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ENCH 501: Transport Processes Quiz #7 **November 23, 2004**

Time Allowed: 30 mins. **Name:**

"Water walls" are now becoming common at airports. These are vertical or inclined walls over which water flows down and splashes into a pool. Some of the water evaporate and maintain the humidity of the space. The arrangement is esthetically pleasing and the sound of flowing water is calming on the nerves. You have been asked to help design one of these.

The wall is to be 2m wide and 3m along the flow direction, and inclined at an angle of 15° to the vertical. The condition you were given is that the wall shear stress must have a minimum value of 15.13 Pa such that slime and other dirt which would ordinarily grow or accumulate in the water cannot form attachments at the wall.

(a) Estimate the ***minimum volumetric supply rate*** of water to the top of the wall, and the ***thickness*** of the water film. Assume the flow is laminar.

(b) Check the validity of the assumption that the flow is laminar.

Data: Properties of water $\mu = 1.7$ mPa s ; $\rho = 998$ kg/m^3

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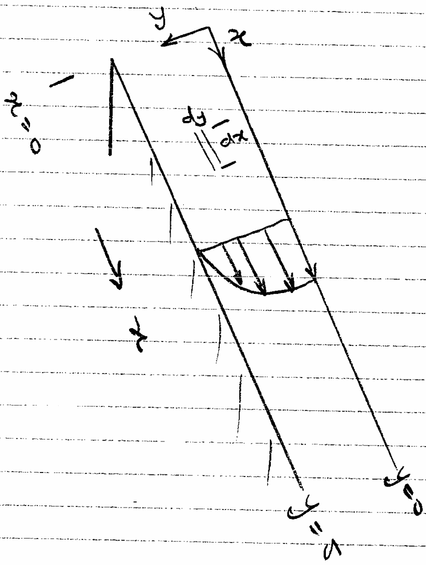

This problem was derived in the Notes.

Force balance over a differential element dx by dy yeilds

$$\frac{d\tau_{xy}}{dy} = +\rho g \cos\beta \qquad (1)$$

where $\tau_{xy} = -\mu \frac{du}{dy}$

On substitution

$$\frac{d^2u}{dy^2} = -\frac{\rho g \cos\beta}{\mu} \qquad \text{eq. 6.2 NOTES}$$

This is subject to the b.c.

$y = 0 \quad \frac{du}{dy} = 0$ (no shear)

$y = \delta \quad u = 0$ (no slip)

The velocity profile is hence

$$u = U_{max}\left(1 - y^2/\delta^2\right); \quad U_{max} = \frac{\rho g \delta^2 \cos\beta}{2\mu} \qquad (2)$$

(a) Integration of eq. (1) yeilds

$$\tau_{xy} = \rho g \cos\beta\, y + C_0 \quad ; C_0 = \text{constant}$$

with b.c. $y = 0,\ \tau_{xy} = 0 \Rightarrow C_0 = 0$

$$\therefore \tau_w = \tau_{xy}\big|_{y=\delta} = \rho g \cos\beta\, \delta \qquad (3)$$

Given $\tau_w = 15.13 \text{ Pa} = 998(9.81)\cos 15^\circ \cdot \delta$

$$\delta = 1.6(10^{-3}) \text{ m}$$

Using equations 6.6 and 6.7 (Notes)

$$Q = \frac{2}{3} U_{max} \cdot W\delta = \bar{u} \cdot (\delta \cdot W) = \frac{\rho g \delta^3 \cos\beta}{3\mu} \cdot W \qquad (4)$$

$$\therefore \quad Q = \frac{998\,(9.81)(1.6)^3(10^{-9})\cos 15^\circ \cdot 2}{3\,(1.7)(10^{-3})} \quad m^3/s$$

$$= 0.01519 \quad m^3/s$$

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(b) Since laminar flow is assumed,

$$\bar{u} = 2/3\, U_{max} = \frac{Q}{W \cdot \delta} = 4.7469 \ m/s$$

(high!)

$$Re = \frac{\Gamma \bar{u} \rho}{\mu} \quad \text{where } \Gamma \text{ is hydraulic diameter}$$

$$= 4\delta$$

$$Re = \frac{4\delta \bar{u} \rho}{\mu} = \frac{4\,(1.6)(10^{-3})(4.7469)(998)}{1.7\,(10^{-3})}$$

$$= 1.7835\,(10^4)$$

The flow is turbulent. Hence the assumption made for evaluating part (a) is invalid.

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