The University of Calgary
Department of Chemical & Petroleum Engineering

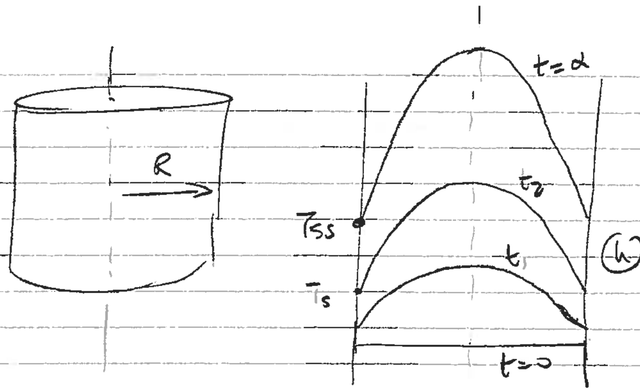
ENCH 501: Transport Phenomena Quiz #7 **November 25, 2008**

Time Allowed: 45 mins. **Name:**

The incandescent light was first created by Humphry Davy in 1802 when he passed current through platinum strips and made them glow. In 1840, Warren de la Rue enclosed the platinum in an evacuated bulb to improve longevity. A Canadian patent was issued to Henry Woodward and Mathew Evans in 1874, but when they could not successfully commercialize their invention, they sold the patent to Thomas Edison in 1879. Edison's bulbs were based on carbon filaments and some lasted 1200 hours. Tungsten's use as filament was described in a Hungarian patent to Sandor Just and Ferenc Hanaman in late 1904 and GE started making and using tungsten filament in bulbs in 1906. The modern bulb has a coil of tungsten supported in a glass envelope containing low pressure inert gas such as argon, neon or nitrogen. The problem is on this technology.

A current is passed through a long metal wire and heat is generated at a constant rate of $g+$ per unit volume everywhere in the wire. The wire was initially at a uniform temperature of T_∞ which is also the constant ambient temperature. If the heat transfer coefficient is h around the wire, derive an expression for the temperature profiles within the wire, T(r,t). Use the ***integral method*** and show all important steps.

#1



(T_∞) Let T_{ss} = steady state surface temp. and $T_s(t)$ is transient surface temperature

Energy Balance eq.

$$\underbrace{g^+(\pi R^2 L)}_{gen} = \underbrace{q_R (2\pi R L)}_{output} + \underbrace{\frac{d}{dt}\left[\int_0^R (T-T_\infty)\rho C_p\, 2\pi L r\, dr\right]}_{accum}$$

$$\text{or} \quad g^+ R^2 = -k\left.\frac{dT}{dr}\right|_R (2R) + \frac{d}{dt}\left[2\rho C_p R^2 \int_0^1 (T-T_\infty)\frac{r}{R}\frac{dr}{R}\right]$$

$$\text{or} \quad \frac{g^+}{2\rho C_p} = -\frac{k}{\rho C_p R}\left.\frac{dT}{dr}\right|_R + \frac{d}{dt}\left[\int_0^1 (T-T_\infty)\frac{r}{R}\frac{dr}{R}\right] \qquad (1)$$

Assume $T(r,t) - T_\infty = \theta(r,t) = Y(r)\,\Gamma(t)$ (2)

where $Y(r)$ is the steady state solution $\Rightarrow \Gamma(\infty) = 1$ and $\Gamma(0) = 0$

Conditions: $r=0 \quad \frac{\partial T}{\partial r} = 0$ (symmetry)

at Steady State (3)

$r = R \quad -k\left.\frac{dT}{dr}\right|_R (2\pi R L) = g^+(\pi R^2)L$ no accum

and $r=R$, $T = T_{ss}$ ($> T_\infty$ when h finite)

i.e $\theta(r,\infty) = Y(r) = a + br + cr^2$ (4)

$$\frac{dY}{dr} = b + 2cr$$

$r=0 \quad \frac{dY}{dr} = 0 \Rightarrow b = 0$

$r=R \quad \frac{dY}{dr} = 2cR = -\frac{g^+R}{2k} \Rightarrow c = -\frac{g^+}{4k}$

at $r = R$ $\quad T_{ss} - T_\alpha = a - \frac{g^+ R^2}{4k}$

$$\therefore \quad Y(r) = \theta(r, \alpha) = (T_{ss} - T_\alpha) + \frac{g^+ R^2}{4k} - \frac{g^+ r^2}{4k}$$

Using the boundary condition (at steady state)

$$-k \left.\frac{dT}{dr}\right|_R (2\pi RL) = h\,(2\pi RL)(T_{ss} - T_\alpha) = g^+ (\pi R^2 L)$$

$$T_{ss} - T_\alpha = \frac{g^+ R}{2h}$$

$$\therefore \quad Y(r) = \frac{g^+ R}{2h} + \frac{g^+ R^2}{4k}\left(1 - \frac{r^2}{R^2}\right)$$

$$\therefore \quad T(r,t) - T_\alpha = \frac{g^+ R^2}{4k}\left\{1 - \frac{r^2}{R^2} + \frac{\cancel{4}^{2}k}{\cancel{2}hR}\right\}\Gamma(t) \qquad (5)$$

$$\text{or} \quad T(r,t) - T_\alpha = \beta\left(1 - \frac{r^2}{R^2} + \frac{2}{Bi}\right)\Gamma(t)$$

where $\beta = \frac{g^+ R^2}{4k}$ and $Bi = \frac{hR}{k}$

If $\eta = r/R$, the integral energy equation is

$$\frac{g^+}{2\rho c_p} = -\frac{\alpha}{R^2}\left.\frac{dT}{d\eta}\right|_{\eta=1} + \frac{d}{dt}\left[\int_0^1 \beta\left(1 - \eta^2 + \frac{2}{Bi}\right)\Gamma\,\eta\,d\eta\right]$$

$$\frac{g^+}{2\rho c_p} = +\frac{\alpha}{R^2}2\beta\Gamma + \frac{d}{dt}\left[\beta\Gamma\int_0^1\left(1 - \eta^2 + \frac{2}{Bi}\right)\eta\,d\eta\right]$$

$$\frac{g^+}{2\rho C_p}\cdot\frac{1}{\beta} = \frac{\cancel{g^+}}{\cancel{2}\rho C_p}\,\frac{\overset{2}{\cancel{4}}k}{\cancel{g^+}R^2} = \frac{2\alpha}{R^2} = \frac{2\alpha}{R^2}\Gamma + \frac{d}{dt}\left[\Gamma\left(\frac{1}{4}+\frac{1}{Bi}\right)\right]$$

$$\left(\frac{1}{4}+\frac{1}{Bi}\right)\frac{d\Gamma}{dt} + \frac{2\alpha}{R^2}(\Gamma-1) = 0 \qquad (6)$$

subject to $t=0$, $\Gamma=0$

Solve

$$\Gamma = 1 - \exp\left[-\frac{2\alpha}{R^2}\left(\frac{4Bi}{4+Bi}\right)t\right] \qquad (7)$$

Subst. (7) in (5)

$$T(r,t) - T_\alpha = \frac{g^+R^2}{4k}\left\{1-\frac{r^2}{R^2}+\frac{2}{Bi}\right\}\left(1-\exp\left[-\frac{2\alpha}{R^2}\cdot\frac{4Bi}{4+Bi}\cdot t\right]\right)$$

(8)

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