**The University of Calgary**
**Department of Chemical and Petroleum Enguneering**

**ENCH 501: Mathematical Methods in Chemical Engineering**
**Quiz #7**

**Student Name**________________________________________

**Time Allowed: 50 minutes** **December 5, 2000**

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Sodium bicarbonate (baking soda) is made by dissolving carbon dioxide in a saturated solution of sodium carbonate (soda ash). Saturated solution at 40°C is supplied to the top of a wetted-wall column and flows on the inside surface. Compressed carbon dioxide is sent in at the bottom. This operation is to be modelled in the laboratory by flowing the solution over a vertical flat plate.

The liquid flows downwards with a thickness $\delta$ over the plate. As the $CO_2$ dissolves and diffuses into the solution, it is consumed by a first order reaction (the rate per unit volume, $r_B = -k_1C_B$; $B=CO_2$). Given that the liquid flow is laminar (no ripples) and the solution initially did not contain dissolved $CO_2$ or bicarbonate (at z=0),

(a) derive the equation and the boundary conditions for the concentration of carbon dioxide as a funtion of position in the film of height L at steady state;

(b) obtain an expression for the amount of bicarbonate in the solution at z=L. For this, assume that the concentration of $CO_2$ in the solution is low.

Assume that the solvent (water) is non-volatile, the total number of moles per unit volume in the solution is essentially constant, and the feed gas, supplied at a high rate, contains 90% $CO_2$ and the remainder is insoluble. State all other assumptions and identify the parameters you need to know to obtain quantitative results for (b).

$Na_2CO_3 + CO_2 + H_2O \rightarrow 2NaHCO_3$

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$CO_2$ + Inert

← $CO_2 \equiv B$

$y_B = 0.9$

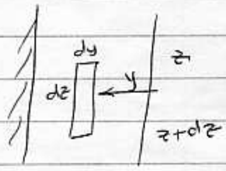
① The velocity distribution in the film is given as: equation 6.4 (Notes)

$$u = \frac{\rho g \delta^2}{2\mu}\left[1 - \left(\frac{y}{\delta}\right)^2\right]$$

② $CO_2$ first dissolves in the solution. Using Henry's law, $x_B\big|_{y=0} = \frac{y_B P}{H}$ ; $y_B = 0.9$ and P is the gas pressure. H is Henry's constant.

③ Identify $A \equiv$ solvent, $B \equiv CO_2$ and $C \equiv$ Bicarbonate.

Material balance on $CO_2$ on differential element - dy dz




Input - convection at z /unit depth

$u\,dy\,C_B\big|_z$

- diffusion at y

$dz\,N_B\big|_y$

Output - at z + dz → $u\,dy\,C_B\big|_{z+dz}$

- at y + dy → $dz\,N_B\big|_{y+dy}$

Generation term → $dy\,dz\,(-k_1 C_B)$

At steady state,

$$-k_1 C_B = \frac{d(u C_B)}{dz} + \frac{dN_B}{dy} \qquad \text{eq. (1)}$$

But $$N_B = -c\mathcal{D}_{AB}\frac{dx_B}{dy} + x_B(N_A + N_B + N_C) \qquad \text{eq. (2)}$$

Since both A and C are non-volatile, $N_A = N_C = 0$

$$N_B = \frac{-c\mathcal{D}_{AB}}{1-x_B}\frac{dx_B}{dy} \qquad \text{if } x_B \text{ is not small.} \qquad \text{eq. (3)}$$

The mass balance equation of $CO_2$ is hence

$$-k_1 \cancel{c} x_B = u\cancel{c}\frac{dx_B}{dz} - \cancel{c}\mathcal{D}_{AB}\frac{d}{dy}\left(\frac{1}{1-x_B}\frac{dx_B}{dy}\right)$$

or

$$\mathcal{D}_{AB}\frac{\partial}{\partial y}\left(\frac{1}{1-x_B}\frac{\partial x_B}{\partial y}\right) - u\frac{\partial x_B}{\partial z} - k_1 x_B = 0$$

where $u = \beta\left(1 - \frac{y^2}{\delta^2}\right)$ ; $\beta = \frac{\rho g \delta^2}{2\mu}$

or

$$\frac{\partial}{\partial y}\left(\frac{1}{1-x_B}\frac{\partial x_B}{\partial y}\right) - \frac{\beta}{\mathcal{D}_{AB}}\left(1 - \frac{y^2}{\delta^2}\right)\frac{\partial x_B}{\partial z} - \frac{k_1 x_B}{\mathcal{D}_{AB}} = 0 \qquad \text{eq. (4)}$$

This is a p.d.e. for $x_B(y, z)$ subject to the conditions:

$z = 0$ $\quad x_B = 0$

$y = 0$ $\quad x_B = x_{B0} = \frac{y_B P}{H} = \text{const}$ $\qquad$ eq. (5)

$y = \delta$ $\quad \frac{dx_B}{dy} = 0$ (impervious)

$\longrightarrow$

(b) If $x_B << 1$, eq. (3) becomes

$$N_B = -c\mathcal{D}_{AB}\frac{dx_B}{dy} \qquad \text{eq. (6)}$$

and eq. (4) becomes

$$\frac{\partial^2 x_B}{\partial y^2} - \frac{\beta}{\mathcal{D}_{AB}}\left(1-\frac{y^2}{\delta^2}\right)\frac{\partial x_B}{\partial z} - \frac{k_1}{\mathcal{D}_{AB}}x_B = 0 \qquad \text{eq. (7)}$$

with the same conditions as eq. (5).

Equation (7) is to be solved for $x_B(y,z)$ and the solution substituted into eq. (6) evaluated at $y=0$. Since 2 moles of bicarbonate are produced per mole $CO_2$ consumed, the total bicarbonate formed is

$$Q = 2\int_{z=0}^{L} N_B\big|_{y=0}\,dz \qquad \text{eq. (8)}$$

To solve eq. (7)

Let $x_B = Y(y)\,Z(z)$ ; $f(y) = +\frac{\beta}{\mathcal{D}_{AB}}\left(1-\frac{y^2}{\delta^2}\right)$

and $\alpha = k_1/\mathcal{D}_{AB}$

$$\frac{Y''}{Y} - f(y)\frac{Z'}{Z} - \alpha = 0$$

$$\frac{1}{f(y)}\left\{\frac{Y''}{Y}-\alpha\right\} = \frac{Z'}{Z} = -\psi^2 \text{, separation constant.}$$

Hence $Z = A e^{-\psi^2 z}$

and $Y'' - \left\{\alpha - \psi^2 f(y)\right\}Y = 0$