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The University of Calgary
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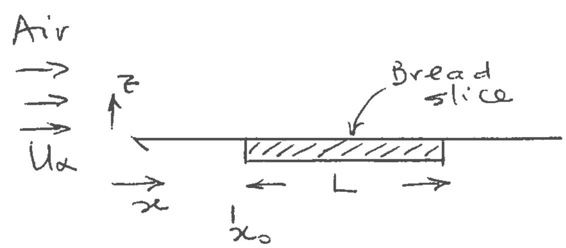
ENCH 501: Transport Phenomena Quiz #6 November 18, 2008

Time Allowed: 45 mins. Name:

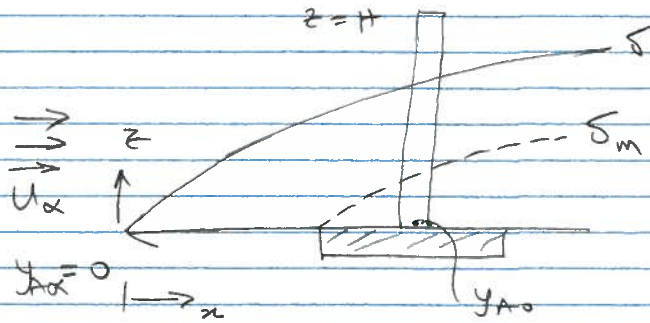
People are often influenced to buy things without their conscious awareness. Stores owners use the senses of sound, smell and sight to attract potential customers into making purchases of things they probably do not urgently need. For example, the aroma of freshly baked bread is observed to induce hunger and thus promote the sale of bakery products, household utensils, appliances and even homes.

A home owner wishing to sell his house to a family about to conduct an inspection placed a rectangular slice of fresh bread in a hollowed out portion of a cutting board on a high shelf (so that it is unobtrusive). The slice of bread fits snugly into the depression in the rectangular wooden board and the top surface of the bread is flush with the surface of the cutting board. The board was then positioned such that air from a vent flowed at a free stream velocity U_∞ over the cutting board. An edge of the bread, width W, is at a distance x_o from the leading edge of the cutting board. The slice of bread has a length L along the air flow direction. Substance A which gives the desired aroma has a constant mole fraction y_{Ao} at the bread's exposed surface while the air does not contain A.

Derive an expression for the mole fraction $y_A(x,z)$ in the material boundary layer over the bread. Use the **integral method** and show all your steps.



ENCH 501 Quiz 6 Solution November 18, 2008



Let A be the substance giving the aroma and B be air flowing over the surface. The flux of air into the surface is zero.

$$\therefore \quad N_A = -c\mathcal{D}_{AB}\frac{dy_A}{dz} + y_A(N_A+N_B)$$

reduces to

$$N_A\Big|_{z=0} = -\frac{c\,\mathcal{D}_{AB}}{1-y_{A0}}\frac{dy_A}{dz}\Big|_{z=0}$$

Consider the differential element of height z = H.

Balance on substance A, at steady state is:

$$\cancel{\int_0^H c\,y_A u\,dz} + N_A\Big|_{z=0} dx = \cancel{\int_0^H c\,y_A u\,dz} + \frac{d}{dx}\left[\int_0^H c\,y_A u\,dz\right]dx$$

or

$$-\frac{\mathcal{D}_{AB}\,c}{1-y_{A0}}\frac{dy_A}{dz}\Big|_{z=0} = \frac{d}{dx}\left[\int_0^{\delta_m} c\,y_A u\,dz\right]$$

where the upper limit of the integral has been changed to δ_m because $y_A = y_{A\infty} = 0$ beyond δ_m.

This is the material balance integral equation.

Boundary conditions: $z = 0 \quad y_A = y_{A0}$

$z = \delta_m \quad y_A = 0$

$z = \delta_m \quad \frac{dy_A}{dz} = 0$

Assume a function:

$$y_A = a + bz + cz^2$$

and apply the b.c.s

$$\frac{y_A}{y_{Ao}} = \left(1 - \frac{z}{\delta_m}\right)^2$$

$$\left.\frac{dy_A}{dz}\right|_{z=0} = -\frac{2\,y_{Ao}}{\delta_m}$$

Substitute into integral equation

$$\frac{2\,D_{AB}}{1-y_{Ao}}\,\frac{1}{\delta_m} = \frac{d}{dx}\left[U_\alpha\,\delta\int_0^{\delta_m/\delta}\left(1-\frac{z}{\delta_m}\right)^2\left(\frac{u}{U_\alpha}\right)d\left(\frac{z}{\delta}\right)\right]$$

Let $\frac{u}{U_\alpha} = \frac{3}{2}\frac{z}{\delta} - \frac{1}{2}\left(\frac{z}{\delta}\right)^3$ from Notes

and $\delta_m = \xi\,\delta$ (with $\xi < 1$)

$$\frac{2\,D_{AB}}{1-y_{Ao}}\,\frac{1}{\delta_m} = \frac{d}{dx}\left[U_\alpha\,\delta\int_0^{\xi}\left(1-\frac{z}{\xi\delta}\right)^2\left(\frac{3}{2}\frac{z}{\delta} - \frac{1}{2}\left(\frac{z}{\delta}\right)^3\right)d\left(\frac{z}{\delta}\right)\right]$$

Let $\eta = z/\delta$

$$\frac{2\,D_{AB}}{1-y_{Ao}}\,\frac{1}{\xi\delta} = \frac{d}{dx}\left[U_\alpha\,\delta\int_0^{\xi}\left(1-\frac{\eta}{\xi}\right)^2\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)d\eta\right]$$

$$= \frac{d}{dx}\left[U_\alpha\,\delta\left\{\frac{1}{8}\xi^2 - \frac{1}{120}\xi^4\right\}\right]$$

Neglect second term in brackets and let

$$\beta = \frac{16\,D_{AB}}{(1-y_{Ao})\,U_\alpha}$$

$$\beta = \xi \delta \frac{d}{dx}\left(\delta \xi^2\right) = \xi^3 \delta \frac{d\delta}{dx} + 2\xi^2 \delta^2 \frac{d\xi}{dx}$$

From previous derivations (in Notes)

$$\delta \frac{d\delta}{dx} = \frac{140}{13}\frac{\nu}{U_\alpha}$$

and $$\delta^2 = \frac{280}{13}\frac{\nu x}{U_\alpha}$$

$$\therefore \quad \beta \frac{13}{140}\frac{U_\alpha}{\nu} = \frac{52 D_{AB}}{35\, \nu (1-y_{A0})} = \xi^3 + 4x \xi^2 \frac{d\xi}{dx} = \varepsilon$$

The solution is

$$\xi^3 = c\, x^{-3/4} + \varepsilon$$

subject to the condition

$$\xi = \frac{\delta_m}{\delta} = 0 \quad \text{at} \quad x = x_0$$

On application of condition to evaluate c

$$\xi = \left(\frac{52}{35}\frac{1}{1-y_{A0}}\frac{D_{AB}}{\nu}\right)^{\frac{1}{3}}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{\frac{1}{3}}$$

Hence

$$\delta_m = \left(\frac{52}{35}\frac{1}{1-y_{A0}}\frac{D_{AB}}{\nu}\right)^{\frac{1}{3}}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{\frac{1}{3}} \cdot \delta$$

where $$\delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}.$$

and

$$\frac{y_A(x,z)}{y_{A0}} = \left(1 - \frac{z}{\delta_m}\right)^2$$