GJ

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ENCH 501: Transport Phenomena Quiz #6 **November 16, 2010**

Time Allowed: 45 mins. **Name:**

Tall buildings with glass walls are attractive. Illuminated billboards with glass covers are in use and the "picture windows" of tourist submarines allow passengers to observe undersea creatures and formations, e.g. off the Grand Cayman Islands. These structures share something in common. Heat from the inside pass through the glass and is removed by the fluid flowing over the surface from the outside. If the heat source is radiant, at a prescribed constant rate, temperature along the length of the plate could vary, thereby creating stress that could lead to catastrophic failure (or sudden shattering) of the glass. The analysis is to explore the conditions for such a situation.

Consider a horizontal glass plate of length L and width W. Radiant heat is beamed at a constant rate q_o (W/m^2) uniformly on the bottom surface. A fluid at T_∞ (lower than the temperature of the glass wall) flows over the top surface, in a direction parallel to the plate, along the length. The Prandtl number for the fluid is >> 1 such that $\delta_t < \delta$.

Use the **integral method** to obtain an expression for the temperature T(x,y) in the fluid within the thermal boundary layer. From this expression, obtain the temperature profile along the upper surface of the plate. Assume that the fluid properties are not temperature dependent.

Hint: Use the velocity profile derived in the Notes. For temperature, identify and use three boundary conditions.

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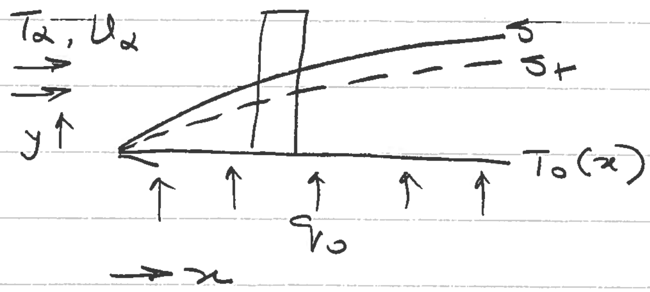


Let the temperature of the upper surface of the plate $= T_0(x)$

Choose a differential element as shown:

The Energy balance equation is Eq. 5.75 in Notes —

$$\alpha \left.\frac{dT}{dy}\right|_{y=0} = \frac{d}{dx}\left[\int_0^H (T_\infty - T)\, u\, dy\right] \; ; \; \alpha = \frac{k}{\rho c_p}$$

The velocity profile is assumed as

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \; ; \; \delta = 4.64\sqrt{\frac{\nu x}{U_\infty}}$$

from Eqs 5.16 and 5.28

For temperature, identify the conditions:

b.c 1 $\quad y = 0 \quad -k\left.\frac{dT}{dy}\right|_{y=0} = q_0 \quad$ or $\quad \frac{dT}{dy} = -\frac{q_0}{k}$

b.c. 2 $\quad y = \delta_t \quad T = T_\infty$

b.c 3 $\quad y = \delta_t \quad \frac{dT}{dy} = 0$

Assume a temperature profile

$$T = a + by + cy^2$$

and apply the conditions.

Differentiate: $\frac{dT}{dy} = b + 2cy$

Apply b.c.1 $\quad b = -q_0/k$

b.c 3 $\quad c = -b/2\delta_t = q_0/2k\delta_t$

bc 2 $\quad T_\alpha = a + b\delta_t + c\delta_t^2$

or $\quad a = T_\alpha - b\delta_t - c\delta_t^2$

$$= T_\alpha + \frac{q_0}{k}\delta_t - \frac{q_0\delta_t^2}{2k\delta_t} = T_\alpha + \frac{q_0\delta_t}{2k}$$

Substitute.

$$T = T_\alpha + \frac{q_0\delta_t}{2k} - \frac{q_0}{k}y + \frac{q_0}{2k}\frac{y^2}{\delta_t}$$

$$T - T_\alpha = \frac{q_0\delta_t}{2k}\left(1 - 2\frac{y}{\delta_t} + \frac{y^2}{\delta_t^2}\right) = \frac{q_0\delta_t}{2k}\left(1 - \frac{y}{\delta_t}\right)^2$$

Substitute for $(T_\alpha - T)$ and u in integral energy equation:

$$\alpha \left.\frac{dT}{dy}\right|_{y=0} = -\frac{d}{dx}\left[u_\alpha\delta\int_{0/\delta}^{\delta_t/\delta}\left(\frac{q_0}{2k}\right)\delta_t\left(1-\frac{y}{\delta_t}\right)^2\frac{u}{u_\alpha}\,d\left(\frac{y}{\delta}\right)\right]$$

$$= -\frac{d}{dx}\left[u_\alpha\delta^2\left(\frac{q_0}{2k}\right)\xi\int_0^{\xi}\left(1-\frac{\eta}{\xi}\right)^2\frac{u}{u_\alpha}\,d\eta\right]$$

where $\eta = \frac{y}{\delta}$ and $\xi = \delta_t/\delta$

Solve the integral

$$\int_0^{\xi}\left(1-\frac{\eta}{\xi}\right)^2\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)d\eta =$$

$$\left.\frac{3}{4}\eta^2 - \eta^3/\xi + \frac{3}{8}\frac{\eta^4}{\xi^2} - \frac{1}{8}\eta^4 + \frac{1}{5}\frac{\eta^5}{\xi} - \frac{1}{12}\frac{\eta^6}{\xi^2}\right|_0^{\xi}$$

$$\frac{3}{4}\xi^2 - \xi^2 + \frac{3}{8}\xi^2 - \frac{1}{8}\xi^4 + \frac{1}{5}\xi^4 - \frac{1}{12}\xi^4$$

$$= \frac{1}{8}\xi^2 - \frac{1}{120}\xi^4$$

$$\alpha \left.\frac{dT}{dy}\right|_{y=0} = -\frac{d}{dx}\left[U_\infty\left(\frac{q_0}{2k}\right)\delta^2\xi\left(\frac{1}{8}\xi^2 - \frac{1}{120}\xi^4\right)\right]$$

$$\alpha\left(-\frac{q_0}{k}\right) = \left(-\frac{q_0}{2k}\right)U_\infty \frac{d}{dx}\left[\delta^2\left(\frac{1}{8}\xi^3 - \cancel{\frac{1}{120}\xi^5}\right)\right]$$

neglect

$$\frac{2\alpha}{U_\infty} = \frac{d}{dx}\left[\frac{1}{8}\delta^2\xi^3\right]$$

But from eq. 5.19 Notes, $\delta^2 = \frac{280}{13}\frac{\nu x}{U_\infty}$

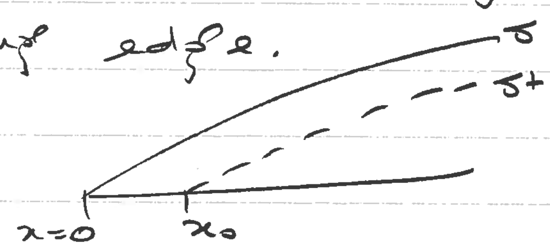
$$\frac{2\alpha}{U_\infty} = \frac{d}{dx}\left[\frac{1}{8}\cdot\frac{280}{13}\frac{\nu x}{U_\infty}\xi^3\right]$$

$$\frac{26}{35}\frac{\alpha}{\nu} = \frac{d}{dx}\left[x\xi^3\right] = \beta \text{, a constant}$$

This is an o.d.e that requires a b.c.

But ξ is indeterminate at $x=0$

so create a new system with an unheated leading edge.



$\therefore$ at $x = x_0$, $\xi = 0$

Solve

$$x\xi^3 = \beta(x - x_0)$$

As $x_0 \to 0$ $\quad \xi^3 = \beta$, a constant or $\xi = \beta^{1/3} = \left(\frac{26}{35}\frac{\alpha}{\nu}\right)^{1/3} = \frac{\delta_t}{\delta}$

Hence the temperature in the thermal b.l. is:

$$T(x,y) - T_\infty = \frac{q_0 \delta_t}{2k}\left(1 - \frac{y}{\delta_t}\right)^2 \; ; \; \delta_t = \left(\frac{26}{35}\frac{\alpha}{\nu}\right)^{1/3}\delta$$

and $\delta = 4.64\sqrt{\frac{\nu x}{U_\infty}}$

$\longrightarrow$

Along the surface of the plate, at $y=0$

$$T(x,0) - T_\infty) = \frac{q_0 \delta_t}{2k}$$

or $$T(x,0) = T_\infty + \frac{q_0}{2k}\left(\frac{26}{35}\frac{\alpha}{\nu}\right)^{1/3}\left(4.64\sqrt{\frac{\nu x}{U_\infty}}\right)$$

$\longrightarrow$

That is the wall temperature increases in proportion to $x^{1/2}$, and the higher q_0 is, the greater the temperature rise along the plate. Thermal stress would be significant under such conditions.

$\longrightarrow$