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ENCH 501: Transport Processes Quiz #7 **November 27, 2007**

Time Allowed: 45 mins. **Name:**

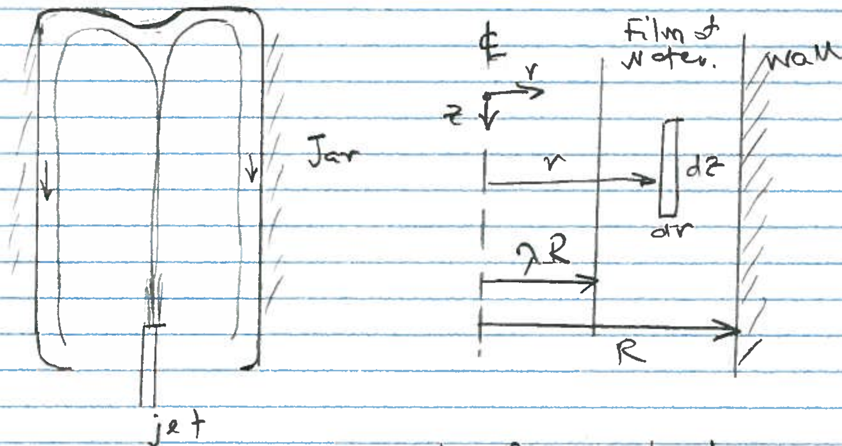
Many glass or plastic bottles and cans are reused without crushing and re-manufacture. This is very good for the environment and it saves costs. After collection, these containers must be washed and thoroughly rinsed out with distilled, de-ionized water before they can be filled with items for human consumption. Cleaning usually involve jets of liquid passed into the inside of the container which is turned upside down. One of the reasons for indenting inwards the bottoms of such containers is to deflect the upwards directed liquid jets to the side so that the liquid runs down on the inside walls. (Others suspect that the indentation is to reduce the volume of the content while giving the appearance that a container is bigger than it actually is!) The liquids are normally passed through a manifold and divided into many lines so that many containers can be washed at once. Any solid particles which lodge in a line would cause the flow rate into that jet to be reduced. Cleaning in that line will then be inadequate and the containers must be rejected as part of quality control.

It is suspected that the jet flow into one manifold line is reduced below required. The container, a jar, is 9 cm inside diameter and the side wall is 17 cm long. The diameter of the mouth is large at 6.5 cm, hence in washing and rinsing, liquids run out easily out of the inverted jar. You observed that a small bubble riding on the free surface of the layer (flowing down the side) traveled through a vertical distance of 10 cm in 2.16ms, and you can assume that the liquid being injected is pure water at 87.8°C, and the thickness of the layer is uniform around the wall of the jar.

a) Assume that the liquid film is thin and estimate the volume rate of injection of water into the jar?
b) Estimate the Reynolds number for flow down the jar wall. Explain your steps.
b) If you do not assume that the film is thin, estimate the volume rate of water flowing down the wall. Are your results the same? Discuss.

Data: Properties of water at 87.8°C
$\rho = 966.7\ kg/m^3$; $\mu = 0.327$ mPa s; k = 0.675 W/m K; and acceleration of gravity, $g = 9.81\ m/s^2$

ENCH 501 Quiz 7 Solution Nov. 27, 2007



Consider a differential element — a ring, within the film of water running down the wall.

A force balance on the element yeilds

$$-\frac{d}{dr}(r\tau_{rz}) + r\rho g = 0$$ Eq. 6.12 Notes

Given a Newtonian fluid, $\tau = -\mu \frac{du}{dy}$

and b.c. $r = R \quad u = 0$

$r = \lambda R \quad \tau = 0, \quad \frac{du}{dr} = 0$

one obtains the solution

$$u = \frac{\rho g R^2}{4\mu}\left\{1 - \frac{r^2}{R^2} + 2\lambda^2 \ln\left(\frac{r}{R}\right)\right\}$$ Eq. 6.17 Notes

The volume rate is given by

$$Q = 2\pi\int_{\lambda R}^{R} u\, r\, dr = \frac{\pi R^4 \rho g}{8\mu}\left\{1 - 4\lambda^2 + \lambda^4(3 - 4\ln\lambda)\right\}$$ Eq. 6.19 Notes.

[For thin films, $Q = 2\pi R\left(\frac{\rho g \delta^3}{3\mu}\right)$ – Eq. 6.22.]

From observation, the bubble at $r = \lambda R$ travelled through 10 cm in 2.16 ms, or $u \approx \frac{0.1}{2.16(10^{-3})} = 46.296\ m/s$

Substitute this into eq. 6.17

$$46.296 = \frac{(966.7)(9.81)\left[(4.5)(10^{-2})\right]^2}{4(3.27)(10^{-4})}\left\{1 - \lambda^2 + 2\lambda^2 \ln\lambda\right\}$$

This gives $\lambda = 0.96$

(c) Substitute this λ value and other data in equation 6.19

$$Q = \frac{\pi (4.5)^4 (10^{-2})^4 \, 966.7 (9.81)}{8 (3.27)(10^{-4})} \cdot f(\lambda)$$

$$f(\lambda) = 1 - 4\lambda^2 + \lambda^4 (3 - 4 \ln \lambda) = 3.2776 (10^{-4})$$

$$\therefore \; Q = 0.015307 \; m^3/s$$

(a) $\delta = (1-\lambda) R = (0.04)(4.5)_{cm} = 1.8_{mm}$

Using eq. 6.22

$$\therefore \; Q_1 = \frac{2\pi (4.5)(10^{-2})(966.7)(9.81)(1.8)^3(10^{-3})^3}{3 (3.27)(10^{-4})}$$

$$= 0.01594 \; m^3/s$$

The two results are close. →

(b) Reynolds # $= \dfrac{\Gamma \bar{u} \rho}{\mu}$; Γ = hydraulic diam $= \dfrac{4 \times \text{X-section area}}{\text{Wetted Perimeter}}$

$$\Gamma = \frac{4 \pi R^2 (1 - \lambda^2)}{2 \pi R}$$

$$= 2 (4.5)(10^{-2})(1 - 0.96^2)$$

$$= 0.0070560 \; m$$

$$\bar{u} = Q/A = \frac{0.01594}{\pi(4.5)^2(10^{-2})^2(1-0.96^2)}$$

$$= 31.9593 \text{ m/s} \quad \left(= \tfrac{2}{3} u|_{r=\lambda R} \right)$$

$$\therefore \quad Re = \frac{0.007056(31.9593)(946.7)}{3.27(10^{-4})}$$

$$= 6.666(10^5)$$

This means flow should be turbulent.

Assumptions made for solving the problem may not be valid.