a.5

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena Quiz #7 **December 06, 2011**

Time Allowed: 45 mins. **Name:**

The quiz is based in part on problem 5.12 in the Notes. It is to explore the performance of odor absorbents such as baking soda that you put in the fridge or in the car.

Trace component of substance A in a gas mixture of a A and B (air) is to be removed. It was suggested that the gas be brought in contact with a large slab (assume semi-infinite) of solid absorbent C. Component B is neither absorbed by nor moving through C. As A diffuses through C, it reacts at a rate proportional to the local concentration of A. The reaction is first order and the rate of removal of A is given by :

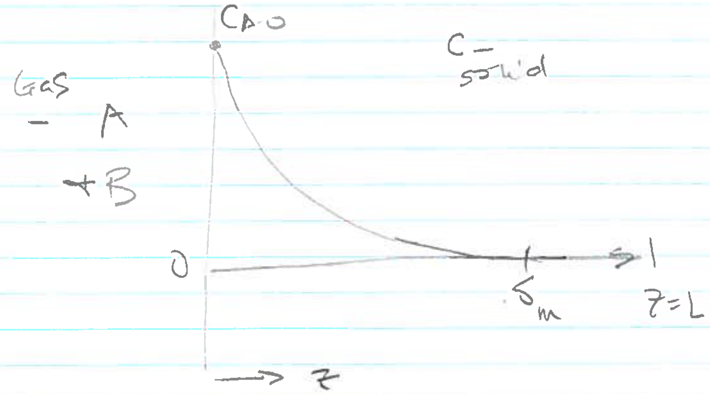
$r_A = k_1 C_A$; where $k_1 = 0.3\ hr^{-1}$

It may be assumed that the concentration of A at the solid side of the gas - solid interface is low and constant at 0.01 moles/m^3. The solid may be assumed to contain no A at the start of the absorption process. The molar concentration of the solid is constant at 40 moles/litre. The diffusivity of A in C is given as 10^{-7} m^2/s. The temperature is constant at 20°C.

a) Use the integral method to determine the time ***t*** from the start of the process that A would have entered into the solid to a depth of 2.5 cm. Show all important steps.

b) At this time, how much total A would have been absorbed by the solid?

ENCH 501 Quiz 7 Solution Dec 6, 2011



Flux of A in C

$$N_A = -cD_{AB}\frac{dx_A}{dz} + x_A(N_A + N_B + N_C)$$

For this problem, x_A is low, ∴ neglect convective term — i.e. since $c = \text{const}$

$$N_A = -D_{AB}\frac{dC_A}{dz}$$

Material balance on component A in $0 \le z < L$

Input + Gen = Output + Accum

$$N_A\Big|_{z=0} - \int_0^L k_1 C_A\,dz = 0 + \frac{d}{dt}\left[\int_0^L C_A\,dz\right]$$

The integral equation is

$$\frac{d}{dt}\left[\int_0^{\delta_m} C_A\,dz\right] = -D_{AB}\frac{dC_A}{dz}\Big|_{z=0} - \int_0^{\delta_m} k_1 C_A\,dz \quad (1)$$

The upper limits of the integrals have been set to δ_m (from L) because $C_A = 0$ at $t=0$ (for all z) and $C_A = 0$ for $z \geq \delta_m$.

The conditions are:

$z=0 \quad C_A = C_{A0} = 0.01$ moles/m^3

$z=\delta_m \quad C_A = 0$

$z=\delta_m \quad \frac{dC_A}{dz} = 0$

assume $C_A = a + bz + cz^2$

and apply b.c

$$C_A = C_{A0} \left(1 - z/\delta_m\right)^2 \qquad (2)$$

substitute this into eq. (1) and simplify to get

$$\frac{d}{dt}\left[C_{A0}\,\delta_m \frac{1}{3}\right] = -D_{AB}\left(-\frac{2C_{A0}}{\delta_m}\right) - \frac{k_1 C_{A0}\,\delta_m}{3}$$

Since C_{A0} = const, cancel out

$$\frac{d\delta_m}{dt} = \frac{6D_{AB}}{\delta_m} - k_1\delta_m$$

$$\delta_m \frac{d\delta_m}{dt} + k_1\delta_m^2 = 6D_{AB}$$

or

$$\frac{d\delta_m^2}{dt} = -2k_1\delta_m^2 + 12D_{AB}$$

subject to the condition $t=0$, $\delta_m = 0$

solve.

$$\ln\left(\frac{12D_{AB}}{12D_{AB} - 2k_1\delta_m^2}\right) = +2k_1 t$$

or

$$\frac{2k_1\delta_m^2}{12D_{AB}} = \frac{k_1\delta_m^2}{6D_{AB}} = 1 - e^{-2k_1 t} \qquad (3)$$

(a) Given $\delta_m = 2.5\,cm$ or $0.025\,m$

$k_1 = 0.3\ hr^{-1} = 8.333(10^{-5})\,s^{-1}$

$D_{AB} = 10^{-7}\ m^2/s$

$$\frac{8.3333(10^{-5})(0.025)^2}{6(10^{-7})} = 1 - e^{-2(8.33)(10^{-5})t}$$

$$t = 544.84\ s$$

(b) How much total A absorbed?

$$\underset{\substack{\text{unit}\\\text{area of}\\\text{surface}}}{\phi} = \int_0^t N_A\Big|_{z=0} dt = \int_0^t -D_{AB}\frac{dC_A}{dz}\Big|_{z=0} dt$$

Given $\dfrac{dC_A}{dz}\Big|_{z=0} = -2\dfrac{C_{Ao}}{\delta_m}$

$$\phi = 2\,C_{Ao}\,D_{AB}\int_0^t \frac{1}{\delta_m}\,dt$$

From eq. (3),

$$\delta_m = \left[\frac{6D_{AB}}{k_1}\left(1 - e^{-2k_1 t}\right)\right]^{1/2}$$

$$= \left[\beta\left(1 - e^{-\varepsilon t}\right)\right]^{1/2}.$$

use integral table

$$Q = 2 C_{AO} D_{AB} \int_0^t \frac{dt}{\left[\beta\left(1 - e^{-\varepsilon t}\right)\right]^{\frac{1}{2}}}$$