The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501 : Mathematical Methods in Chemical Engineering
Quiz #7

Time Allowed: 50 min. **December 4, 2001** *AJ*

Current trends at many airports is to install 'wet walls'. These are miniature waterfalls inside the terminals. Such walls perform many functions. First, it is pleasing to tired eyes and ears - the aesthetics. Splashing water calms the nerves. As with a humidification tower, the wall provides moisture for the space. Some heat will be removed from the water as a small fraction of it evaporates. The water will also absorb from the air chemicals which smell and are emitted by the people, cleaning fluids or the carpets.

One of the chemicals often emitted by the carpets is terephthalic acid, a mild irritant which has a low solubility in water. This is absorbed by the water which is assume non-volatile at given conditions.

Derive the equations (with the boundary conditions), which when solved, allow a determination of the ***steady state rate of uptake*** of the acid by the water.

Information and Hints:

1. Assume the wall is inclined at angle β to the horizon, the length of the wall is L (along the incline) and the width is W, and the volume rate of water supply to the wall is Q. The acid does not react with water and its concentrations in air and in the water are very low.

2. State all your assumptions and indicate how you will obtain parameters such as the concentration or mole fraction of the acid at the water surface (x_{A0}), given the mole fraction of terephthalic acid in air as y_A and atmospheric pressure as P.

3. Even though the geometry and operations are different, the logic of setting the problem up is similar to that of heating up a fluid in laminar flow in a pipe.

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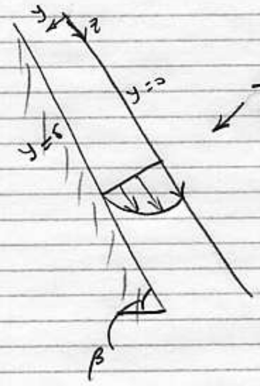


Set up the problem by taking a differential element in the film — perform a force balance to obtain the velocity distribution and a mass balance on the acid (TA).

Given a flow rate of Q and an average velocity $\bar{u}$, the film thickness δ is given from

$$Q = \bar{u} W \delta \quad \text{or} \quad \frac{Q}{W} = \bar{u}\delta$$

$$\frac{Q}{W} = \frac{\rho g \delta^3 \sin\beta}{3\mu}$$; same as eq. 6.7 but note cos is changed to sin because β is to the horizon for this problem.

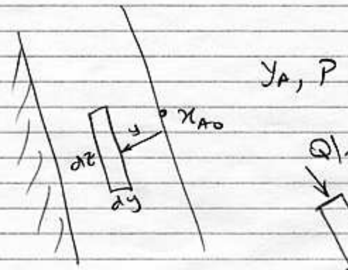
$$\therefore \bar{u} = \frac{Q/W}{\delta} = \frac{\rho g \delta^2 \sin\beta}{3\mu}$$

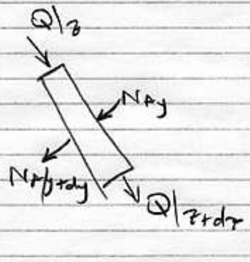
$$= \frac{2}{3} U_{max}$$ from eq. 6.6 and 6.5 Notes.

From equation 6.4

$$u = U_{max}\left(1 - \left(\frac{y}{\delta}\right)^2\right)$$ →

Now set up material balance.





$x_{Ao} = \frac{y_A P}{H}$ where H is Henry's constant.

This is the surface concentration

On differential element, perform a material balance on A

Input – diffusion $N_{Ay}(W\,dz)$

– convection $(W\,dy)\,u C_A$

Output – diffusion $N_{Ay}(W\,dz) + \frac{dN_{Ay}}{dy}\,dy\,W\,dz$

– convection $W\,dy\,u C_A + W\,dy\,\frac{d}{dz}(u C_A)\,dz$

At steady state, there is no accumulation at any position. Also no generation, because no reaction:

$$\frac{dN_{Ay}}{dy} + \frac{d(uC_A)}{dz} = 0$$ where $u = f(r)$ not z

Also $N_{Ay} = -c D_{AB}\frac{dx_A}{dy} + x_A(N_A + N_B)$ where B is water

x_A given $<< 1$ ∴ neglect 2nd term r.h.s.

$c \approx$ moles water/volume mixture = const.

Substitute

$$-D_{AB}\,c\,\frac{d^2 x_A}{dy^2} + u\frac{dC_A}{dz} = 0$$

or $$D_{AB} \frac{\partial^2 C_A}{\partial y^2} = u \frac{\partial C_A}{\partial z}$$ is the equation that gives $C_A(y,z)$

subject to $z=0$ $\quad C_A = 0$

$y=0$ $\quad C_A = C x_{A0}$ (solubility)

$y=\delta$ $\quad \frac{dC_A}{dz} = 0$ (impervious wall)

Once $C_A(y,z)$ is known, the total Terephthalic acid taken up can be determined by one of 2 ways —

① integrate for C_A over y at $z=L$, i.e.

$$\overline{C_A} = \frac{1}{\bar{u}\,\delta}\int_0^\delta C_A\, u\, dy$$ — note this is bulk mean concentration!

Total TA uptake rate = $Q\,\bar{C}_A$

② Integrate flux at $y=0$ from $z=0$ to L

Total T_A uptake = $$W\int_0^L N_A\Big|_{y=0} dz$$

where $$N_A\Big|_{y=0} = -D_{AB}\frac{dC_A}{dy}\Big|_{y=0}$$

and $W dz$ is differential area.