GJ

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ENCH 501: Transport Processes   Quiz #8   December 7, 2004

Time Allowed: 50 mins.   Name:

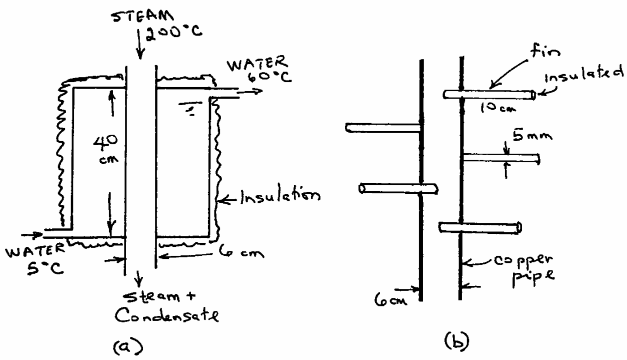
"Steam boxes" are used to recover heat from low energy steam (which would otherwise be discarded) into water. A steam box is as shown in sketch (a) below. The box is an insulated metal chest. A 6 cm o.d. copper pipe traverses the box. The length of the pipe within the box is 40 cm. Through the inside of the pipe, steam at 200°C is passed at a high rate. It will be assumed that the outside wall of the pipe is maintained constant at this temperature. Into the box, water at 5°C is admitted at a rate of 0.03 m^3/hr. This water rapidly mixes with the water in the box, and the water leaves the box at a temperature of 60°C.

The rate of water flow through the box is to be increased 10 times. The inlet and effluent temperatures of the water are to remain unchanged. It has been suggested that copper fins, 5 mm diameter and 10 cm long, be attached to the surface of the copper pipe (sketch b). The extended surfaces will help increase the heat transfer rate enough to achieve the desired result. Assume the exposed tips of the fins are insulated.

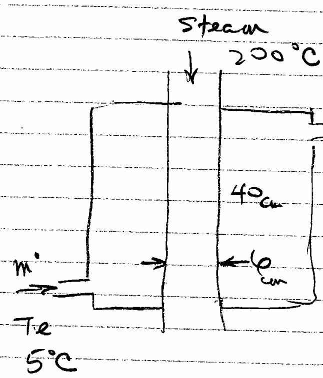
Determine how many fins would be required.
Show all your steps. State assumptions.

Data:
Properties of water at 60°C.
ρ = 983.2 kg/m^3 ; C_p = 4.185 kJ/kg K

Properties of copper
k = 386 W/m K ; Cp = 0.383 kJ/kg K



ENCH 501 Quiz #8 Solution Dec. 7, 2004



Energy Balance at steady state

Energy gained by water = Heat loss by steam

$$Q = hA(T_s - T_\alpha) = \dot{m}\, C_p (T_w - T_e)$$

where all terms except h are known.

☐ No fins.

$$\left[\frac{0.03\,(983.2)}{3600}\right](4185)(60-5) = h \cdot \left[\pi(0.06)(0.4)\right] \times [200-60]$$

kg/s water; C_p; $T_w - T_e$; area; $T_s - T_\alpha$

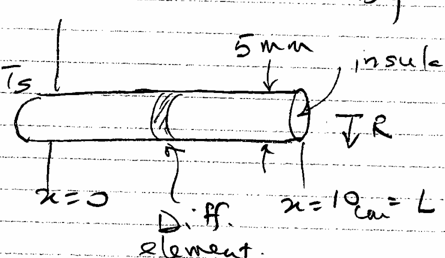
$$(0.008193)(4185)(55) = h(0.0754)(140)$$

$$1885.9 = h\,(10.556)$$

$$\therefore\ h = 178.64\ W/m^2K.$$

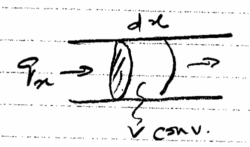
Assume this is always the value in the steam box.

☐ With Fins. — Evaluate the rate of heat transfer by each fin, with base temp = 200°C.



Consider a fin as shown.

Energy Balance for a differential element



$$(\pi R^2 q_x)_x = (\pi R^2 q_x)\Big|_{x+dx} + h(2\pi R\,dx)(T - T_\alpha)$$

where T_α = ambient temp. = 60°C.

$$0 = \pi R^2 \frac{dq_x}{dx} dx + h(2\pi R dx)(T - T_\alpha)$$

where $q_x = -k \frac{dT}{dx}$.

Substitute

$$\pi R^2 k \frac{d^2T}{dx^2} dx = h(2\pi R dx)(T - T_\alpha)$$

or

$$kR \frac{d^2T}{dx^2} = 2h(T - T_\alpha)$$

As was done in Notes, page 161, non-dimensionalize eq. with:

$$\theta = \frac{T - T_\alpha}{T_s - T_\alpha}, \quad \xi = \frac{x}{L}, \quad N^2 = \frac{2hL^2}{kR}$$

$$\therefore \quad \frac{d^2\theta}{d\xi^2} = N^2\theta$$ with the conditions

$\xi = 0 \quad \theta = 1$ (base)

* ($L = 10$ cm) $\quad \xi = 1 \quad \frac{d\theta}{d\xi} = 0$ (insulated)

The solution is same as eq. 6.83 (Notes)

$$\frac{T - T_\alpha}{T_s - T_\alpha} = \frac{\cosh N(\xi - 1)}{\cosh N}$$

The Rate of heat transfer from fin to water can be estimated by using a term similar to the numerator of eq. 6.85, i.e.

$$E = \int_0^L h(\pi D\, dx)(T - T_\alpha) = \pi D h \int_0^L (T - T_\alpha)\, dx$$

$$= \pi D h L (T_s - T_\alpha) \int_0^1 \left(\frac{T - T_\alpha}{T_s - T_\alpha}\right) d\left(\frac{x}{L}\right)$$

$$= \frac{\pi D h L (T_s - T_\alpha)}{\cosh N} \int_0^1 \cosh N(\xi - 1)\, d\xi$$

From Math tables

$$\int \cosh ax\, dx = \frac{\sinh ax}{a}; \quad \text{also } \sinh(-x) = -\sinh(x)$$

$$E = \frac{(\pi D L) h (T_s - T_a) \tanh N}{N}$$

$$N = \left[\frac{2hL^2}{Rk}\right]^{\frac{1}{2}} = \left[\frac{2(178.66)(0.1)^2}{(2.5)(10^{-3})386}\right]^{\frac{1}{2}}$$

$$= 1.924$$

$$E = \pi(5)(10^{-3})(0.1)(178.66)(200-60)\frac{0.9583}{1.924}$$

$$= 19.568 \text{ W } (J/s)$$

Let the number of fins to be used = n

The heat gained by $10(0.03)\ m^3/hr$ of water =

heat released by all the fins + heat from pipe area not covered at base by fins.

i.e.

$$10(1885.9) = n(19.568) + 178.66\left(0.0754 - n\left(\pi\frac{5^2(10^{-3})^2}{4}\right)(140)\right)$$

or

$$9(1885.9) = n(19.568 - 0.4911)$$

$$\therefore n = \frac{9(1885.9)}{19.077} = 889.7$$

$\therefore$ 890 fins are required to be attached to the pipe. These fins cover 23.18% of the surface area of the pipe in the steam box.