ENCH 501 Transport Phenomena University of Calgary Name________________________________

<u>**Quiz #6**</u> / **Time Allowed**: 50 minutes Use of cheat sheet allowed. December 1, 2015 aJ

Alternate energy has recently been prominently in the news. Solar power generation is one of prime schemes. The method involves exposing photo-voltaic collector panels to the sun and storing some of the electricity produced in batteries. The technique works where there are extended periods of intense sunlight, and these often happen to be arid areas such as deserts. At such locations, there are often strong winds that are laden with fine dust particles. These would settle on the collector as fouling and considerably reduce the efficiency of the power generation. Solar panels in urban areas are also fouled by dust, smog, industrial chemicals and microorganisms that are transported by the wind. The rate of fouling of a panel is to be analyzed.

A flat panel, width W and length L, has air flowing at a steady free stream velocity of U_α over the surface in a direction normal to the width. The air is laden with very fine dust particles at a concentration of $\psi_{A\alpha}$ (number of particles per unit volume of suspension). Both a velocity boundary layer δ and a concentration boundary layer δ_m are formed, with $\delta_m < \delta$. The fine particles diffuse towards the panel surface and, as soon as they touch the surface, they stick and are transformed into part of the scale. The concentration of particles at the surface is therefore zero, i.e. $z = 0$, $\psi_A = 0$. The diffusivity of the particles (A) through air (B) is D_{AB}. The total concentration of A and B (number of dust particles and number of molecules in air per unit volume) is effectively constant at ψ. Thus $\psi_A = \psi y_A$ where y_A is mole fraction of dust particles in air.

a) Obtain an expression for the concentration profile of the dust particles in the boundary layer on the plate. Use the **integral method** and show your steps.

b) Derive an expression for the rate at which dust particles would deposit on the panel in the first length ℓ from the leading edge?

Note: 1 mole = 6.023 (10^{23}) molecules or particles.

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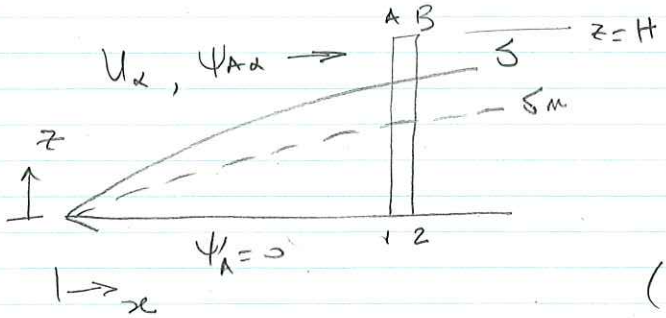


The number of dust particles is analogous to number of moles.
(1 mole = $6.023(10^{23})$ particles)

(a)

Material balance is done over the control volume 1AB2.

The input terms, per width w, are

(A1) $w\int_0^H \psi_A u\, dy$

(12) from $N_A = -\psi \mathcal{D}_{AB} \dfrac{dy_A}{dz} + y_A(N_A + N_B)$

at $z=0$, $N_B = 0$ $\therefore N_A\Big|_{z=0} = \dfrac{-\psi \mathcal{D}_{AB}}{1-y_A}\dfrac{dy_A}{dz}\Big|_{z=0}$

$\therefore$ Input at 12 is

$$-\frac{\psi D_{AB}}{1-y_{A0}}\frac{dy_A}{dz}\Big|_{z=0} \cdot W\,dx$$

Note ① $y_{A0} = 0$

② $\dfrac{dy_A}{dz}$ is +ve, $\therefore$ the flux is towards the wall, i.e. negative input

The output terms are

(B2) $w\int_0^H \psi_A u\, dz + \dfrac{d}{dx}\left[w\int_0^H \psi_A u\, dz\right]$

(AB) $$W\left[-\frac{d}{dx}\left[\int_0^H u\,dz\right]dx\right]\cdot \psi_{A\alpha}$$

The balance is hence

$$W\int_0^H \psi_A u\,dz - \psi D_{AB}\frac{dy_A}{dz}\bigg|_{z=0}\cdot W\,dx =$$

$$W\int_0^H \psi_A u\,dz + \frac{d}{dx}\left[W\int_0^H \psi_A u\,dz\right]dx$$

$$- \frac{d}{dx}\left[\int_0^H \psi_{A\alpha} u\,dz\right]\cdot W\,dx.$$

or

$$-\psi D_{AB}\frac{dy_A}{dz}\bigg|_{z=0} = -\frac{d}{dx}\left[\int_0^H (\psi_{A\alpha}-\psi_A)u\,dz\right]$$

$$\psi D_{AB}\frac{dy_A}{dz}\bigg|_{z=0} = \frac{d}{dx}\left[\psi\int_0^{\delta_m}(y_{A\alpha}-y_A)u\,dz\right]$$

This is the
Integral mass balance eq.

Conditions : $z=0 \quad y_A = 0$

$z=\delta_m \quad y_A = y_{A\alpha}$

$z=\delta m \quad \frac{dy_A}{dz} = 0$

Assume eq. $y_A = a + bz + cz^2$

Apply conditions

$$\frac{y_A}{y_{A\alpha}} = 2\left(\frac{z}{\delta_m}\right) - \left(\frac{z}{\delta_m}\right)^2$$

Use velocity profile

$$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{z}{\delta}\right) - \frac{1}{2}\left(\frac{z}{\delta}\right)^3 \quad \text{From Notes}$$

substitute into integral eq.

$$\frac{2 D_{AB}\, y_{A\alpha}}{\delta_m} = \frac{d}{dx}\left[\int_0^{\delta_m} y_{A\alpha}\left(1 - \frac{y_A}{y_{A\alpha}}\right) u\, dz\right]$$

Let $\xi = \delta_m/\delta \quad (< 1)$

$$\frac{2 D_{AB}}{\delta_m} = \frac{d}{dx}\left[U_\alpha \delta \int_0^{\xi} \left(1 - 2\left(\frac{z}{\delta\xi}\right) + \left(\frac{z}{\delta\xi}\right)^2\right) \cdot \left\{\frac{3}{2}\left(\frac{z}{\delta}\right) - \frac{1}{2}\left(\frac{z}{\delta}\right)^3\right\} d\left(\frac{z}{\delta}\right)\right]$$

Let $\eta = z/\delta$

$$\frac{2 D_{AB}}{U_\alpha \delta_m} = \frac{d}{dx}\left[\delta \int_0^{\xi} \left(1 - \frac{2\eta}{\xi} + \frac{\eta^2}{\xi^2}\right)\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right) d\eta\right]$$

$$\frac{2\,D_{AB}}{U_\alpha\,\delta_m} = \frac{d}{dx}\left[\delta\left(\frac{1}{8}\xi^2 - \cancel{\frac{1}{120}\xi^4}\right)\right]$$

$$\frac{16\,D_{AB}}{U_\alpha} = \delta\xi\frac{d}{dx}\left(\delta\xi^2\right)$$

$$= 2\delta^2\xi^2\frac{d\xi}{dx} + \xi^3\delta\frac{d\delta}{dx}.$$

From Notes

$$\delta\frac{d\delta}{dx} = \frac{140}{13}\frac{\nu}{U_\alpha} \quad \text{and} \quad \delta^2 = \frac{280}{13}\frac{\nu x}{U_\alpha}.$$

$$\therefore\ \frac{16\,D_{AB}}{U_\alpha} = \frac{280}{13}\frac{\nu x}{U_\alpha}\,2\xi^2\frac{d\xi}{dx} + \frac{140}{13}\frac{\nu}{U_\alpha}\xi^3$$

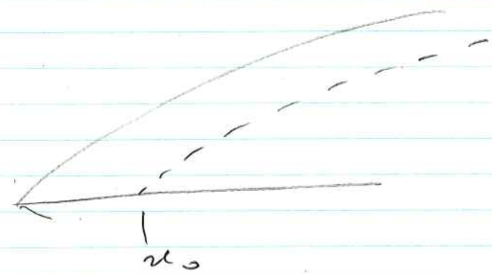
$$\frac{16\,D_{AB}}{\nu}\cdot\frac{13}{140} = 4x\,\xi^2\frac{d\xi}{dx} + \xi^3$$

$$\beta = \frac{52\,D_{AB}}{35\,\nu} = \frac{4}{3}x\frac{d\xi^3}{dx} + \xi^3$$

solve

$$\xi^3 = C x^{-3/4} + \beta$$

Condition; define a region w/o mass transfer.



at $x = x_o$, $\delta_m = 0$

$\xi = 0$

Apply this condition

$$\xi = \beta^{\frac{1}{3}}\left(1 - \left(\frac{x_o}{x}\right)^{3/4}\right)^{\frac{1}{3}}$$

Let $x_o = 0$

$$\xi = \frac{\delta_m}{\delta} = \beta^{\frac{1}{3}}$$

$\therefore$ Concentration profile at any x is

$$\frac{y_A}{y_{A\alpha}} = 2\left(\frac{z}{\delta\beta^{\frac{1}{3}}}\right) - \left(\frac{z}{\delta\beta^{\frac{1}{3}}}\right)^2$$

$\longrightarrow$

(b) The rate of deposition $0 \le x \le l$ is

$$\dot{Q} = \int_0^l N_A\Big|_{z=0} dx = \int_0^l -\psi D_{AB} \frac{dy_A}{dz}\Big|_{z=0} dx$$

$$\dot{Q} = -\psi D_{AB} \int_0^{l} 2\, y_{A\alpha} / \delta_m \; dx$$

$$= -2\psi D_{AB}\, y_{A\alpha} \int_0^{l} \frac{dx}{\beta^{1/3}\delta} \quad ; \quad \delta = 4.64 \sqrt{\frac{\nu}{u_\alpha}}\, x^{1/2}$$

$$= \frac{-2\psi D_{AB}\, y_{A\alpha}}{\beta^{1/3}\, 4.64 \sqrt{\nu/u_\alpha}} \int_0^{l} \frac{dx}{x^{1/2}}$$

$$= \frac{-2 D_{AB}\, \psi_{A\alpha}}{\left(\frac{52}{35}\right)^{1/3} D_{AB}^{1/3}\, \nu^{-1/3}\, 4.64\, \nu^{1/2}\, u_\alpha^{-1/2}} \cdot 2 l^{1/2}$$

Rate of deposition

$$\dot{Q} = 0.755\, D_{AB}^{2/3}\, \nu^{-1/6}\, u_\alpha^{1/2}\, l^{1/2}$$

$\longrightarrow$