G.J.

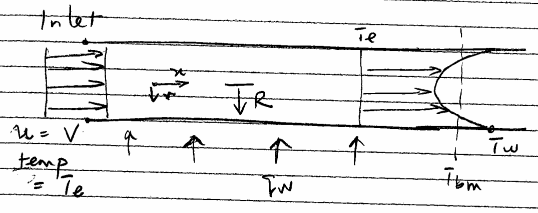
The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #8 **December 2, 2003**

Time Allowed: 45 mins. **Name:**

For fully developed laminar flow through a tube of inside diameter D, the Nusselt number (hD/k) for constant heat flux along the tube wall and for fully developed temperature profiles is given as 4.364 or 48/11. If the flow is such that a flat velocity profile (plug flow) can be assumed, derive a value for the Nusselt number for the same conditions as above. Show all your steps and state your assumptions.

ENCH 501 Solution - Quiz #8 Dec. 2, 2003



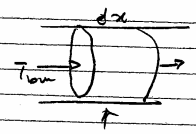
Vel. = V

For heating, $\frac{dT}{dr}\Big|_R$ is +ve.

The energy balance equation (Notes: eq. 6.95) is

$$\frac{1}{ur}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial x}, \quad \alpha = k/\rho C_p \qquad (1)$$

Also from an energy balance around a disc



$$dQ_x = 2\pi R\,dx\,q_w = (\pi R^2 V)\rho\, C_p\, dT_{bm}$$

or

$$\frac{dT_{bm}}{dx} = \frac{2q_w}{RV\rho C_p} = \text{const.} \qquad (2)$$

Fully developed temperature profile

$$\frac{T_w - T}{T_w - T_{bm}} \neq f(x) \quad \text{or} \quad \frac{d}{dx}\left(\frac{T_w - T}{T_w - T_{bm}}\right) = 0 \qquad (3)$$

Expand this expression to obtain

$$\frac{dT}{dx} = \frac{dT_w}{dx} - \frac{T_w - T}{T_w - T_{bm}}\frac{dT_w}{dx} + \frac{T_w - T}{T_w - T_{bm}}\frac{dT_{bm}}{dx} \qquad (4)$$

But the condition of heat transfer into the fluid is given by

$$q_w = +k\frac{dT}{dr}\Big|_R = h(T_w - T_{bm}) = \text{const.}$$

Re-arrange

$$\frac{h}{k} = \frac{+\partial T/\partial r|_R}{T_w - T_{bm}} = \frac{d}{dr}\left[\frac{T_w - T}{T_w - T_{bm}}\right]\Bigg|_R \qquad (5)$$

Since the temperature profile is fully developed, the r.h.s. of equation (5) equals a constant. That is, h = constant. In turn, this means $(T_w - T_{bm})$ = const.

$$\therefore \quad \frac{dT_w}{dx} = \frac{dT_{bm}}{dx} = \text{const.} \quad (\text{given in eq. (2)})$$

When these are used in eq. (4), we obtain

$$\frac{dT}{dx} = \frac{dT_w}{dx} = \frac{dT_{bm}}{dx} = \text{const.}$$

Equation (1) hereby reduces to an o.d.e., or

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \left[\frac{1}{\alpha}\frac{dT}{dx}V\right] r = \beta r \qquad (6)$$

Integrate

$$r\frac{dT}{dr} = \frac{\beta r^2}{2} + C_1$$

Using symmetry condition, $r=0$, $\frac{dT}{dr}=0$, we get

$$C_1 = 0$$

$$\therefore \quad \frac{dT}{dr} = \frac{\beta r}{2} \quad (7) \qquad \text{and} \quad T = \beta\frac{r^2}{4} + C_2$$

where $C_2 = T_o$ or temp. along the tube axis.

$$T = T_o + \beta r^2/4 \qquad (T_o \text{ yet unknown}) \quad (8)$$

Evaluate the terms for eq. (5)

$$r = R, \quad T = T_w(x) \quad \text{or} \quad T_w = T_o + \beta R^2/4 \qquad (9)$$

$$T_{bm} = \frac{\int_0^R uTr\,dr}{\int_0^R ur\,dr} = \frac{\int_0^R \left(T_o r + \frac{\beta r^3}{4}\right)dr}{\frac{1}{2}R^2}$$

$$T_{bm} = \frac{2}{R^2}\left[T_0\frac{r^2}{2} + \beta\frac{r^4}{16}\right]_0^R = \frac{2}{R^2}\left[T_0\frac{R^2}{2} + \beta\frac{R^4}{16}\right]$$

$$= T_0 + \beta\frac{R^2}{8} \qquad (10)$$

and $$\left.\frac{dT}{dr}\right|_R = \beta\,R/2 \qquad \text{(from equation (7))} \qquad (11)$$

Substitute into equation (5)

$$\frac{h}{k} = \frac{+\beta\,R/2}{\beta\left[\frac{R^2}{4} - \frac{R^2}{8}\right]} = \frac{1}{2}\,\frac{8}{R}$$

Nusselt number, $\frac{hD}{k} = 8.$ $\longrightarrow$