Nov. 20

University of Calgary - Department of Chemical & Petroleum Engineering

Fall 2012

GJ

ENCH 501 Transport Phenomena **Quiz #6** **Name** ______________________

Time Allowed: 45 minutes

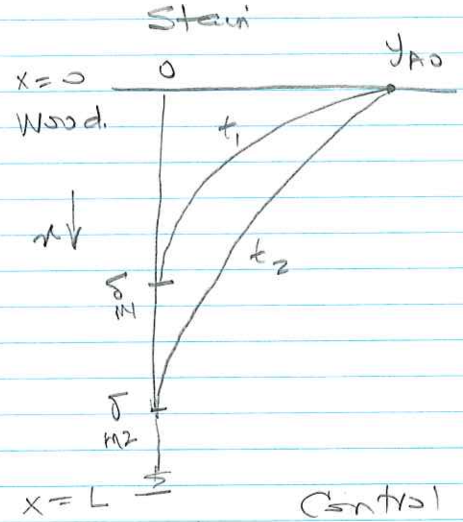
Stains are applied to wood to enhance appearance and to preserve the wood from the elements. They are usually brushed or rolled on the surfaces to be treated as liquid. Stains are composed of a solvent and a dye or a suspension of pigments. For the present problem, no pigments are present, and the stain (dye and solvent) is compound A that diffuses into the wood. Assume that A does not react with the wood. The pores of the wood are sealed so that there is no capillarity effect.

A flat wood surface is coated with the stain. The concentration of A at the wood-liquid boundary is given as a constant Cy_{AO} (moles/m^3), where C is a constant equal the total number of moles of cellulose and other substances in the wood and y_{AO} is the moles fraction at the wood side of the interface. The wood was initially free of A. If the diffusivity of A in the wood is 6 (10^{-8}) m^2/s and y_{AO} equals 0.3, use the ***integral method*** to determine

a) how deep would the stain have penetrated into the wood after 50 minutes. Show your derivations and state all assumptions.
b) How much stain would have been absorbed by a surface that is 1 m^2 by this time?

Data: Assume C is 55.6 moles/m^3.

ENCH 501 Quiz #6 Solution Nov. 20, 2012



Let A = stain + B = wood.

$N_B = 0$ and y_A is not small.

$$N_A = -cD_{AB}\frac{dy_A}{dx} + y_A(N_A + \underbrace{N_B}_{0})$$

$$\therefore\ N_A = \frac{-cD_{AB}}{1-y_A}\frac{dy_A}{dx}$$

Control volume $0 \le x < L$

Material balance

$$\text{Input} + \underbrace{\cancel{\text{Gen}}}_{0} = \underbrace{\cancel{\text{Output}}}_{0} + \text{Accum.}$$

$$N_A\Big|_{x=0} \qquad\qquad \frac{d}{dt}\left[\int_0^L C\,y_A\,dx\right]$$

$\therefore$ Integral Mass Transfer Eq.

$$\left(-\frac{cD_{AB}}{1-y_A}\frac{dy_A}{dx}\right)_{x=0} = \frac{d}{dt}\left[\int_0^{\delta_m} c\,y_A\,dx\right]$$

B.C. Conditions: $x = 0 \quad y_A = y_{A0}$

$y = \delta_m \quad y_A = 0$

$y = \delta_m \quad \frac{dy_A}{dx} = 0$

Assume $y_A = a + bx + cx^2$

$$\frac{dy_A}{dx} = b + 2cx$$

Apply b.c.s $\frac{y_A}{y_{A0}} = \left(1 - \frac{x}{\delta_m}\right)^2$

Substitute into integral eq

$$-\frac{c\,\mathcal{D}_{AB}}{1-y_{A0}}\left(-\frac{2y_{A0}}{\delta_m}\right) = \frac{d}{dt}\left[c\,y_{A0}\,\delta_m\int_0^1\left(1-\frac{x}{\delta_m}\right)^2 d\left(\frac{x}{\delta_m}\right)\right]$$

$$2\frac{\cancel{c}\,\mathcal{D}_{AB}}{1-y_{A0}}\frac{\cancel{y_{A0}}}{\delta_m} = \frac{d}{dt}\left[\cancel{c}\,\cancel{y_{A0}}\,\frac{\delta_m}{3}\right]$$

$$\frac{6\,D_{AB}}{1-y_{A0}} = \frac{\delta_m\,d\delta_m}{dt} = \frac{1}{2}\frac{d\delta_m^2}{dt}$$

$$\frac{d\delta_m^2}{dt} = \frac{12\,\mathcal{D}_{AB}}{1-y_{A0}} = \beta$$

Integrate $\delta_m^2 = \beta t + \underbrace{\cancel{c}}_{0}$ | use i.c. $t=0$, $\delta_m = 0$

$$\delta_m = \sqrt{\frac{12\,\mathcal{D}_{AB}\,t}{1-y_{A0}}}$$

(a) When $t = 50$ min's or $(50\times 60) = 3000$ s

$$\delta_m = \sqrt{\frac{12(6)(10^{-8})(3000)}{1-0.3}} = 5.555(10^{-2})\ m$$

$\longrightarrow$ 5.6 cm

(b) The total amount of A absorbed / m^2

$$\varphi\Big|_t = \int_0^{\delta_m} c\,y_A\,dx \quad \text{when } t = 3000\text{ s.}$$

$$Q = c\,\delta_m\, y_{A0} \int_0^1 \left(1 - \frac{x}{\delta_m}\right)^2 d\left(\frac{x}{\delta_m}\right)$$

$$= \frac{1}{3}\, c\, \delta_m\, y_{A0}$$

$$= \frac{1}{3}(55.6)(5.5549)(10^{-2})(0.3) \text{ moles}$$

$$= 0.30885 \text{ moles}$$

$\longrightarrow$