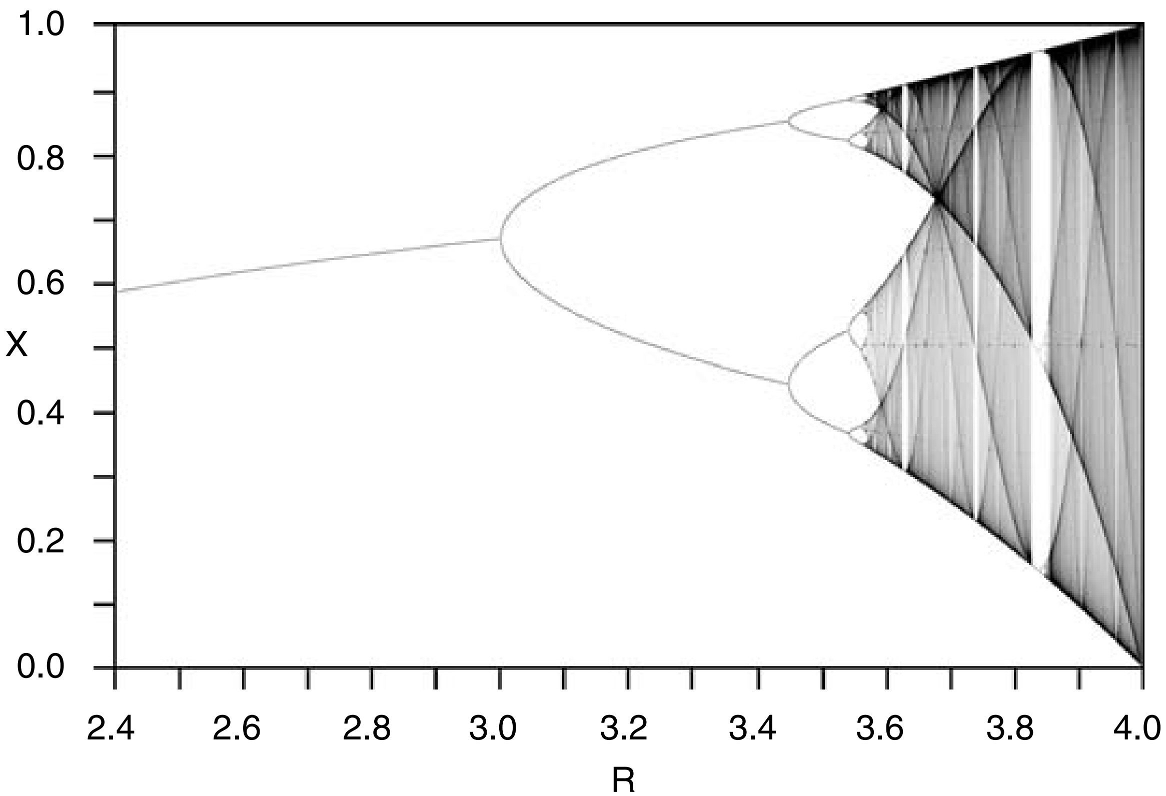

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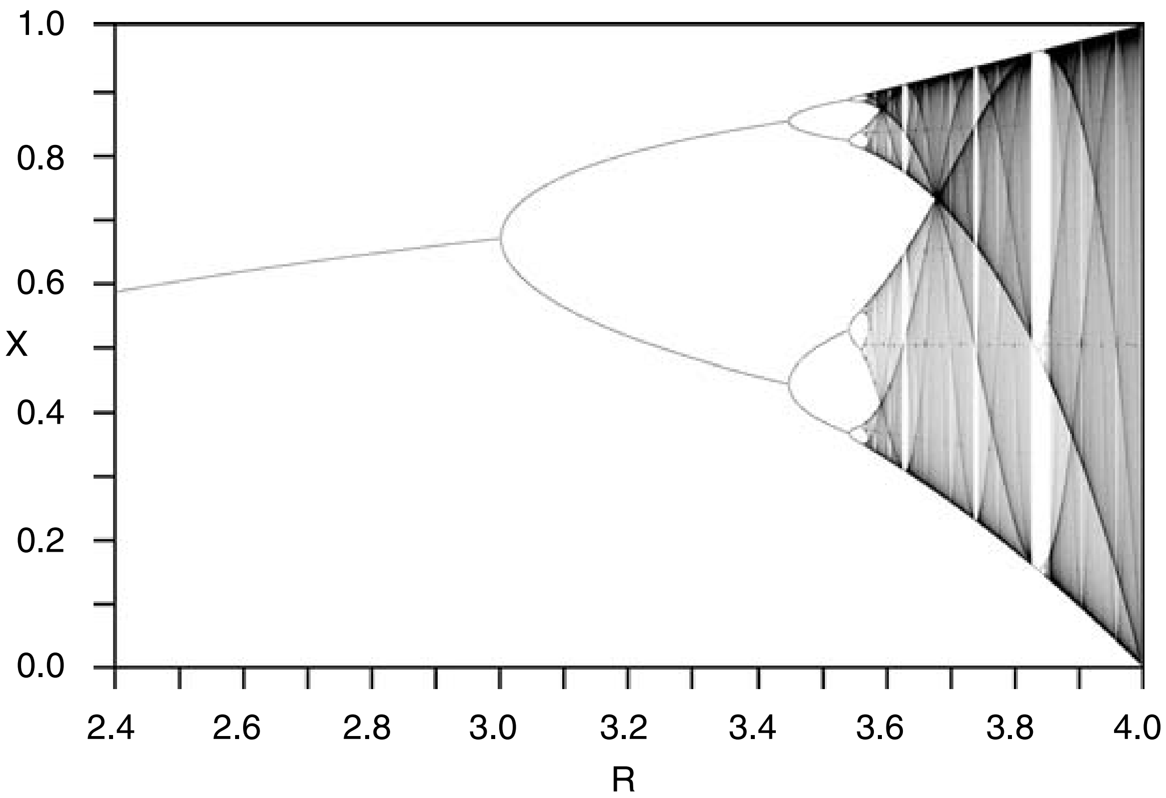

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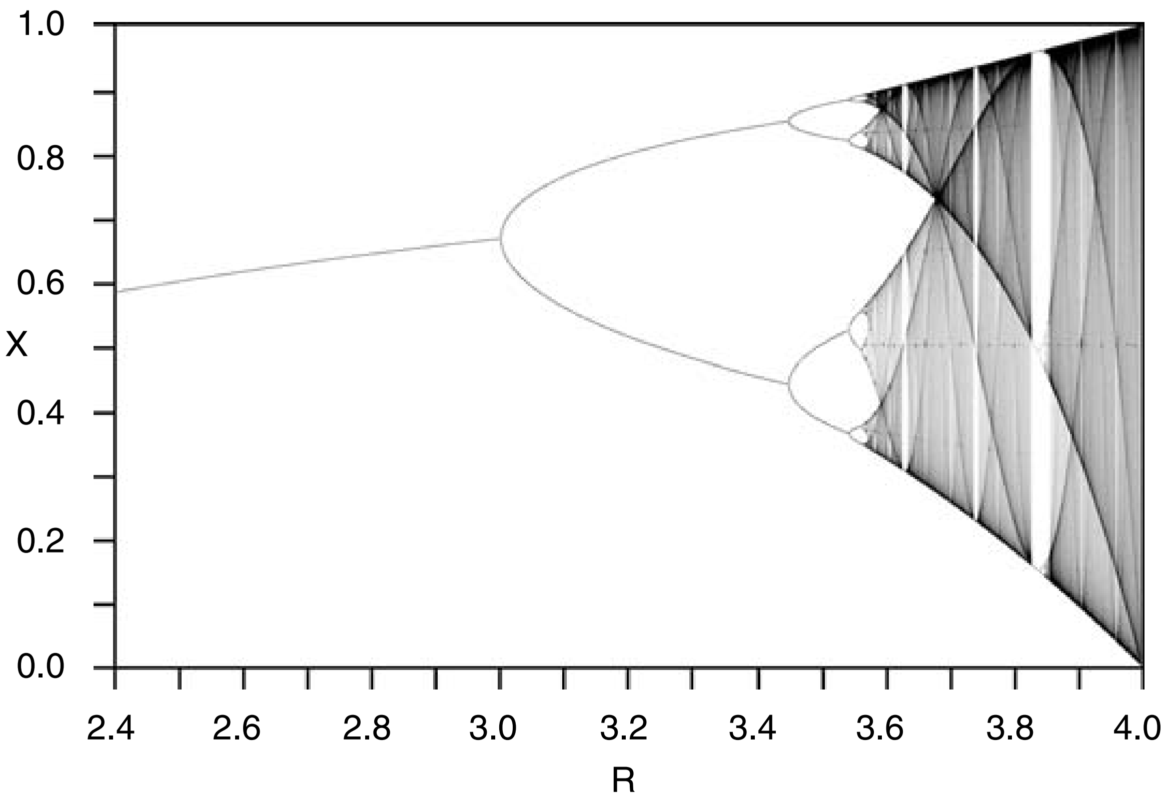

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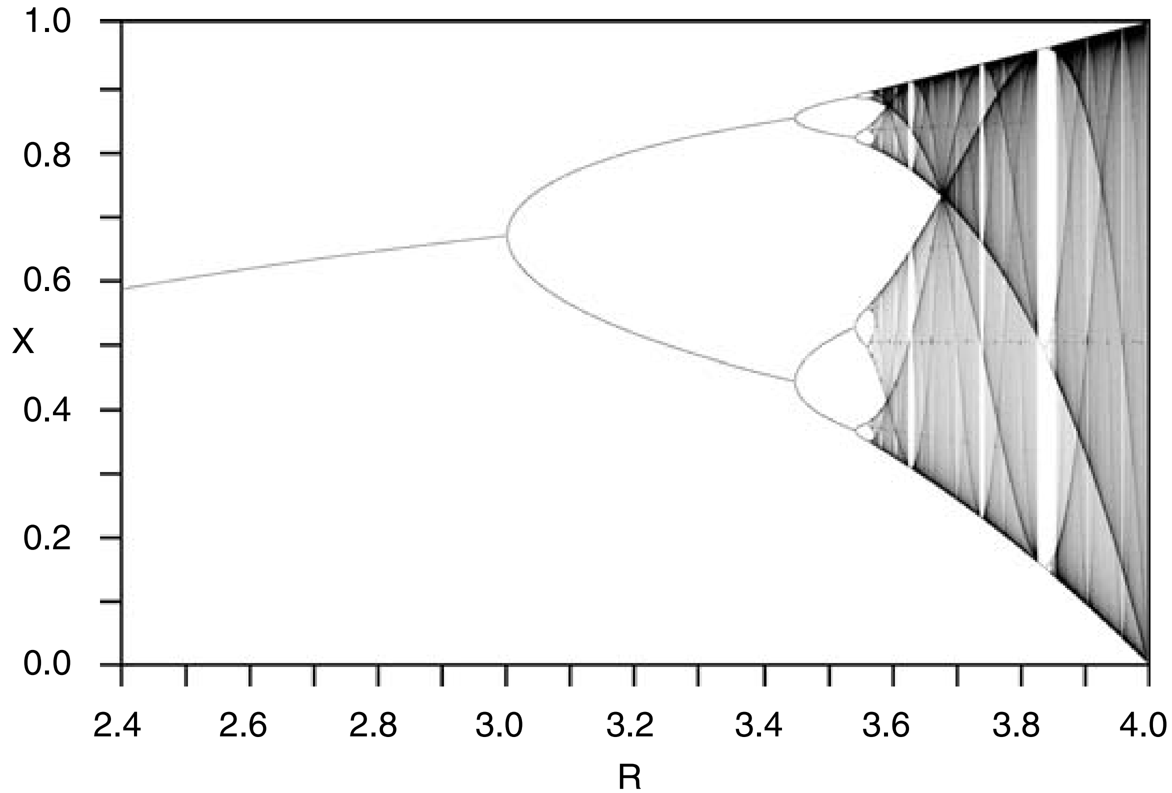
RENORMALIZATION

Scaling and Fractality

Look closely at bifurcation diagram. As you move from left (large) to right (small), what do you notice?



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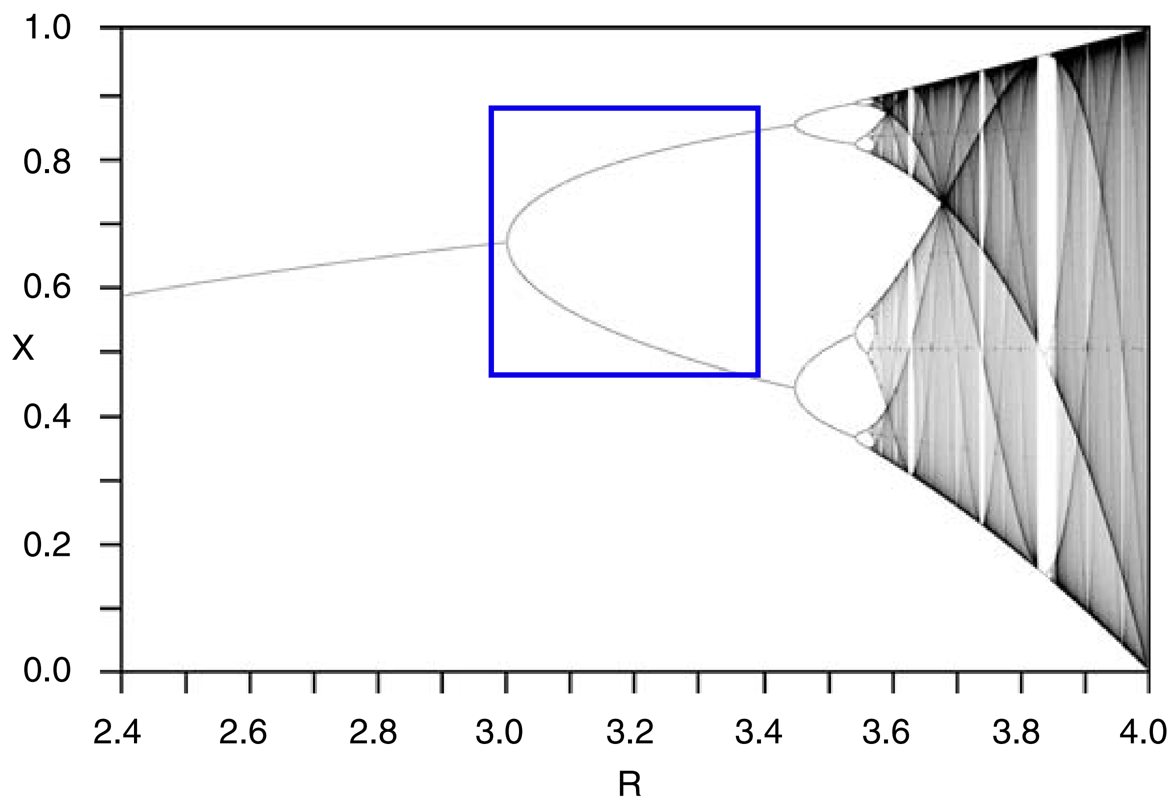

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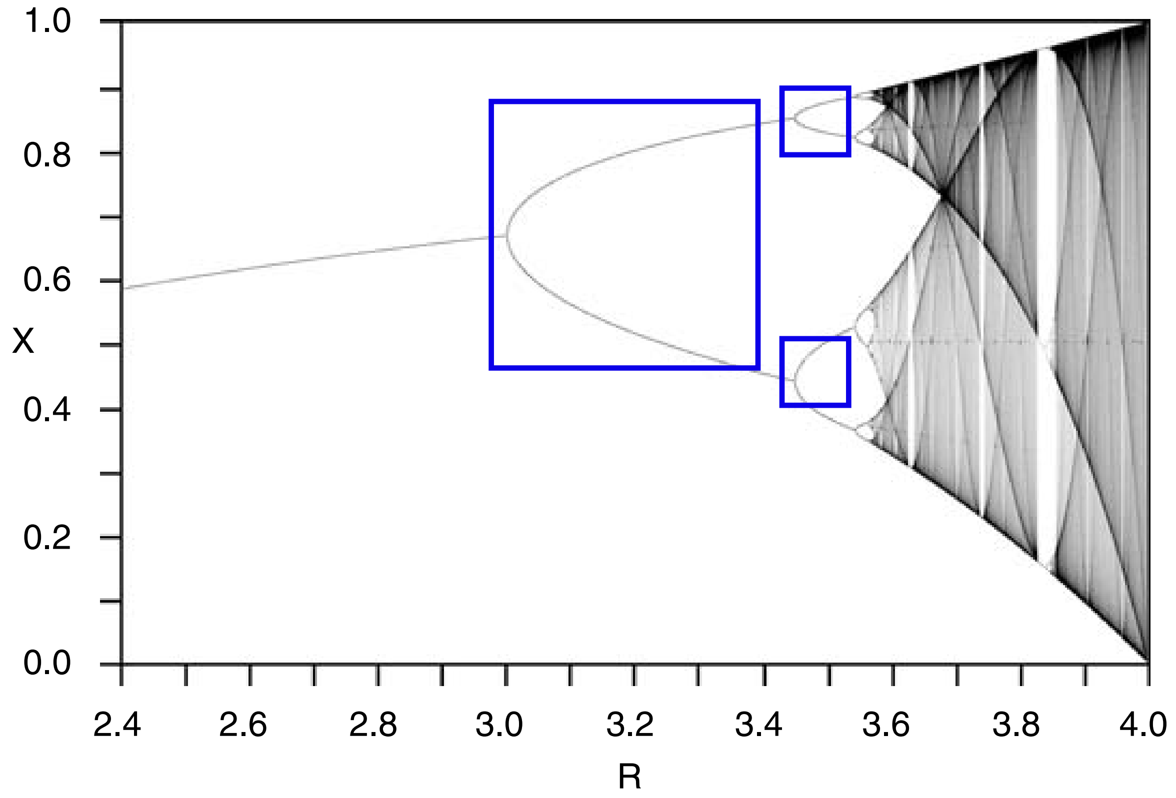

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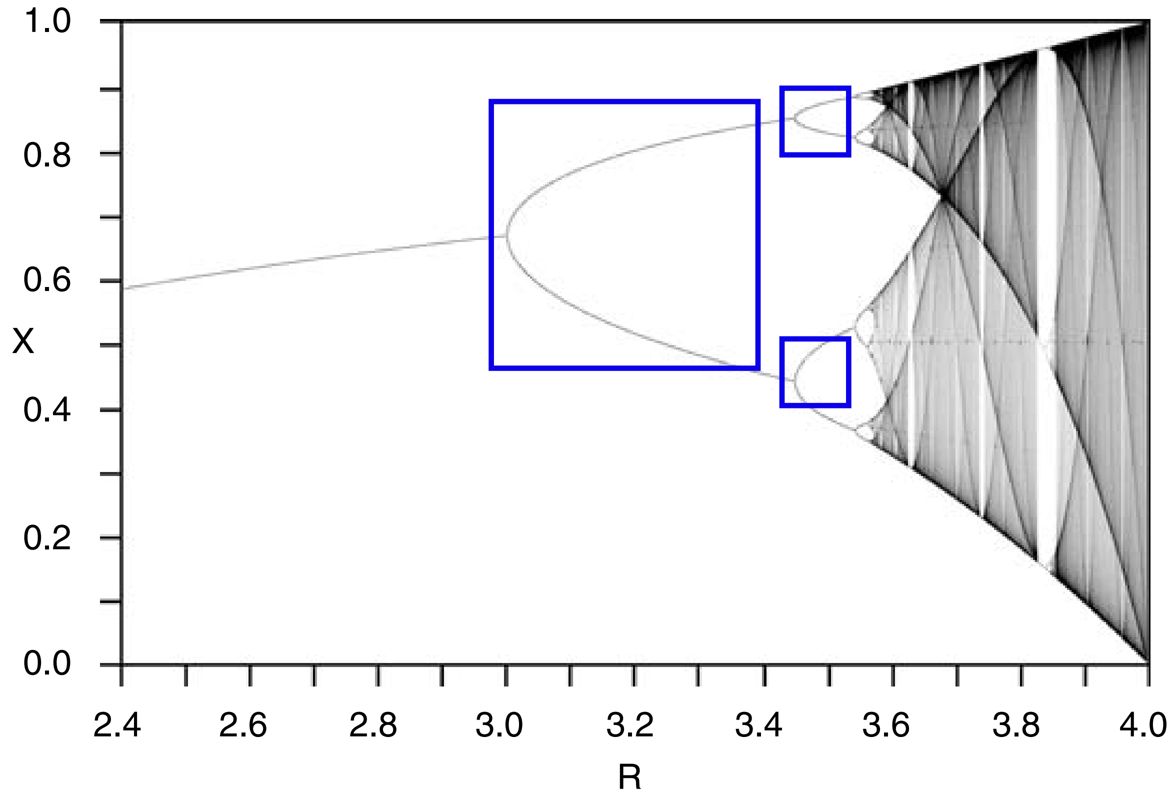

Self-similarity

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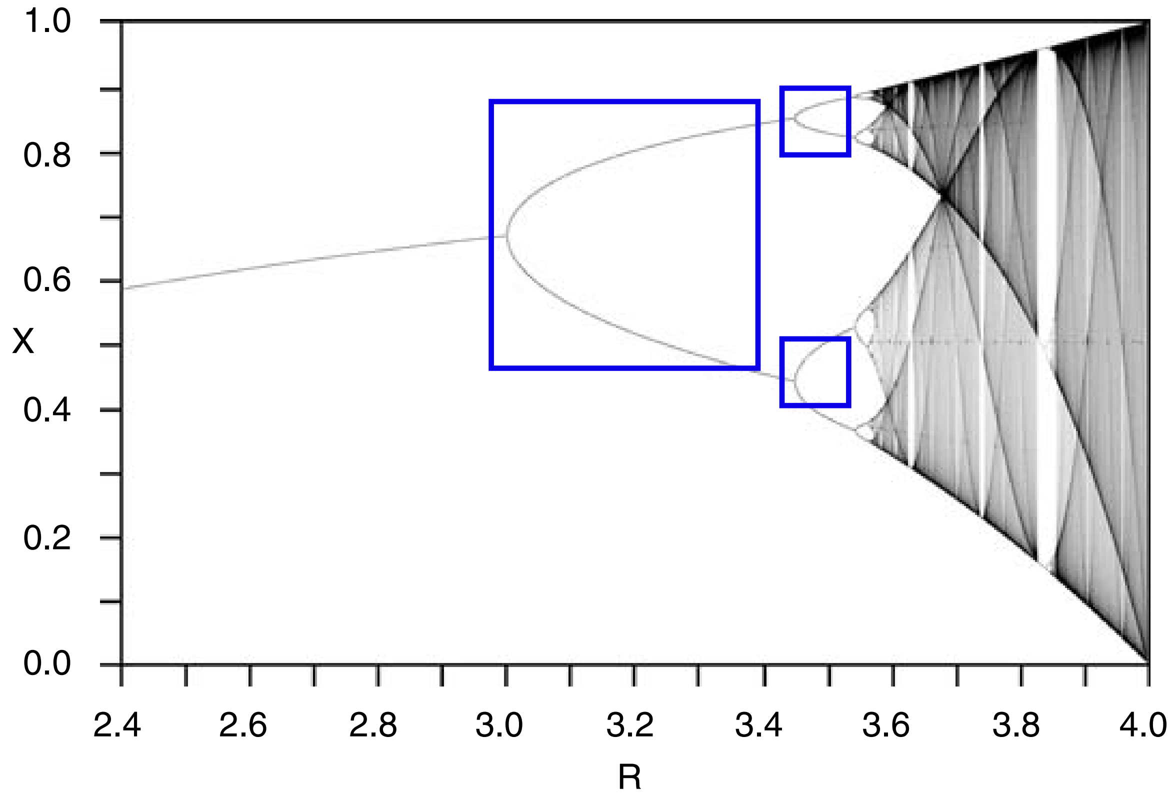

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Geometric self-similarity at different length scales

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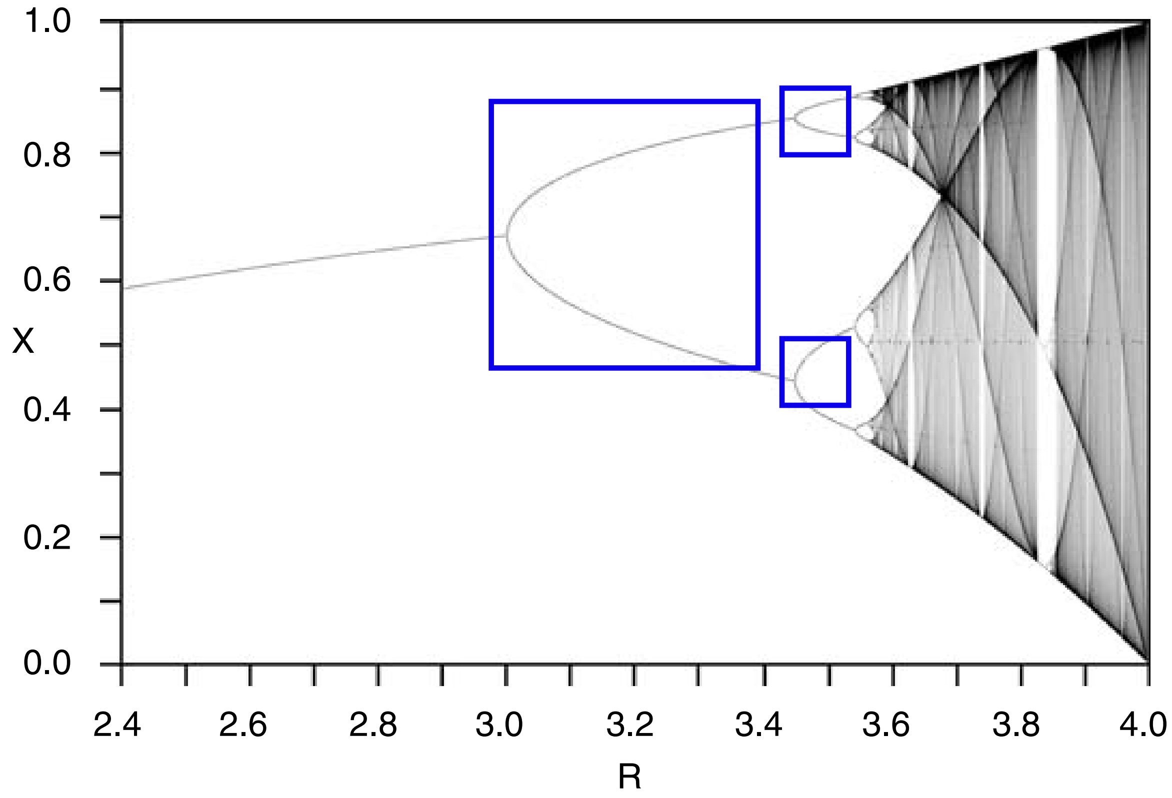

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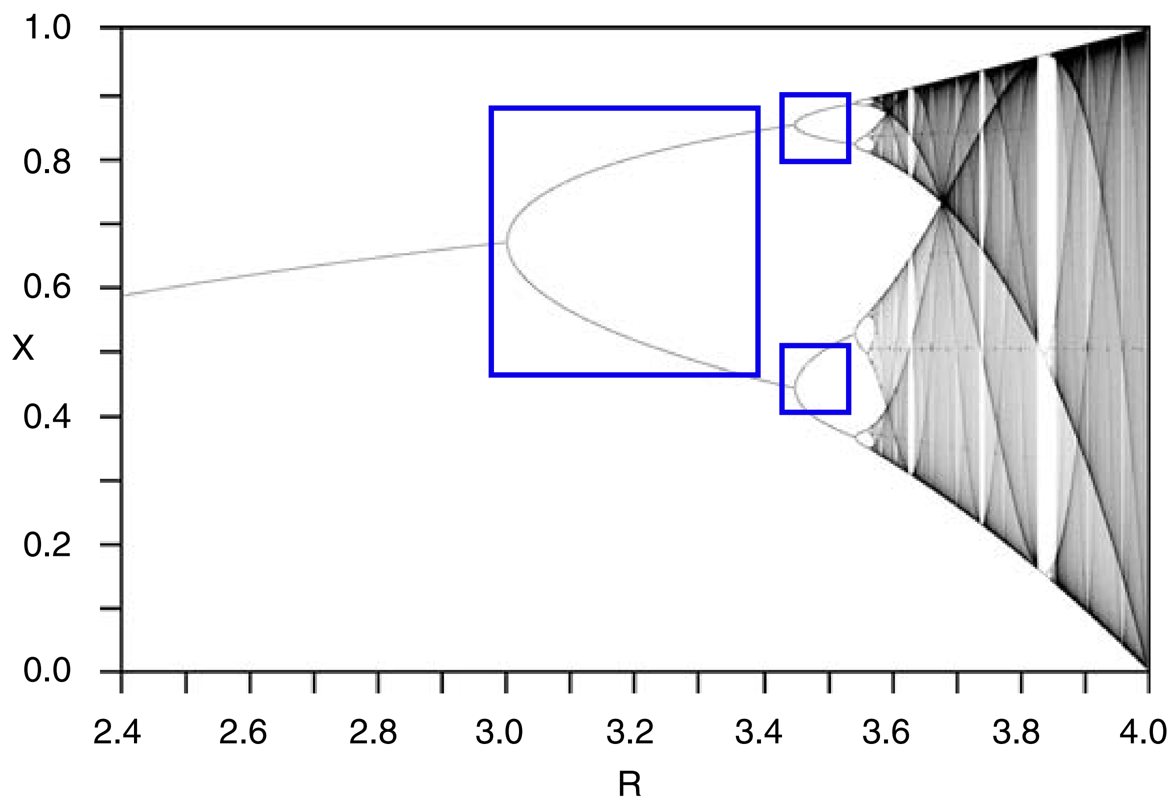

Self-similarity

Geometric self-similarity at different length scales → **Fractality**

Fractals and Dimension

What does *dimension* actually mean?

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Start with a line...

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Bisect it... ______ ______

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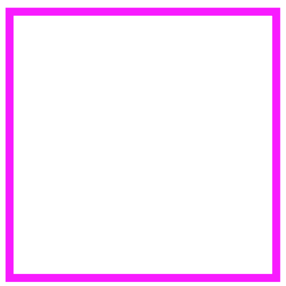

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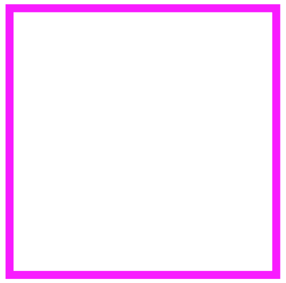
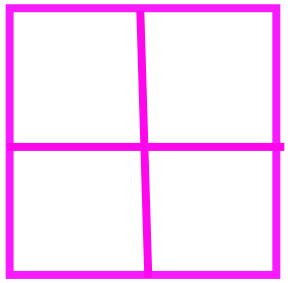


Bisect each side...



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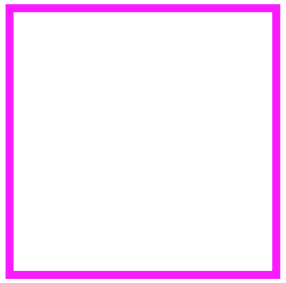

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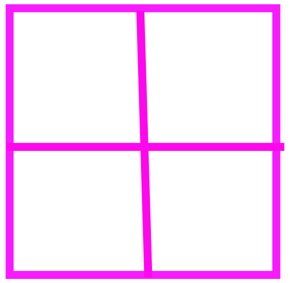

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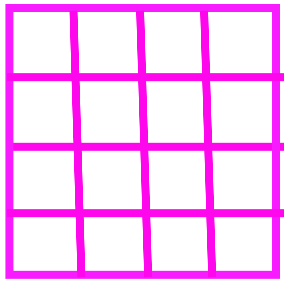

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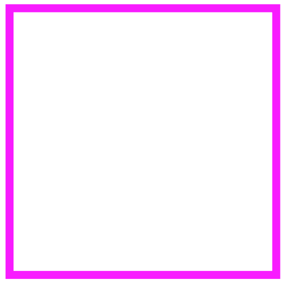

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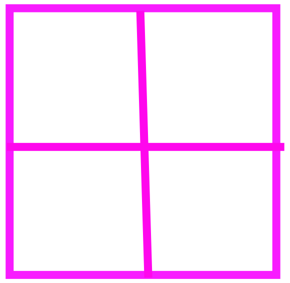

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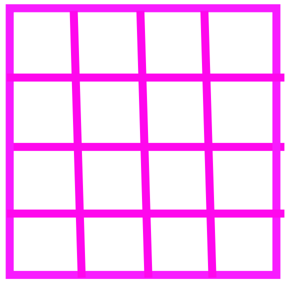

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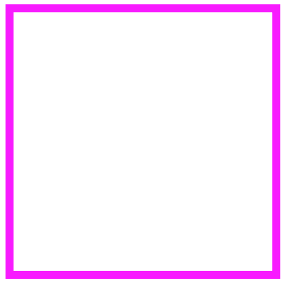

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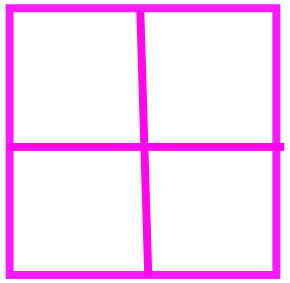

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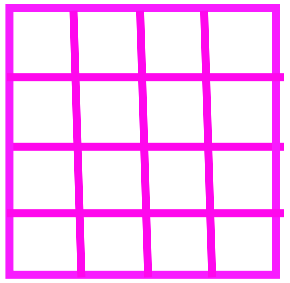

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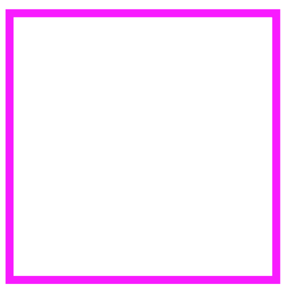

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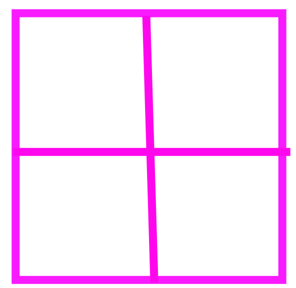

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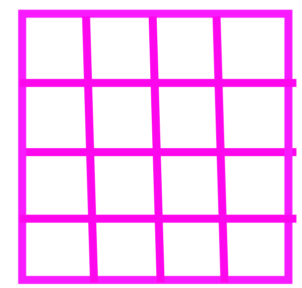

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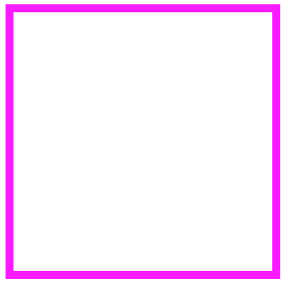

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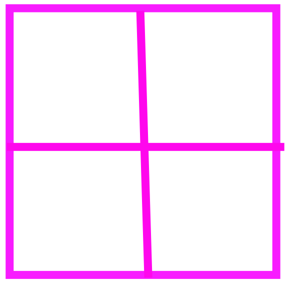

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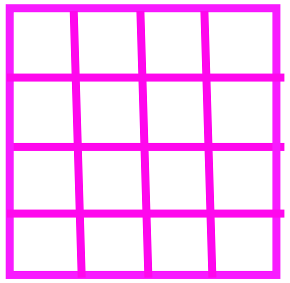

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So what's the pattern?

Each level (*scaling*) is made up of smaller copies of the previous level, where the number of copies is:

$$N=2^{dimension}$$

Or more generally, $N=x^{dimension}$ *where* x is the number of divisions of a length-scale.

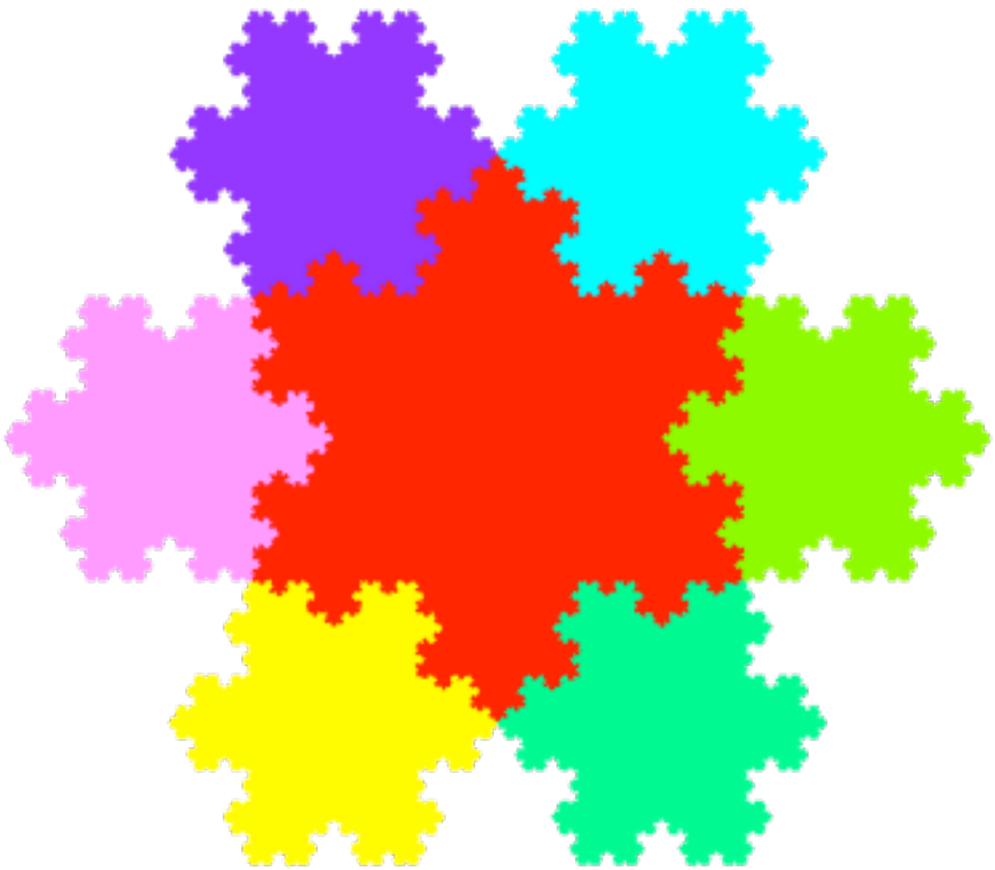
A Strange Dimension...

We weren't surprised by the dimension of *regular geometric* objects (lines, squares, cubes, etc.). What about the dimension of something a little less "regular"?

The Koch Snowflake

The Koch curve can be constructed by starting with an equilateral triangle, then recursively altering each line segment as follows:

1. divide the line segment into three segments of equal length.
2. draw an equilateral triangle that has the middle segment from step 1 as its base and points outward.
3. remove the line segment that is the base of the triangle from step 2



Calculating the Dimension of the Koch Curve

1) Draw a few iterations (3 or 4) of the Koch curve

2) How many times smaller are the line segments of the current level than were the line segments of the previous level? (How many divisions were made for each line?)

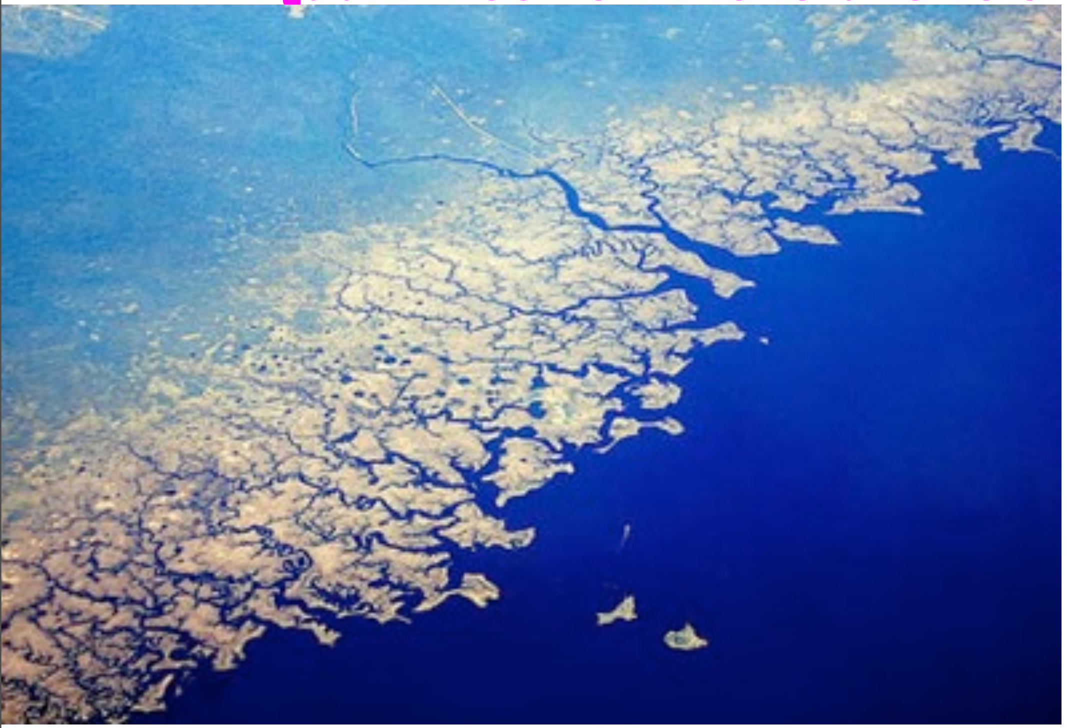
3) How many "copies" of the previous level does the current level contain?

4) Using the relation $N=x^{dimension}$, calculate the approximate dimension of the Koch curve. What do you notice?!

Fractal Dimension and Complexity

The "strange" results you found for the Koch curve are an example of *fractal dimension*.

Quantifies how the total size of an object will change as magnification changes



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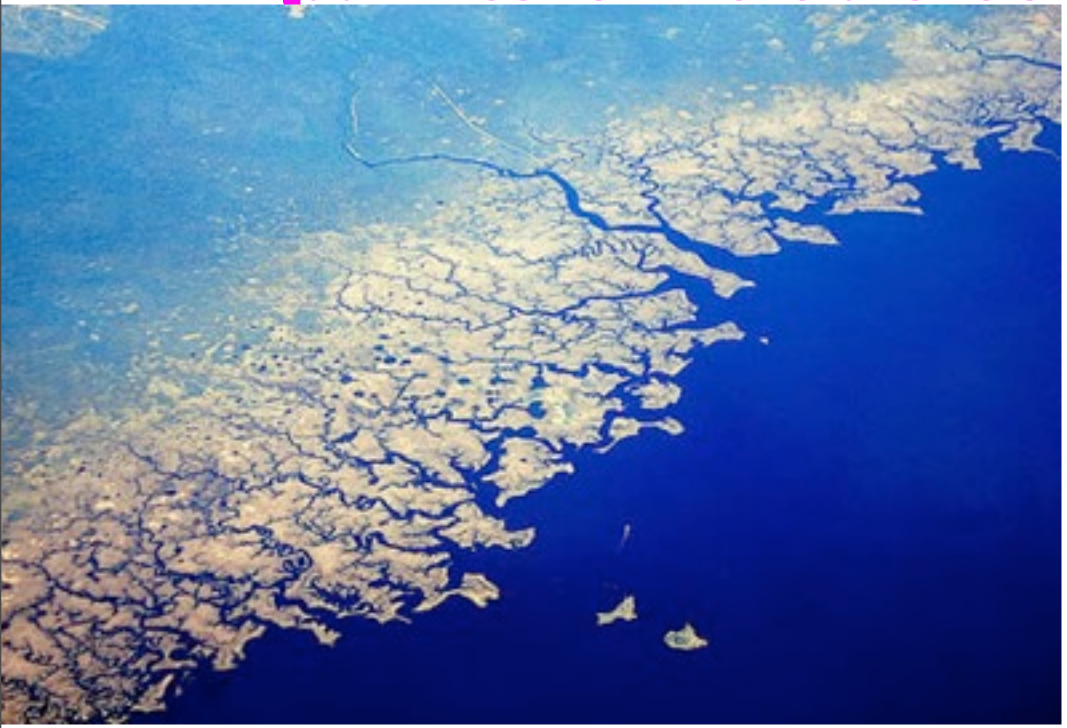

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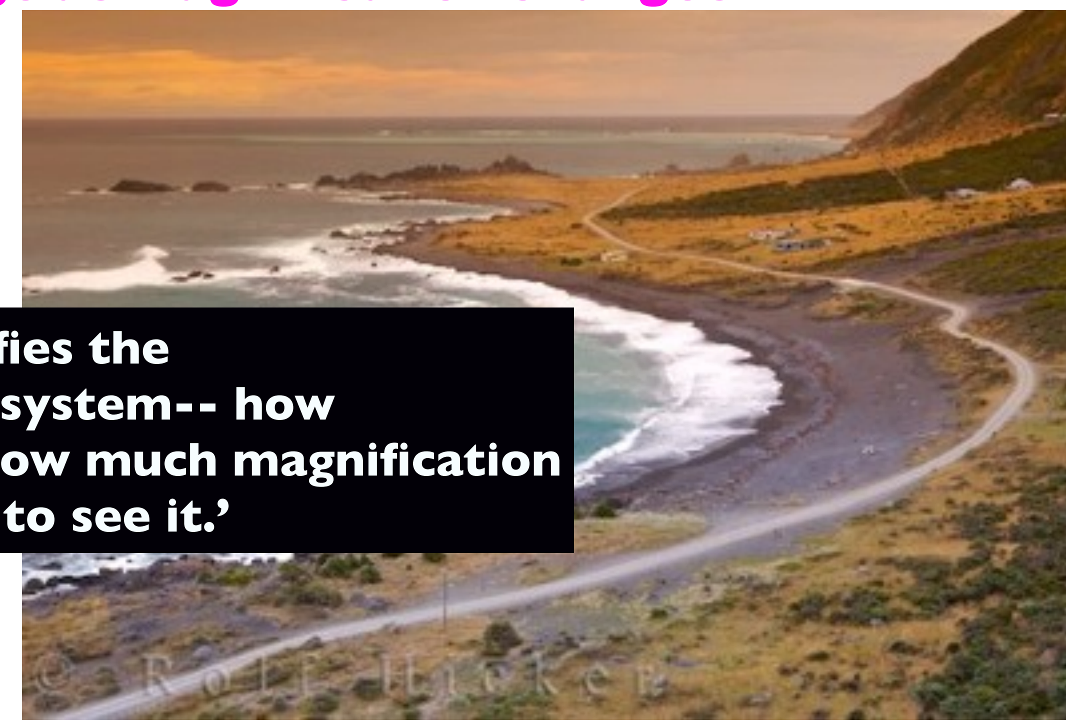

‘Fractal dimension quantifies the “cascade of detail” in a complex system-- how interesting that detail is as a function of how much magnification you have to do at each level to see it.’

Summing up...

- **Nonlinear dynamics are a manifestation of self-organization and lead to emergent properties/behaviors in complex systems**

- **Study/classify systems based on types of universal behaviours/scalings they show**

- **One very useful measure of complexity is fractal dimension, which shows (roughly) how detail *scales* with magnification**