**ENCH 501** F2018 **Quiz #6** Name:________________________________________

December 4, 2018 Time Allowed: 45 minutes aJ

In deriving the problem of flow down an inclined wall with a free surface, it is usually assumed that the air is still or moving so slowly that the velocity gradients at both sides of the air-liquid boundary are very low. Thus, the shear stress at the interface is considered negligible. This boundary condition allows a determination of the velocity profile in the liquid film. If air is flowing at a fast rate over the air-liquid interface in the direction of liquid flow or in the opposite direction, the shear stress at the boundary would not vanish. The interface moves at higher (or lower) velocities than if the air is still over a falling film. Air flowing over oceans at high velocities cause waves to form at shorelines.

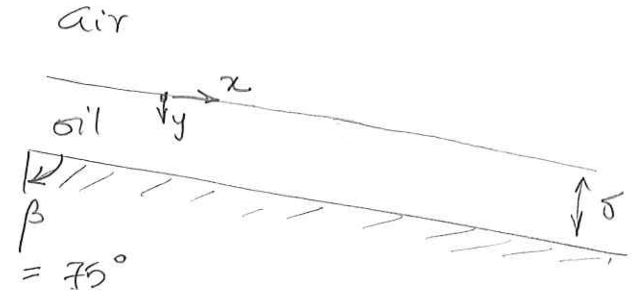
Consider an unused engine oil layer at 15°C flowing down a wall inclined at an angle of 75° to the vertical. The thickness of the layer is uniform along the length of the wall at 1.25 cm. No ripples or waves are present at the interface.

**a)** (4 pts) Estimate the velocity at the air-oil interface and the volumetric flow rate of the oil (per meter width of the wall) if the air is not moving over the interface. Derivation is not necessary.

**b)** (6 pts) If air is flowing along the same direction as the oil and a speck of dust at the interface is moving at 0.24 m/s, what is the volumetric rate of the flow of the oil per meter width of the wall? Compare the flow rates for parts a and b. Are the volumetric ratios the same as the ratio of surface velocities? Show your derivations.

**Data**: Properties of unused engine oil at 15°C are $\rho$ = 889 kg/m$^3$, $\mu$= 1.06 Pa.s; the acceleration of gravity is 9.81 m/s$^2$.

ENCH 501 Quiz 6 F18 Solution G. Ogie




for still air, this problem has been solved.

The velocity profile is – (equation 7.4)

$$u = \frac{\rho g \delta^2 \cos\beta}{2\mu}\left[1 - (y/\delta)^2\right]$$

(a) At the interface $y = 0$

$$\therefore \quad u = U_{max} = \frac{\rho g \delta^2 \cos\beta}{2\mu} = \frac{889(9.81)(1.25)^2(10^{-4})\cdot \cos 75}{2\,(1.06)}$$

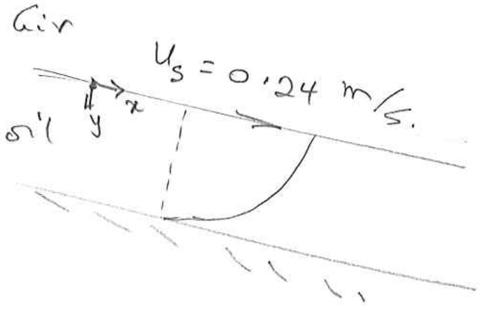
$$= 0.1664 \text{ m/s} \longrightarrow$$

The volume rate is given by (eq. 7.7)

$$Q = \frac{\rho g \delta^3 \cos\beta}{3\mu} = \frac{889(9.81)(0.0125)^3 \cos 75}{3\,(1.06)}$$

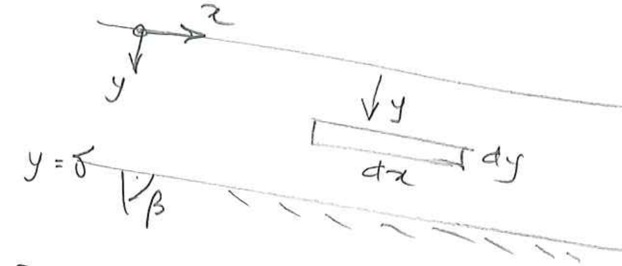
$$= 1.386\,(10^{-3})\ \text{m}^3 / \text{m width of wall} \longrightarrow$$

(b)



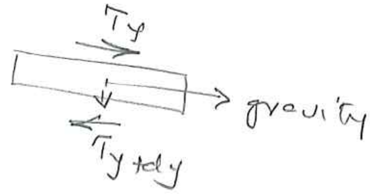

The air caused the liquid film to move faster.

Consider a differential element within the layer, $dx \times dy$




The net force = Rate of $\Delta$ momentum = 0

Resolving the forces




$$(\tau\, dx)_y - (\tau\, dx)\Big|_{y+dy} + dx\, dy\, \rho g \cos\beta = 0$$

with $\tau = -\mu \dfrac{du}{dy}$ for a Newtonian fluid

On substitution

$$\frac{d^2u}{dy^2} = -\frac{\rho g \cos\beta}{\mu} = \lambda = -\frac{(889)(9.81)\cos 75}{1.06}$$

$$= -2129.416 \ m/s^2$$

Integrate once

$$\frac{du}{dy} = \lambda y + C_1$$

Integrate a second time

$$u = \lambda \frac{y^2}{2} + C_1 y + C_2$$

The b.c.s are

$y = 0 \qquad u = U_s$

$y = \delta \qquad u = 0$

Substitute b.c. , $C_2 = U_s$

and $0 = \frac{\lambda}{2}\delta^2 + C_1\delta + U_s$

$$C_1 = -\frac{1}{\delta}\left(\frac{\lambda}{2}\delta^2 + U_s\right) = -\frac{\lambda}{2}\delta - \frac{U_s}{\delta}$$

$$\therefore \quad u = \frac{\lambda}{2}y^2 - \left(\frac{\lambda}{2}\delta + \frac{U_s}{\delta}\right)y + U_s$$

$$= a_0 y^2 - a_1 y + U_s$$

The volume rate is given by / per unit width

$$Q = \int_0^\delta u\,dy = \int_0^\delta (a_0 y^2 - a_1 y + U_s)\,dy$$

$$= \frac{a_0}{3}y^3 - \frac{a_1}{2}y^2 + U_s y \Big|_0^\delta$$

$$= \frac{a_0}{3}\delta^3 - \frac{a_1}{2}\delta^2 + U_s\delta$$

$$= \frac{\lambda}{6}\delta^3 - \frac{1}{2}\left(\frac{\lambda}{2}\delta + \frac{U_s}{\delta}\right)\delta^2 + U_s\delta$$

Substitute $\lambda = -2129.416$ $m/s^2$ ; $\delta = 0.0125\,m$ and $U_s = 0.24\,m/s$

$Q = 1.847(10^{-3})\ m^3/s$ or $1.332$ times for still air

and $\frac{u_{y=0}\ \text{flow}}{u_{y=0}\ \text{still}} = 1.4423$ $\therefore$ Ratios different. $\longrightarrow$

Extra

$\psi$

□ Check if assumptions of laminar flow without ripples are satisfied

$$D_H = 4\delta \quad ; \quad \bar{u} = \frac{2}{3} u_{max}$$

$$Re = \frac{D_H \bar{u} \rho}{\mu} = \frac{4(0.0125)\frac{2}{3}(0.1664)(889)}{1.06}$$

$$= 4.65$$

$\therefore$ laminar w/o ripples ✓

for still air.

When air is moving the interface as well as gravity

$$\bar{u} = \frac{1}{\delta}\int_0^\delta u\,dy \neq \frac{2}{3} u_{max}$$

On substitution

$$\bar{u} = Q/\delta = \frac{1.847(10^{-3})}{0.0125} = 0.1478 \text{ m/s}$$

and $$Re = \frac{4(0.0125)(0.1478)(889)}{1.06} = 6.196$$

$\therefore$ laminar, maybe occasional ripple

□ The shear stress at the interface is obtained

From $$\tau\big|_{y=0} = -\mu \frac{du}{dy}\bigg|_{y=0} = -\mu(-a_1) =$$

$$\mu\left(\frac{\gamma}{2}\delta + \frac{u_s}{\delta}\right) = 6.2446 \text{ Pa}$$

$\longrightarrow$