GJ

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #6 **November 21, 2006**

Time Allowed: 45 mins. **Name:**

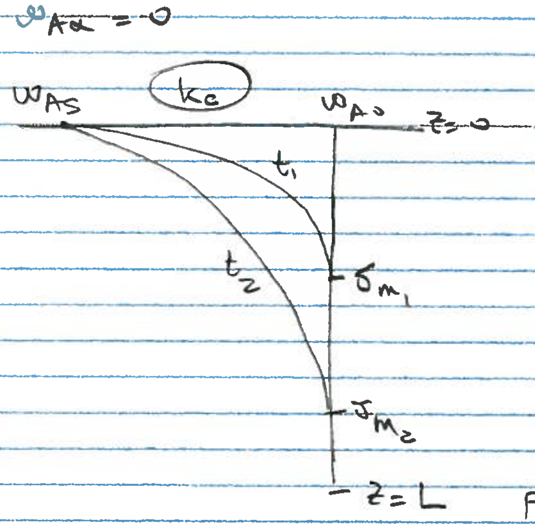
Volatile organic compounds (VOC) constitute a class of indoor pollutants. They are released from materials and surfaces such as synthetic carpets, linoneum, lacquers, paints, adhesives and varnishes, to name a few. VOC acts with other pollutants such as molds to cause 94% of all respiratory problems, cause irritation of the eye, nose and throat, headaches and damage to the liver, kidney and central nervous systems.

A full can of oil-based paint was left inadvertently open after the content had been thoroughly stirred. The diameter of the open can surface is 16cm and the depth of the paint is 20cm. One of the components of the paint is varsol or naptha, a solvent. Varsol was initially present as 1% (mass basis) in the paint which has a constant density (ρ) of 1,400 kg/m^3. The diffusivity of varsol in the paint is $2(10^{-8})$ m^2/s. Convection currents in the air above the can be assumed strong such that the mass transfer coefficient (k_c) at the paint-air boundary is very large and the air above the can contains a negligible amount of naptha.

a) Use the ***integral method*** to derive an expression for the concentration profile for naptha $\omega_A(z,t)$ in the paint. You may assume that the paint is stationary, no other components are volatile, the domain is semi-infinite, $\rho_A \approx \rho\ \omega_A$ and ω_A is sufficiently small that the convective term of flux may be neglected. Show all your steps.

b) Estimate the fraction of the original varsol that would be left in the can after 24 hours of exposure.

ENCH 501 Quiz 6 Solution. Nov. 21, 2006



This is diffusion in a stationary medium. Because the mass transfer coefficient k_c is very large, the concentration of varsol at the paint surface $\omega_{AS} = 0$. For the problem, $\omega_{A0} = 0.01$

Let varsol or naptha be component A in paint B.

Material balance on A in region $0 \le z \le L$

$$\underbrace{\text{Input}}_{0} + \underbrace{\text{Generation}}_{0} = \text{Output} + \text{Accum.}$$

$$\text{output} = n_A\Big|_{z=0} \cdot S = S\left[-\rho \mathcal{D}_{AB}\frac{d\omega_A}{dz} + \omega_A\underbrace{(n_A+n_B)}_{\text{neglect}}\right]$$

$$\text{Accum} = -\frac{d}{dt}\left[\int_0^L \rho\,\omega_A\,dz\,S\right]$$

where S = cross sectional area.

and the -ve is because naptha is lost

Hence

$$0 = -\rho\mathcal{D}_{AB}\frac{d\omega_A}{dz}\Big|_{z=0}\cdot S - \frac{d}{dt}\left[\int_0^L \rho\,\omega_A\,dz\,S\right]$$

Since S and ρ are constants

$$\mathcal{D}_{AB}\frac{d\omega_A}{dz}\Big|_{z=0} = -\frac{d}{dt}\left[\int_0^L \omega_A\,dz\right] \quad (1)$$

This is the mass transfer integral equation

Boundary conditions

$z = 0 \qquad \omega_A = \omega_{As} = 0 \qquad$ b.c. 1

$z = \delta_m \qquad \omega_A = \omega_{Ao} \qquad$ b.c. 2

$z = \delta_m \qquad \dfrac{\partial \omega_A}{\partial z} = 0 \qquad$ b.c. 3

Assume an expression for ω_A

$\omega_A = a + bz + cz^2 \quad - \quad a = 0$, from b.c. 1

$\dfrac{d\omega_A}{dz} = \cancel{b + 2cz} \quad - \quad 0 = b + 2c\delta_m$, b.c. 3

$\omega_{Ao} = b\delta_m + c\delta_m^2$, b.c. 2

$\therefore \quad c = -\omega_{Ao}/\delta_m^2 \quad , \quad b = \dfrac{2\,\omega_{Ao}}{\delta_m}$

$$\therefore \quad \omega_A = \omega_{Ao}\left(\frac{2}{\delta_m}z - \frac{z^2}{\delta_m^2}\right) \qquad (2)$$

Substitute eq. (2) into (1)

$$D_{AB}\left(\frac{2\,\omega_{Ao}}{\delta_m}\right) = -\frac{d}{dt}\left[\int_0^{\delta_m} \omega_{Ao}\left(\frac{2z}{\delta_m} - \frac{z^2}{\delta_m^2}\right)dz\right]$$

$$- \frac{d}{dt}\left[\int_{\delta_m}^{L} \omega_{Ao}\, dz\right]$$

$$= -\omega_{Ao}\frac{d}{dt}\left[\frac{z^2}{\delta_m} - \frac{z^3}{3\delta_m^2}\right]_0^{\delta_m} + \omega_{Ao}\frac{d\delta_m}{dt}$$

$$\frac{2D_{AB}}{\delta_m} = -\frac{d}{dt}\left[\frac{2\delta_m}{3}\right] + \frac{d\delta_m}{dt}$$

$$= \frac{1}{3}\frac{d\delta_m}{dt}$$

That is,

$$\frac{d\delta_m^2}{dt} = 12 D_{AB}$$

with condition $t=0$, $\delta_m = 0$

$$\therefore \quad \delta_m = \sqrt{12 D_{AB} t} \qquad (3)$$

The required profile is Eqs (2) & (3) combined.

$$w_{A0} - w_A = w_{A0}\left(1 - \frac{2z}{\delta_m} + \frac{z^2}{\delta_m^2}\right)$$

or

$$\frac{w_{A0} - w_A}{w_{A0}} = \left(1 - \frac{z}{\sqrt{12 D_{AB} t}}\right)^2 \longrightarrow$$

(b) The amount of varsol in the can at any instant is given by

$$Q_t = \int_0^{\delta_m} \rho w_A \, dz \cdot S + \int_{\delta_m}^{L} \rho w_{A0} \, dz \cdot S$$

The original amount (S = cross-sectional area)

$$Q_0 = \int_0^L \rho w_{A0} \, dz \cdot S = \rho w_{A0} S \cdot L$$

At 24 hours, $\delta_m = \sqrt{12 D_{AB} t}$

$$= \sqrt{12(2)(10^{-8})\,24\,(3600)} = 0.144 \text{ m}$$

$$\Phi_t = S\int_0^{\delta_m} \rho w_{A0}\left(\frac{2z}{\delta_m} - \frac{z^2}{\delta_m^2}\right) dz + \rho w_{A0} S (L-\delta_m)$$

$$= S\delta_m \rho w_{A0} \int_0^1 (2\eta - \eta^2)\, d\eta + \rho w_{A0} S (L-\delta_m)$$

where $\eta = z/\delta_m$

$$= S\delta_m \rho w_{A0}\left[\eta^2 - \tfrac{1}{3}\eta^3\right]_0^1 + \rho w_{A0} S (L-\delta_m)$$

$$= \tfrac{2}{3} S\delta_m \rho w_{A0} + \rho w_{A0} S (L-\delta_m)$$

$$\therefore \frac{\Phi_t}{\Phi_0} = \frac{\tfrac{2}{3}\cancel{S}\delta_m \cancel{\rho}\cancel{w_{A0}} + \cancel{\rho}\cancel{w_{A0}}\cancel{S}(L-\delta_m)}{\cancel{\rho}\cancel{w_{A0}}\cancel{S}L}$$

$$= \frac{\tfrac{2}{3}\delta_m + L - \delta_m}{L} = 1 - \frac{1}{3}\frac{\delta_m}{L}$$

$$= 1 - \frac{1}{3}\frac{(0.144)}{0.2} = 0.76$$

That is 76% of the varsol is left in the can after 24 hrs.