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ENCH 501: Transport Phenomena Quiz #6 **November 17, 2009**

Time Allowed: 45 mins. **Name:**

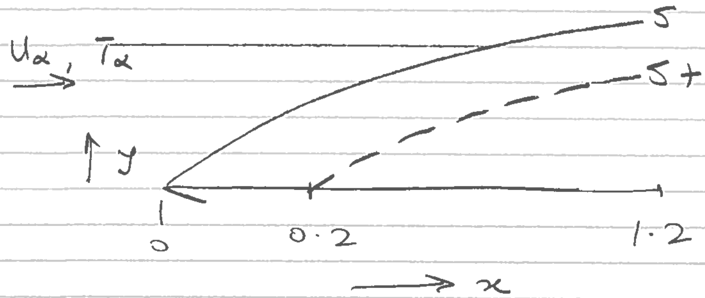
Energy conservation is in everyone's interest. It is suspected that commercial plattens (hot surfaces) for preparing foods such as pancakes at restaurants waste energy, particularly when a fan is used to move air towards the chef so that he or she is not too hot. This problem is to estimate the rate of energy loss from one such unit.

A rectangular hot plate is 1.2 m long and 0.8m wide. The first 20 cm of the plate, along its length, is unheated. The remaining section of the surface is maintained at a constant temperature of 150°C. Air at 27°C blows at a speed of 0.72 m/s towards the plate (and the chef standing at the side). The air flows over and parallel to the long side of the plate, from the unheated section side.

a) What are the momentum, displacement and thermal boundary layer thicknesses at the end of the plate - i.e. at the opposite side of the leading edge?
b) Estimate the rate of heat transfer from the plate into the air. Show important steps, including any analytical or numerical integration.

Data: Properties of air at 27°C (assume independent of temperature in the boundary layer)
$\rho = 1.1774\ kg/m^3$; $\mu = 1.8462\ (10^{-2})\ mPa\ s$; $k = 0.02624\ W/m\ K$; $C_p = 1.0057\ kJ/kg\ K$

ENCH 501 Quiz 6 Solution Nov. 17, 2009



The momentum and thermal boundary layers develop as shown in sketch.

The first step is to establish that the b.l. is laminar

$$Re_x\Big|_L = \frac{U_\infty L \rho}{\mu} = \frac{(0.72)(1.2)(1.1774)}{1.8462(10^{-5})} = 5.51(10^4)$$

This is less than $5(10^5)$, ∴ b.l. is laminar.

(a) The momentum thickness, δ_2, can be estimated as derived in the Notes, p 88-90

$$\delta = 4.64\sqrt{\frac{\nu x}{U_\infty}} \quad ; \quad \nu = \frac{\mu}{\rho} = 1.568(10^{-5})\ m^2/s$$

$$x = L = 1.2\ m, \quad U_\infty = 0.72 \quad ; \quad \delta_2 = \underset{\text{Notes}}{0.1392\,\delta}$$

Boundary layer thickness, δ (Eq. 5.25)
$$\delta = 4.64\sqrt{\frac{(1.568)(10^{-5})(1.2)}{0.72}} = 2.372(10^{-2})\ m$$

⇒ Momentum thickness $= 3.302(10^{-3})\ m$ ⟶

Displacement thickness, δ_1 (Eq. 5.30)
$$\delta_1 \doteq \frac{3}{8}\delta = 8.895(10^{-3})\ m$$
$\sim$ 8.9 mm ⟶

Thermal b.l. thickness (Eq. 5.83)
$$\frac{\delta_t}{\delta} = \xi = \left(\frac{13}{14}\frac{\alpha}{\nu}\right)^{1/3}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$$

$$\alpha = \frac{k}{\rho c_p} = \frac{0.02624}{1.1774(1.0057)(10^3)} = 2.216(10^{-5})\ m^2/s$$

$$\xi = \frac{\delta_t}{\delta} = \left[\frac{13}{14}\frac{(2.216)(10^{-5})}{(1.568)(10^{-5})}\right]^{\frac{1}{3}} \left[1 - \left(\frac{0.2}{1.2}\right)^{3/4}\right]^{\frac{1}{3}}$$

at $x = L$

$$= 0.9899$$

$$\therefore \quad \delta_t = (0.9899)(2.372)(10^{-2})\ m$$

$$= 2.348\,(10^{-2})\ m \quad \text{or} \quad 2.348\ cm$$

$\longrightarrow$

(b) The energy transfer rate from the plate to air can be estimated from

$$\dot{Q} = \int_{x_0}^{L} q_x W\,dx \qquad \text{where} \qquad q_x = -k\left.\frac{dT}{dy}\right|_{\substack{y=0\\x}}$$

$$\text{or} \quad = h_x(T_0 - T_\alpha)$$

$$\therefore \quad \dot{Q} = W\int_{x_0}^{L} h_x(T_0 - T_\alpha)\,dx$$

$$= W(T_0 - T_\alpha)\int_{x_0}^{L} h_x\,dx$$

from Notes, eq. 5.86

$$h_x = 0.332\,k\,P_r^{\frac{1}{3}}\left(\frac{U_\alpha}{\nu x}\right)^{\frac{1}{2}}\left[1 - \left(\frac{x_0}{x}\right)^{\frac{3}{4}}\right]^{-\frac{1}{3}}$$

$$= \beta\left(\frac{x_0}{x}\right)^{\frac{1}{2}}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

$$\text{where} \quad \beta = 0.332\,k\,P_r^{\frac{1}{3}}\left(\frac{U_\alpha}{\nu x_0}\right)^{\frac{1}{2}}$$

$$\therefore \quad \dot{Q} = W(T_0 - T_\alpha)\beta\int_{x_0}^{L}\left(\frac{x_0}{x}\right)^{\frac{1}{2}}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-\frac{1}{3}} dx$$

$$\text{Let} \quad Y = \frac{x}{x_0}$$

$$Q' = W(T_o - T_\alpha)\beta \int_{x_o}^{L} Y^{-\frac{1}{2}}\left[1 - Y^{-\frac{3}{4}}\right]^{-\frac{1}{3}} x_o \, d\frac{x}{x_o}$$

$$= W(T_o - T_\alpha)\beta\, x_o \int_{1}^{L/x_o} Y^{-\frac{1}{2}}\left[1 - Y^{-3/4}\right]^{-\frac{1}{3}} dY$$

Given $\frac{L}{x_o} = 6$

$$Q' = W(T_o - T_\alpha)\beta\, x_o \int_1^6 \frac{dY}{Y^{\frac{1}{2}}\left[1 - Y^{-3/4}\right]^{\frac{1}{3}}}$$

x, m	Y	$f(Y)\ \left(= \frac{1}{Y^{\frac{1}{2}}\left[1-Y^{-3/4}\right]^{\frac{1}{3}}}\right)$	
$x_o = 0.2$	1	α	
0.2002	1.001	11.0015	0.0724
0.202	1.01	5.0973	0.334
0.2262	1.1	2.3246	1.476
0.4	2	0.9554	0.8277
0.6	3	0.6999	0.6391
0.8	4	0.5783	0.5409
1.0	5	0.5034	0.4775
1.2	6	0.4515	

Use Trapezoidal Rule

$\Sigma = 4.3675$

$$Q' \simeq (0.8)(150 - 27)\beta(0.2)(4.3675) \quad J/s \text{ or } W$$

$$\beta = 0.332(0.02624)\left(\frac{1.568}{2.216}\right)^{\frac{1}{3}}\left(\frac{0.72}{1.568(10^{-5})(0.2)}\right)^{\frac{1}{2}}$$

$$= 3.7197$$

$$Q' \doteq 319.72 \text{ W} \longrightarrow$$

Rate of heat loss from plate.