Dec 4

GJ

ENCH 501 Transport Phenomena **Quiz #7** **Name** ____________________

Time Allowed: 45 minutes

Pipelines used to transport liquids and gases over long distances accumulate debris that include materials from construction, particles that settle from the from the fluid, waxes that precipitate from oils, hydrates formed from certain gases and moisture, and corrosion products. Pipes are inspected for structural integrity and accumulated debris cleaned out, as part of maintenance, with a device called the PIG. This is a cylinder (supported by plastic rings) that is inserted and pushed or pulled through the pipe to scrape off scales and sediments, or take measurements of pipe wall thickness. Pigs have different shapes and designs. For the current problem, we consider the pig to be a cylinder of length **L** and radius **κR**, concentrically placed in a straight, horizontal pipe with an internal radius **R**. An annular space is present between the pig and the pipe wall.

An oil, viscosity **μ** and density **ρ**, fills the pipe but there is no flow because a valve downstream is shut. A string attached to the front (and centre) of the pig is used to pull the pig at a constant velocity **V** through the oil. The pressure in front of the pig $\mathbf{P_1}$ is higher than the pressure at the rear $\mathbf{P_0}$ (just as for a truck moving through air).

a) Derive a relationship for the velocity profile **u(r)** in the annular gap. Show all your steps.
b) At what distance from the pipe wall is the velocity minimum? Sketch the velocity profile.

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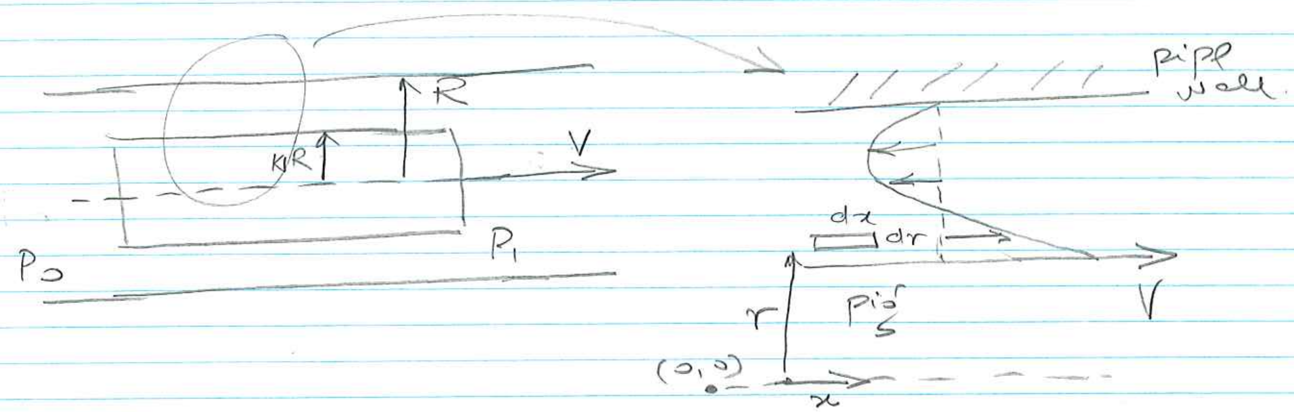
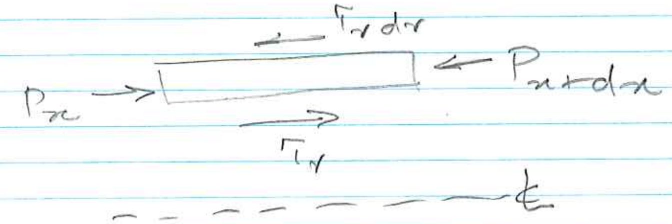


Consider a differential element, a ring at distances r and x from the origin

Newton's Law: Net Force = Rate of Δ Momentum

$= 0$



Force balance (horizontal pipe, no gravity)

$$\left(2\pi r\,dx\,\tau\right)_r - \left(2\pi r\,dx\,\tau\right)_{r+dr} + P_x\,2\pi r\,dr$$

$$- P_{x+dx}\left(2\pi r\,dr\right) = 0$$

or

$$-\frac{d}{dr}\left(2\pi r\,dx\,\tau\right)dr - \frac{dP}{dx}dx\left(2\pi r\,dr\right) = 0$$

with $\tau = -\mu\frac{du}{dr}$ for Newtonian fluids,

$$\mu\frac{d}{dr}\left(r\frac{du}{dr}\right) = r\frac{dP}{dx} \qquad (1)$$

Integrate once

$$r \frac{du}{dr} = \frac{1}{2\mu} \frac{dP}{dx} r^2 + C_1 \qquad (2)$$

If the velocity is maximum or minimum at $r = \lambda R$, then $\frac{du}{dr} = 0$ at this pt.

Apply this relationship gives that

$$C_1 = -\frac{1}{2\mu} \frac{dP}{dx} (\lambda R)^2 \qquad (3)$$

Integrate equation 2

$$u = \frac{1}{4\mu} \frac{dP}{dx} r^2 + C_1 \ln r + C_2$$

A boundary condition, $r = R$, $u = 0$ is applied.

$$C_2 = -\frac{1}{4\mu} \frac{dP}{dx} R^2 - C_1 \ln R$$

$$\therefore \quad u = \frac{1}{4\mu} \frac{dP}{dx} (r^2 - R^2) + C_1 \ln \left(\frac{r}{R} \right) \qquad (4)$$

Apply second condition, $r = K_1 R$, $u = V$

$$V = -\frac{R^2}{4\mu} \frac{dP}{dx} (1 - K_1^2) - \frac{1}{2\mu} \frac{dP}{dx} (\lambda R)^2 \ln K_1$$

$$\therefore \quad \lambda^2 = \frac{V + \frac{R^2}{4\mu}\frac{dP}{dx}(1-\kappa^2)}{-\frac{1}{2\mu}\frac{dP}{dx}R^2 \ln \kappa} \qquad (5)$$

Substitute eq. (3) + (5) into (4)

$$u = -\frac{R^2}{4\mu}\frac{dP}{dx}\left(1 - \frac{r^2}{R^2}\right) + \frac{V + \frac{R^2}{4\mu}\frac{dP}{dx}(1-\kappa^2)}{\ln \kappa} \cdot \ln\frac{r}{R}$$

(a)

$$u = V\frac{\ln(r/R)}{\ln \kappa} - \frac{R^2}{4\mu}\frac{dP}{dx}\left\{1 - \frac{r^2}{R^2} - \frac{1-\kappa^2}{\ln \kappa}\ln\frac{r}{R}\right\}$$

$\longrightarrow$

Check:

When $r = R$, $u = 0$

$r = \kappa R$ $u = V$

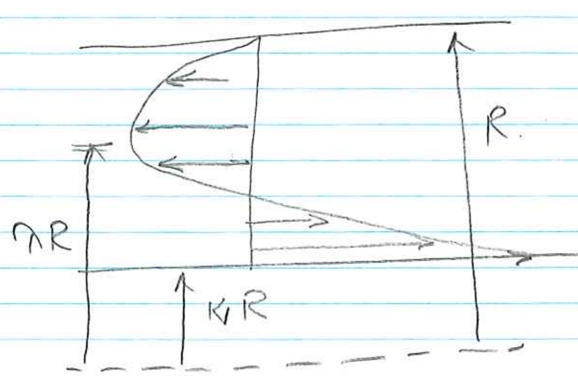
Also

$$\frac{dP}{dx} = \frac{P_1 - P_0}{L}$$

(b) Equation (5) gives the location of max. or min, at $r = \lambda R$.

Distance from the wall is $R - \lambda R = R(1-\lambda)$

$\longrightarrow$



Profile.