aJ

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ENCH 501: Transport Phenomena Quiz #8 December 7, 2010

Time Allowed: 50 mins. Name:

In oil production, water is often produced with the oil. The liquids need to be separated, with the water disposed off locally while the oil is pumped to storage or refining facilities.

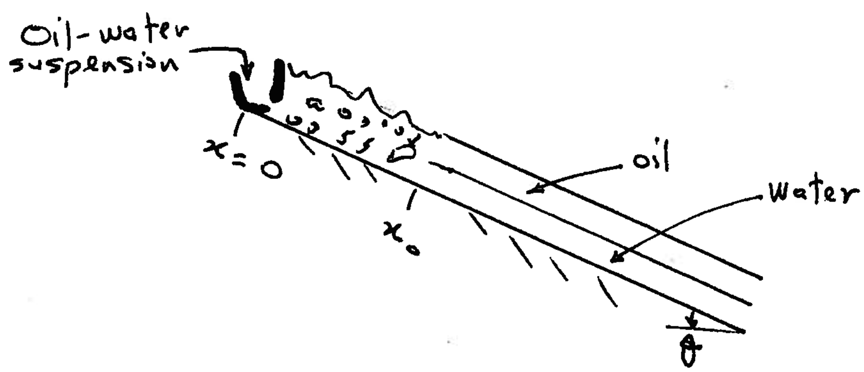
A 4 m wide wall is inclined to the horizontal at 5°. A suspension of light crude oil and water from a nearby field is distributed through a perforated trough at the top (see sketch) and the water and oil are separated into two layers as they flow down. At a distance of 1m from the trough, the liquids are considered fully separated and the oil rides on the water. Thereafter, the thicknesses of the two layers are considered constant, the oil layer is 1.2 cm deep and the water layer is 1mm deep. The flow streams are laminar. The properties of the oil and water are given in the table below. Assume the air above the oil is hardly moving.

Estimate
a) the velocity at the oil-water interface, and
b) the ratio of the oil-to-water volumetric rates.

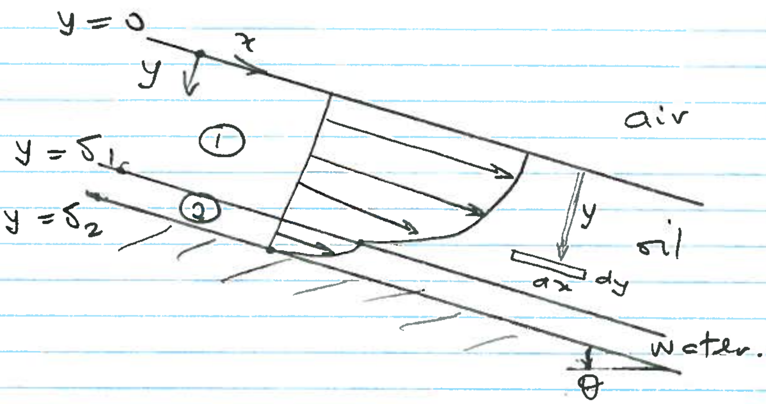
State your assumptions and show all important steps.

Data: Properties of the liquids:
Oil - $\rho = 890\ kg/m^3$; $\mu = 1.6$ mPa s ; Water - $\rho = 1000\ kg/m^3$, $\mu = 0.92$ mPa s

Hint: A force balance equation is needed for each layer. There are two boundary conditions at the oil-water interface. Use the b.c. that the oil and water velocities are the same at the interface with the equation for the oil, and the b.c. that the shear stress will be the same for both liquids at the interface with the equation for the water.



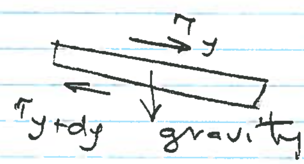
ENCH 501 Quiz #8 Solution Dec. 7, 2010



Locate origin at air-oil interface as shown in diagram. i.e. $y=0$ at free surface

Let oil ≡ 1, water ≡ 2

Consider a differential element dx-dy as shown. Do a force balance on the element, unit width



$$\tau\,dx\big|_y - (\tau\,dx)\big|_{y+dy} + dx\,dy\,\rho g \sin\theta = 0$$

or

$$-\frac{d\tau}{dy}\,dx\,dy + dx\,dy\,\rho g \sin\theta = 0$$

or

$$\frac{d\tau}{dy} = \rho g \sin\theta$$

For a Newtonian fluid, $\tau = -\mu \frac{du}{dy}$

$$\therefore \quad \frac{d^2u}{dy^2} = \frac{-\rho g \sin\theta}{\mu} \quad (1)$$

is the momentum equation — valid for both layers.

Layer 1 boundary conditions

(2) $y = 0$ $\quad \frac{du}{dy} = 0$ (no shear, free surface)

(3) $y = \delta_1$ $\quad u_1 = u_2$ (2 liquids have same velocity)

Layer 2 boundary conditions

(4) $y = \delta_1$, $\mu \frac{du}{dy}\Big|_1 = \mu \frac{du}{dy}\Big|_2$ (same shear)

or $\frac{du}{dy}\Big|_2 = \frac{\mu_1}{\mu_2} \frac{du}{dy}\Big|_1$

and $y = \delta_2$, $u = 0$ (no slip) (5)

□ Let the velocity profile in the oil $= u_1(y)$

The force balance equation is:

$$\frac{d^2u_1}{dy^2} = \beta_1 \quad ; \quad \beta_1 = \frac{-\rho_1 g \sin\beta}{\mu_1}$$

integrate

$$\frac{du_1}{dy} = \beta_1 y + C_1 \qquad (6)$$

Use eq. (2) — $y = 0$, $\frac{du_1}{dy} = 0$ $\Rightarrow$ $C_1 = 0$

Integrate again

$$u_1 = \frac{\beta_1}{2} y^2 + C_2 \qquad (7)$$

use eq. (3), $y = \delta_1$, $u_1 = u_2(\delta_1) = u_{2b}$ (vel. at boundary)

This is not yet known.

$$C_2 = u_{2b} - \frac{\beta_1}{2}\delta_1^2$$

Substitute into (7)

$$u_1(y) = u_{2b} + \frac{\beta_1}{2}(y^2 - \delta_1^2) \qquad (8)$$

$\longrightarrow$

☐ Let the velocity profile in the water = $u_2(y)$

The force balance equation is:

$$\frac{d^2u_2}{dy^2} = \beta_2 \; ; \quad \beta_2 = -\frac{\rho_2 g \sin\beta}{\mu_2}$$

Integrate once

use eq. (4)

$$\text{(8)} \quad \frac{du_2}{dy} = \beta_2 y + c_3 \; ; \quad y = \delta_1 , \; \frac{du_2}{dy} = \frac{\mu_1}{\mu_2}\frac{du_1}{dy}$$

since from eq. (6) $\left.\frac{du_1}{dy}\right|_{\delta_1} = \beta_1 \delta_1$

$$\left.\frac{du_2}{dy}\right|_{\delta_1} = \frac{\mu_1}{\mu_2}\beta_1\delta_1$$

Substitute

$$c_3 = \frac{\mu_1}{\mu_2}\beta_1\delta_1 - \beta_2\delta_1$$

$$\therefore \quad \frac{du_2}{dy} = \beta_2 y + \underbrace{\frac{\mu_1}{\mu_2}\beta_1\delta_1 - \beta_2\delta_1}_{c_3} \qquad \text{(9)}$$

Integrate again

$$u_2 = \frac{\beta_2 y^2}{2} + c_3 y + c_4 \qquad \text{(10)}$$

use eq. (5), $y = \delta_2$, $u_2 = 0$

and $c_4 = -\frac{\beta_2}{2}\delta_2^2 - c_3\delta_2$

Hence eq. (10) becomes

$$u_2 = \frac{\beta_2 y^2}{2} + c_3(y - \delta_2) - \frac{\beta_2}{2}\delta_2^2 \qquad \text{(11)}$$

(a) Velocity at the oil-water boundary, u_{2b}

This can be obtained from eq. (11) at $y = \delta_1$

$$u_{2b} = \frac{\beta_2 \delta_1^2}{2} + \left(\frac{\mu_1}{\mu_2}\beta_1\delta_1 - \beta_2\delta_1\right)(\delta_1 - \delta_2) - \frac{\beta_2}{2}\delta_2^2$$

$$u_{2b} = \frac{\beta_2}{2}\left(\delta_1^2 - \delta_2^2\right) + \frac{\mu_1}{\mu_2}\delta_1(\beta_1 - \beta_2)(\delta_1 - \delta_2) \quad (12)$$

$$\beta_1 = -\frac{\rho_1 g \sin\beta}{\mu_1} = \frac{-890(9.81)\sin 5^\circ}{1.6(10^{-3})} = -4.7559(10^5)$$

$$\beta_2 = -\frac{\rho_2 g \sin\beta}{\mu_2} = -\frac{1000(9.81)\sin 5^\circ}{0.92(10^{-3})} = -9.2935(10^5)$$

$\delta_1 = 0.012$ m and $\delta_2 = 0.013$ m

(remember – δ_2 is distance from air-oil surface to wall.)

Substitute into (12)

$$u_{2b} = 11.617 - 9.4698 = 2.1472 \text{ m/s}$$

$\longrightarrow$

(b) Ratio of volume rates

$$\phi = \frac{\dot{V}_{oil}}{\dot{V}_{water}} = \frac{\int_0^{\delta_1} u_1\,dy}{\int_{\delta_1}^{\delta_2} u_2\,dy} \; ; \quad \text{on the basis of unit channel width}$$

Substitute equations (8) and (11)

$$\phi = \frac{\int_0^{\delta_1}\left[\left(u_{2b} - \frac{\beta_1}{2}\delta_1^2\right) + \frac{\beta_1}{2}y^2\right]dy}{\int_{\delta_1}^{\delta_2}\left[\frac{\beta_2}{2}y^2 + c_3 y - \left(c_3\delta_2 + \frac{\beta_2}{2}\delta_2^2\right)\right]dy}$$

$$= \frac{\left[\left(u_{2b} - \frac{\beta_1}{2}\delta_1^2\right)y + \frac{\beta_1}{6}y^3\right]_0^{\delta_1}}{\left[\frac{\beta_2}{6}y^3 + \frac{c_3}{2}y^2 - \left(c_3\delta_2 + \frac{\beta_2}{2}\delta_2^2\right)y\right]_{\delta_1}^{\delta_2}}$$

$$= \frac{u_{2b} - \frac{\beta_1}{2}\delta_1^3 + \frac{\beta_1}{6}\delta_1^3}{\frac{\beta_2}{6}\left(\delta_2^3 - \delta_1^3\right) + \frac{c_3}{2}\left(\delta_2^2 - \delta_1^2\right) - \left(c_3\delta_2 + \frac{\beta_2}{2}\delta_2^2\right)\left(\delta_2 - \delta_1\right)}$$

Given $c_3 = \frac{\mu_1}{\mu_2}\beta_1\delta_1 - \beta_2\delta_1 = \left(\frac{\mu_1}{\mu_2}\beta_1 - \beta_2\right)\delta_1 = 1.2268(10^3)$

$$\phi = \frac{u_{2b} - \frac{\beta_1}{3}\delta_1^3}{\frac{\beta_2}{6}\left(\delta_2^3 - \delta_1^3\right) + \frac{c_3}{2}\left(\delta_2^2 - \delta_1^2\right) - \left(c_3\delta_2 + \frac{\beta_2}{2}\delta_2^2\right)\left(\delta_2 - \delta_1\right)}$$

Subst. values

$$\phi = 426.39$$