ENCH 501 F2017 **Quiz #6** Name:____________________________

November 28, 2017 Time Allowed: 45 minutes aJ

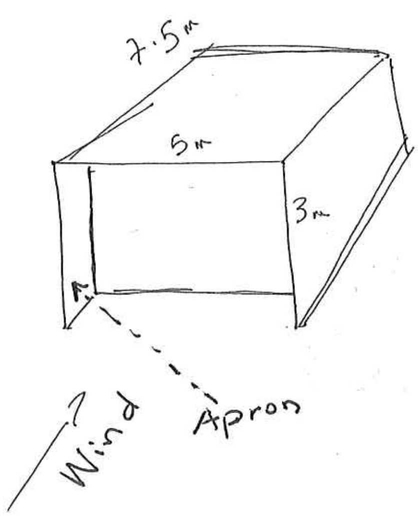
The rates of heat loss from buildings depend on the wind to which they are exposed. Houses isolated on hill tops or cabins for research and exploration in remote places such as the polar regions require more energy supply to maintain comfortable temperatures for people than shelters in cities and protected sites.

Consider a cabin in the Antarctica. It is an uninsulated, rectangular, steel structure that is 7 x 5m base and 3m tall. The temperature on the inside is maintained at a constant 18°C using a space heater and a fan. The circulation within the cabin is considered strong such that the walls ate maintained at the same temperature as the air inside the space. The outside air is at -50°C and a steady wind at 2.16 km/hr is blowing over the cabin. The bottom of the cabin is on a pad on the ground. One of the researchers on site remarked that the amount of energy required to maintain the inside temperature seem to depend on the direction of the wind. You are requested to check the validity of the remark.

a) (10 pts) Estimate the rates of heat loss from the walls parallel to the direction of air flow for two wind directions – normal to the 5m and to the 7m sides. Show your steps. Comment on your results.

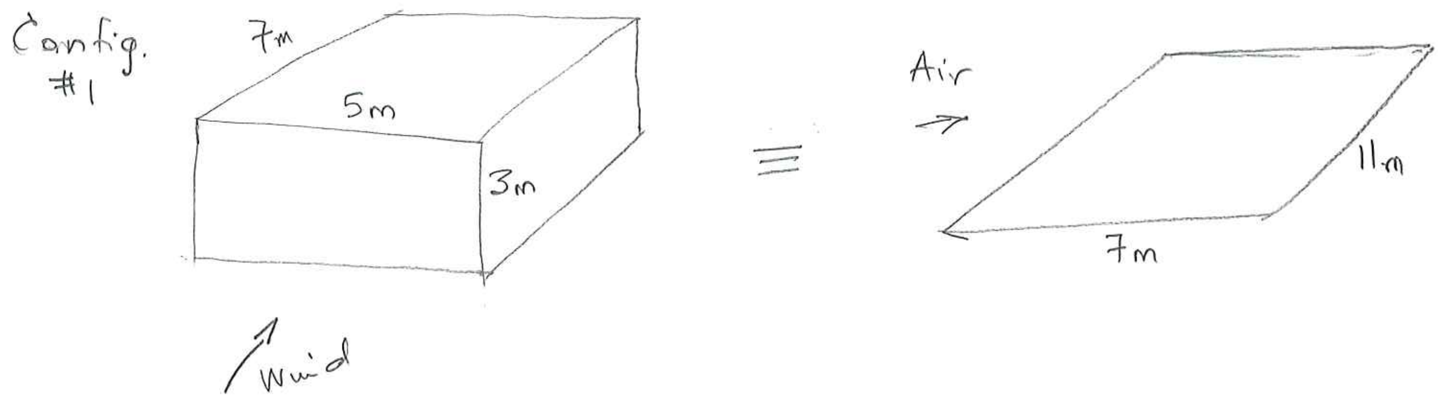
b) (Bonus 2 pts) If an 0.5m wide and rigid apron is attached to the side facing the wind for flow normal to the 5m side, as per the sketch attached, how will your results change?

Data: Properties of air in the boundary layer are assumed constant at an effective temperature of -23°C – density = 1.4119 kg/m^3; dynamic viscosity = 1.606 (10^{-2}) mPa s; kinematic viscosity = 1.137 (10^{-5}) m^2/s; specific heat = 1.003 kJ/kg K; thermal conductivity = 0.0223 W/m K; Prandtl number = 0.724



ENCH 501 Quiz #6 Solution F17 G. Achari

Config. #1



The two sides and the top of the cabin is equivalent to a plate 11m wide and 7m length from the leading edge.
The first step is to show that the boundary layer is laminar.

$$Re_L = \frac{U_\infty L}{\nu} \; ; \; U_\infty = 2.16 \frac{km}{hr} = 0.6 \; m/s$$

$$L = 7m \quad \text{and} \quad \nu = 1.137(10^{-5}) \; m^2/s$$

$$= 3.694(10^5) < 5(10^5) \; \therefore \text{laminar.}$$

max. b.l. thickness, δ, is given by

$$\delta_L = 4.64\sqrt{\frac{\nu L}{U_\infty}} = 4.64\left[\frac{1.137(10^{-5}) \cdot 7}{0.6}\right]^{\frac{1}{2}}$$

$$= 5.344(10^{-2}) \; m$$

Thermal b.l. thickness

$$\delta_t\Big|_L = \delta_L \cdot \xi = \frac{1}{1.026} Pr^{-\frac{1}{3}} \delta_L = \frac{1}{1.026}(0.724)^{-\frac{1}{3}} \delta_L$$

$$= 1.08543 \; \delta_L = 5.801(10^{-2}) m \quad \text{(check later)}$$

For this case, $\delta_t > \delta$ – hence analysis may * not be valid as the assumption that $\xi < 1$ was in the derivation. ξ is > 1 in this case.

II Continuing with the calculations with previous derivation,

$$h_x = 0.332\, k\, Pr^{\frac{1}{3}} \left(\frac{U_\infty}{\nu x} \right)^{\frac{1}{2}} = 0.332\, \frac{k}{x}\, Pr^{\frac{1}{3}} \left(\frac{U_\infty x}{\nu} \right)^{\frac{1}{2}}$$

$$h_L = 0.332\, \frac{k}{L} \left(Pr^{\frac{1}{3}} \right) Re_L^{\frac{1}{2}} = 0.332 \left(\frac{0.0223}{7} \right) (0.724)^{\frac{1}{3}} \cdot \left[3.6939 (10^5) \right]^{\frac{1}{2}}$$

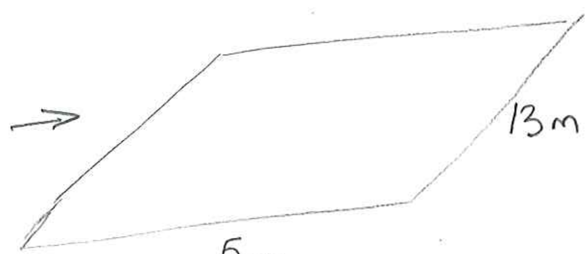
$$= 0.5772 \quad W/m^2K$$

The average heat transfer coefficient $0 \le x \le L$

$$\bar{h}_L = 2 h_L = 1.1544 \quad W/m^2K$$

$$\therefore \quad Q = \bar{h}_L A (T_s - T_\infty) = 6.0446 (10^3) \; W \longrightarrow$$

Config #2



For this configuration, L equals 5 m.

$$Re_L = \frac{0.6 \times 5}{1.137 (10^{-5})} = 2.6385 (10^5), \text{ laminar}$$

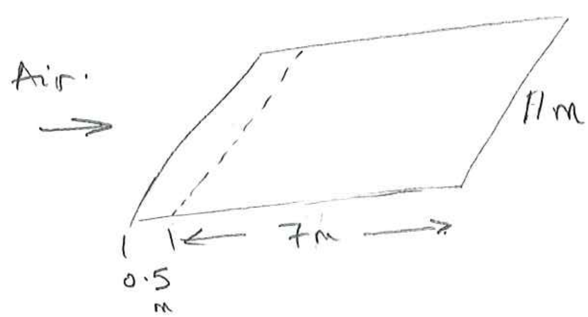
$$h_L = 0.332 \left(\frac{0.0223}{5} \right) (0.724)^{\frac{1}{3}} \left[2.6385 (10^5) \right]^{\frac{1}{2}}$$

$$= 0.683 \quad W/m^2K$$

$$\bar{h}_L = 1.366 \; W/m^2K \quad \text{and} \quad Q = 6.0375 (10^3) \; W \longrightarrow$$

The heat loss is marginally smaller for configuration 2 – for which h_L is higher but the area is smaller.

Bonus Question



For this configuration, there is an unheated leading edge.

The analysis for this gives the heat transfer coefficient as

$$h_x = 0.332\, k\, P_r^{\frac{1}{3}} \left[\frac{U_\infty}{\nu x}\right]^{\frac{1}{2}} \left[1 - \left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]^{-\frac{1}{3}}$$

where $x_o = 0.5\text{m}$

The average $\bar{h}_L$ is given by

$$\bar{h}_L = \frac{1}{L - x_o} \int_{x_o}^{L} h_x \, dx = \frac{\beta}{L - x_o} \int_{x_o}^{L} x^{-\frac{1}{2}} \left[1 - \left(\frac{x_o}{x}\right)^{\frac{3}{4}}\right]^{-\frac{1}{3}} dx$$

where $\beta = 0.332\, k\, P_r^{\frac{1}{3}} \left(\frac{U_\infty}{\nu}\right)^{\frac{1}{2}}$, $x_o = 0.5\text{m}$ and $L = 7.5\text{m}$.

$$\bar{h}_L = \frac{\beta}{L - x_o} \int_{x_o}^{L} \frac{dx}{x^{\frac{1}{2}}\left[\left(1 - \varepsilon x^{-3/4}\right)\right]^{\frac{1}{3}}}$$

This can be solved numerically using Matlab

$$\bar{h}_L = 1.08765 \ \text{W/m}^2\text{K}.$$

$$Q = \bar{h}_L (A)(\Delta T)$$
$$= 1.08765(77)(68) = 5.695(10^3)\ W$$

The rate of heat loss is lower with the apron than for the corresponding configurations without the apron.

[] Check both parts (a) and (b) for validity

Part (a)
Eq. 6.109 (Notes) is the result from substituting equations for velocity and temperature into the integral equation

i.e. $\theta_\alpha U_\alpha \frac{d}{dx}\left[\delta\left(\frac{3}{20}\xi^2 - \frac{3}{280}\xi^4\right)\right] = \frac{3}{2}\frac{\alpha\theta_\alpha}{\delta\xi}$

when $\xi < 1$, the second term in the brackets of l.h.s. is neglected.

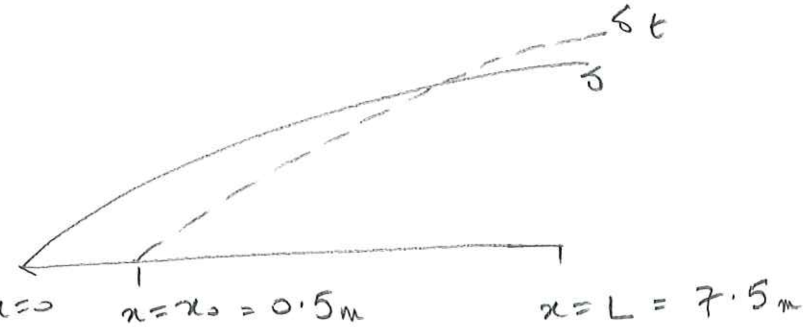
For configuration # 1 without apron

$\xi = 1.08543 \quad \therefore \quad \xi^2 = 1.1782$ and $\xi^4 = 1.3881$

The first term is ~ 11.88 times > the second term.

Dropping the second terms creates an error but the analysis appears valid.

For Part (b), the check is whether $\delta_t > \delta$ in $x_o \leq x \leq L$



at $x=L$, $\delta_L = 4.64\sqrt{\dfrac{1.137(10^{-5})\times 7.5}{0.6}}$

$= 5.5316\,(10^{-2})\,m$

At $x=L$

$$\xi = \frac{\delta_t}{\delta} = \frac{1}{1.026} P_r^{-1/3}\left[1-\left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$$

$$= \frac{1}{1.026}(0.724)^{-1/3}\left[1-\left(\frac{0.5}{7.5}\right)^{3/4}\right]^{1/3}$$

$$= 1.0357$$

This means δ_t crosses δ and it is larger downstream. Same considerations as for part (a) apply.

If the apron had been 1.5 m (unheated leading edge), then $L = 8.5\,m$ and $Re_L = 4.485\,(10^5)$

At L, $\xi = \frac{1}{1.026}(0.724)^{-1/3}\left[1-\left(\frac{1.5}{8.5}\right)^{3/4}\right]^{1/3} = 0.9763$

Hence, for $0 \le x \le 8.5\,m$, $\delta_t < \delta$

For this situation, $\bar{h}_L = 0.805\ W/m^2K$

and $Q = 4.215\,(10^3)\ W$

$\Rightarrow$ The wider the apron, the less heat is lost!