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ENCH 501: Transport Phenomena Quiz #7 **November 30, 2010**

Time Allowed: 50 mins. **Name:**

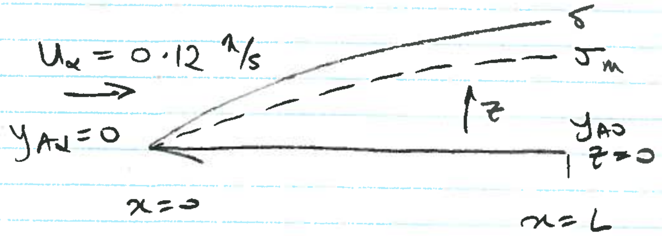
Non-stick cookware (pots, pans, skillets, griddles, cookie sheets etc.), clothes irons and ironing boards, self-cleaning ovens, curling irons and many other appliances are coated with resins of polytetrafluoroethylene (PTFE) that is generally known as Teflon. The compound is safe for use, but when it is overheated, it may emit fumes of monomers and polymer fragments of PTFE at temperatures as low as 120°C. Above 260°C, PTFE starts to degrade. The fumes may cause flu-like symptoms in humans but they are very toxic to birds, especially pets.

A new rectangular ironing board, 30cm wide by 1.1m long, is being used and the heat from the iron caused PTFE monomer (TFE) to be emitted. (The time for ironing clothes is much shorter than the intervals in between. Thus assume that clothes and the iron are not on the board.) The concentration of TFE at the board surface is low but constant at 10^{-3} moles/liter in air. The temperature and pressure of the air in the room are 18°C and 0.94 atm. Air is blown by a small fan over the board, along the direction of its long side, at a steady rate of 0.12 m/s. Assume the room air does not contain TFE.

Estimate the total amount (in moles) of TFE that would have been released into the room in one (1) hour. Use the **integral** method and show all important steps. State your assumptions.

Data: Diffusivity of TFE in air is $1.2(10^{-5})$ m^2/s; Universal gas constant is 0.08205 (liter atm)/(mol K); Viscosity of air 0.018 mPa s; Density of air 1.18 kg/m^3.

ENCH 501 Quiz 7 Solution Nov. 30, 2010



$L = 1.1\,m$

Schmidt # $= Sc = \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$

For this problem,

$$Sc = \frac{1.8(10^{-5})}{1.18(1.2)(10^{-5})} = 1.27$$

$\therefore\ \delta > \delta_m.$

Also, for flow over a flat board, determine whether b.l. is laminar.

$$Re_L = \frac{0.12(1.1)(1.18)}{1.8(10^{-5})} = 8{,}653 < 5(10^5)$$

$\therefore$ laminar.

Let the concentration of TFE (A) at the boundary of the board $= y_{A0} = C_{A0}/C_T$

C_{A0} is given as 10^{-3} moles/litre

$C_T = P/RT$ (from ideal gas law) $= \frac{0.94}{0.08205(291.15)}$

$= 0.03935$ mols/litre

$\therefore\ y_{A0} = 0.0254 \ll 1$

The flux of A due to molecular diffusion in the z-direction may be defined as

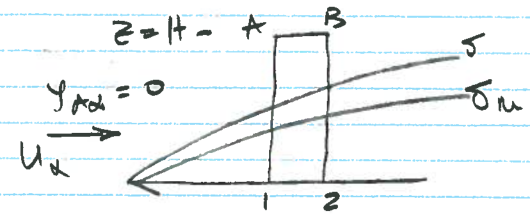
$$N_A = -cD_{AB}\frac{dy_A}{dz} + y_A(N_A + N_B)$$

where subscript B refers to air.

Since $y_{A0} \ll 1$, the convective term will be neglected. Hence

$$N_A = -cD_{AB}\frac{dy_A}{dz} \quad ; \quad c = C_T$$

By the integral method, we consider a control volume 1AB2 — as shown



The balance on A in the control volume is given by

In at 1-2 $\quad N_A\big|_{z=0}(dx\,W)$

where W is width of board.

In at A-1 $\quad \displaystyle\int_{z=0}^{H} c\,y_A\,u\,(W\,dz)$

Out at B-2 $\quad \displaystyle\int_{z=0}^{H} c\,y_A\,u\,(W\,dz) + \frac{d}{dx}\left[\int_0^H c\,y_A\,u\,W\,dz\right]dx$

Out at AB $\quad -\dfrac{d}{dx}\left[\displaystyle\int_0^H u\,dz\right]dx\,(y_{A\infty}C) = 0 \quad \because y_{A\infty} = 0$

The integral mass balance equation on A is:

$$N_A\Big|_{z=0}(dx\,W) = \frac{d}{dx}\left[\int_0^H c\,y_A\,u\,W\,dz\right]dx$$

or $$N_A\Big|_{z=0} = -cD_{AB}\frac{dy_A}{dz}\Big|_{z=0} = \frac{d}{dx}\left[\int_0^{\delta_m} c\,y_A\,u\,dz\right]$$

The boundary conditions for the problem are:

$z = 0 \qquad y_A = y_{A0}$, a constant

$$z = \delta_m \qquad y_A = y_{A\infty} = 0$$

$$z = \delta_m \qquad \frac{dy_A}{dz} = 0$$

Choose an equation

$$y_A = a + bz + cz^2$$

apply b.c. to obtain

$$\frac{y_A}{y_{Ao}} = \left(1 - \frac{z}{\delta_m}\right)^2 \quad ; \quad \frac{dy_A}{dz} = -\frac{2y_{Ao}}{\delta_m}\left(1 - \frac{z}{\delta_m}\right)$$

substitute into the integral equation

$$-c\mathcal{D}_{AB}\left(-\frac{2y_{Ao}}{\delta_m}\right) = \frac{d}{dx}\left[\int_0^{\delta_m} c\, y_{Ao}\left(1 - \frac{z}{\delta_m}\right)^2 u\, dz\right]$$

From Notes, use

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{z}{\delta}\right) - \frac{1}{2}\left(\frac{z}{\delta}\right)^3 \quad \text{for vel. profile}$$

$$\frac{2\mathcal{D}_{AB}}{\delta_m} = \frac{d}{dx}\left[\int_0^{\delta_m} U_\infty\left(1 - \frac{2z}{\delta_m} + \frac{z^2}{\delta_m^2}\right)\left(\frac{3}{2}\left(\frac{z}{\delta}\right) - \frac{1}{2}\left(\frac{z}{\delta}\right)^3\right)dz\right]$$

Let $\xi = \delta_m/\delta$ and $\eta = z/\delta$

$$\frac{2\mathcal{D}_{AB}}{\delta_m} = \frac{d}{dx}\left[\int_0^{\delta_m}\left(1 - \frac{2\eta}{\xi} + \frac{\eta^2}{\xi^2}\right)\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)\delta\,\frac{dz}{\delta}\right]$$

$$= \frac{d}{dx}\left[\delta\int_0^{\delta_m/\delta}\left(1 - \frac{2\eta}{\xi} + \frac{\eta^2}{\xi^2}\right)\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)d\eta\right]$$

Hence

$$\frac{2 D_{AB}}{\delta \xi} = \frac{d}{dx}\left[\delta U_\infty \left\{ \frac{1}{8}\xi^2 - \cancel{\frac{1}{120}\xi^4} \right\}\right]$$

neglect , $\xi < 1$

$$\therefore \quad \frac{16 D_{AB}}{U_\infty \delta \xi} = \frac{d}{dx}\left[\delta \xi^2\right] = \delta \frac{d\xi^2}{dx} + \xi^2 \frac{d\delta}{dx}$$

or $$\frac{16 D_{AB}}{U_\infty} = \delta^2 \xi \frac{d\xi^2}{dx} + \xi^3 \delta \frac{d\delta}{dx}.$$

From Notes:

Eq. 5.118 $$\delta \frac{d\delta}{dx} = \frac{140}{13}\left(\frac{\nu}{U_\infty}\right)$$

and Eq. 5.119 $$\delta^2 = \frac{280}{13} \frac{\nu x}{U_\infty}.$$

Substitute

$$\frac{16 D_{AB}}{U_\infty} = \frac{280}{13}\frac{\nu x}{U_\infty} 2\xi^2 \frac{d\xi}{dx} + \xi^3\left(\frac{140}{13}\frac{\nu}{U_\infty}\right)$$

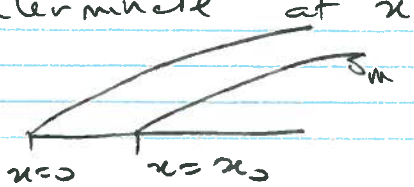
Re-arrange

$$\frac{52}{35}\frac{D_{AB}}{\nu} = 4x\xi^2\frac{d\xi}{dx} + \xi^3 = \beta$$

Solve.

$$\xi^3 = C x^{-3/4} + \beta$$

Since ξ is indeterminate at $x=0$, use a no flux leading edge



$\therefore \xi = 0$ at $x = x_0$

Hence $\xi = \frac{\delta_m}{\delta} = \beta^{1/3}\left[1 - \left(\frac{x_o}{x}\right)^{3/4}\right]^{1/3}$

when $x_o \longrightarrow 0$ $\xi = \beta^{1/3}$, a constant

Hence $y_A = y_{Ao}\left(1 - \frac{z}{\delta\beta^{1/3}}\right)^2$; $\delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}$

The local flux at the wall has been given as

$$N_A\Big|_{z=0} = -cD_{AB}\frac{dy_A}{dz}\Big|_{z=0} = (-cD_{AB})\left(-\frac{2y_{Ao}}{\delta_m}\right)$$

$$= 2cy_{Ao}\frac{D_{AB}}{\delta_m}$$

$\therefore$ Rate of removal of A from the board is

$$\dot{Q} = \int_0^L N_A\Big|_{z=0}(W\,dx) \quad \text{moles/s}$$

$$= \int_0^L 2cy_{Ao}D_{AB}W\left(\frac{1}{\beta^{1/3}\varepsilon}\right)\frac{dx}{x^{1/2}} \quad ; \quad \varepsilon = 4.64\sqrt{\frac{\nu}{U_\alpha}}$$

$$= \frac{2cy_{Ao}D_{AB}W}{\beta^{1/3}\varepsilon}\cdot 2x^{1/2}\Big|_0^L \quad ; \quad L = 1.1\,m$$

$W = 0.3\,m$

$cy_{Ao} = 10^{-3}$ kmols/m^3

Substitute values

$$\beta^{1/3} = \left[\frac{52}{35}\frac{(1.2)(10^{-5})(1.18)}{1.8(10^{-5})}\right]^{1/3} = 1.05335; \quad \varepsilon = 0.05232$$

$$\dot{Q} = 2.741(10^{-7}) \text{ kmols/s}$$

$\therefore$ For one hour

$$Q = \dot{Q}(3600) = 0.987 \text{ moles}$$