The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena Quiz #8 December 2, 2008

Time Allowed: 45 mins. **Name:**

In oil production, hydrocarbon gases often evolve from the liquid or are produced with the oil from reservoirs. In some operations, the volume fraction that is gas in the fluid mixture is substantial and, during flow in a pipe, the gas forms long bubbles which travel in the middle of the pipe, surrounded by liquid flowing in the same direction under the imposed pressure gradient. For the problem of interest, it will be assumed that the oil contains surface active agents which coat the bubble surfaces and thus make the boundary act as if it is rigid.

In a 6 cm inside diameter pipe oriented at 30° to the horizon, a very long bubble is observed to be traveling at 1.2m/s. The bubble diameter is 4.8 cm and the bubble is located axis-symmetrically in the pipe. The pressure drop per meter along the flow direction in the pipe is given as 4.4 kPa/m.

a) (***10 pts***) Estimate the volume flow rate of the oil.

b) Bonus (***2 pts***) Is the average velocity of the oil different from the displacement rate for the bubble?

Data: Properties of oil - $\rho = 890$ kg/m^3; $\mu = 1.06$ Pa s

SOME INTEGRALS

QJ

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

$$\int xe^{ax}\,dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$$

$$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a}\left(x^2 - \frac{2x}{a} + \frac{2}{a^2}\right)$$

$$\int \ln x\,dx = x\ln x - x$$

$$\int x\ln x\,dx = \frac{x^2}{2}(\ln x - \tfrac{1}{2})$$

$$\int x^m \ln x\,dx = \frac{x^{m+1}}{m+1}\left(\ln x - \frac{1}{m+1}\right)$$ [If $m = -1$ see 14.528.]

$$\int \frac{\ln x}{x}\,dx = \frac{1}{2}\ln^2 x$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln(ax+b)$$

$$\int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$$

$$\int \frac{x^2\,dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3}\ln(ax+b)$$

$$\int \frac{x^3\,dx}{ax+b} = \frac{(ax+b)^3}{3a^4} - \frac{3b(ax+b)^2}{2a^4} + \frac{3b^2(ax+b)}{a^4} - \frac{b^3}{a^4}\ln(ax+b)$$

$$\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left(\frac{x}{ax+b}\right)$$

$$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left(\frac{ax+b}{x}\right)$$

$$\int \frac{dx}{x^3(ax+b)} = \frac{2ax-b}{2b^2x^2} + \frac{a^2}{b^3}\ln\left(\frac{x}{ax+b}\right)$$

$$\int \frac{dx}{(ax+b)^2} = \frac{-1}{a(ax+b)}$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right) \quad \text{or} \quad \frac{1}{a}\tanh^{-1}\frac{x}{a}$$

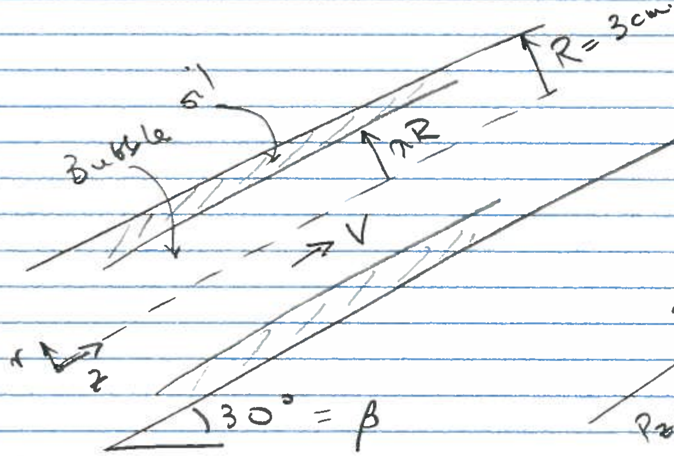
$$\int \frac{x\,dx}{a^2-x^2} = -\frac{1}{2}\ln(a^2-x^2)$$

$$\int \frac{x^2\,dx}{a^2-x^2} = -x + \frac{a}{2}\ln\left(\frac{a+x}{a-x}\right)$$

$$\int \frac{x^3\,dx}{a^2-x^2} = -\frac{x^2}{2} - \frac{a^2}{2}\ln(a^2-x^2)$$

$$\int \frac{dx}{x(a^2-x^2)} = \frac{1}{2a^2}\ln\left(\frac{x^2}{a^2-x^2}\right)$$

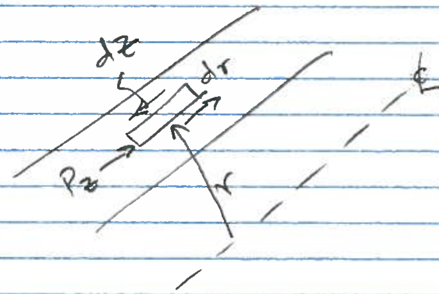
ENCH 501 Quiz 8 Solution Dec. 2, 2008



Given

$$\lambda = \frac{4.8}{6} = 0.8$$

$$V = 1.2 \text{ m/s}$$



Force balance on differential ring at r

$$(2\pi r\,dz\,\tau)_r - (2\pi r\,dz\,\tau)_{r+dr} + P_z(2\pi r\,dr) -$$

$$P_{z+dz}(2\pi r\,dr) - (2\pi r\,dr\,dz)\rho g \sin\beta = 0$$

$$\frac{d(r\tau)}{dr} + r\frac{dP}{dz} + r(\rho g \sin\beta) = 0$$

With

$$\tau = -\mu\frac{du}{dr}, \text{ we obtain}$$

$$\mu\frac{d}{dr}\left(r\frac{du}{dr}\right) = r\left[\frac{dP}{dz} + \rho g \sin\beta\right]$$

$$\frac{d}{dr}\left(r\frac{du}{dr}\right) = -\frac{\Delta}{\mu L}\cdot r \quad ; \quad \frac{\Delta}{L} = -\left[\frac{dP}{dz} + \rho g \sin\beta\right]$$

Integrate

$$r\frac{du}{dr} = -\frac{\Delta}{\mu L}\frac{r^2}{2} + C_1$$

or $\frac{du}{dr} = -\frac{\Delta}{2\mu L} r + \frac{c_1}{r}$

Integrate again

$$u = -\frac{\Delta}{4\mu L} r^2 + c_1 \ln r + c_2$$

subject to the conditions

at $r = \lambda R$ $\quad u = V$ (bubble)

$r = R$ $\quad u = 0$ (no slip)

apply conditions

$$u = \frac{\Delta}{L}\left(\frac{R^2}{4\mu}\right)\left(1 - r^2/R^2\right) + \frac{1}{\ln \lambda}\left\{V - \frac{\Delta}{L}\frac{R^2}{4\mu}(1-\lambda^2)\right\} \ln(r/R)$$

$$u = \alpha\left(1 - r^2/R^2\right) + \beta \ln(r/R)$$

The volume rate of oil

$$Q = \int_{\lambda R}^{R} u\, 2\pi r\, dr = 2\pi \int_{\lambda R}^{R} r u\, dr$$

$$= 2\pi \int_{\lambda R}^{R} \left\{\alpha r - \alpha \frac{r^3}{R^2} + \beta r \ln\left(\frac{r}{R}\right)\right\} dr$$

$$= 2\pi R^2 \int_{\lambda}^{1} \alpha\left(\frac{r}{R}\right) - \alpha\left(\frac{r}{R}\right)^3 + \beta\left(\frac{r}{R}\right)\ln\left(\frac{r}{R}\right) d\left(\frac{r}{R}\right)$$

Let $\eta = r/R$

$$Q = 2\pi R^2 \int_{\lambda}^{1} \left\{ \alpha\eta - \alpha\eta^3 + \beta\eta\ln\eta \right\} d\eta$$

$$= 2\pi R^2 \left\{ \alpha\frac{\eta^2}{2} - \alpha\frac{\eta^4}{4} + \beta\left[\frac{\eta^2}{2}\left(\ln\eta - \frac{1}{2}\right)\right] \right\} \Bigg|_{\lambda}^{1}$$

(a)

$$Q = 2\pi R^2 \left\{ \frac{\alpha}{4} - \frac{\beta}{4} - \frac{\alpha\lambda^2}{2} + \frac{\alpha\lambda^4}{4} - \beta\left\{\frac{\lambda^2}{2}\left(\ln(\lambda) - \frac{1}{2}\right)\right\} \right\}$$

now substitute numbers

$$\alpha = \frac{\chi}{L}\left(\frac{R^2}{4\mu}\right) = \frac{34.55\,(0.03)^2}{4(1.06)} = 7.334(10^{-3})$$

$$\beta = \frac{1}{\ln(0.8)}\left\{1.2 - 7.334(10^{-3})(1 - 0.8^2)\right\} = -5.3659$$

$$\lambda = 0.8 \quad \text{and} \quad R = 0.03\text{ m}$$

$$\therefore \quad Q = 2\pi(0.03)^2\left\{1.3433 - 0.002347 + 0.000751 - 1.2417\right\}$$

$$Q = 2\pi(0.03)^2(0.1) = 5.66(10^{-4})\,\text{m}^3/\text{s}$$

(b) Bonus $\bar{u} = \frac{Q}{A} = \frac{Q}{\pi R^2(1-\lambda^2)} = \frac{5.66(10^{-4})}{\pi(0.03)^2(1-0.8^2)}$

$$\bar{u} = 0.556\text{ m/s}$$

This is slower than the translation rate for bubble.