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ENCH 501: Transport Processes Quiz #6 November 22, 2005

Time Allowed: 45 mins. Name:

Wood is a porous material which consists of cells, the walls of which are made of cellulose, hemicelluloses and lignin. The free space or void volume (10 -23.4%) is, in the living plant, occupied by water and gases. Wood used in construction of homes is derived mostly from tree trunks, cut and dried to remove free water from the pores. The cell walls are, however, still damp from water 'bound' to cellulosic substances. Wood for construction of foundations, decks, fences and other domestic and industrial structures are normally infused with either preservatives (chromated copper arsenate, alkaline copper quarternary or copper azole) against fungal rot and termites or with fire retardants. The wood to be treated is submerged in a pool of a solution of the chemicals in water and kept under pressure of between 6.5 and 14 atm. The applied pressure rapidly causes the pores to be filled with water, assumed pure at t=0. As the chemical compound in the solution outside the wood diffuses into the water in the pore spaces, a portion reacts with and is bound to the cellulose. It is assumed that only the chemical compound diffuses into the wood within which the water is stationary.

A thick slab of wood is to be treated by submergence in a pool of chromated copper arsenate (CCA) solution in water. The concentration of CCA in the pool is $C_{A\alpha}$ and the mass transfer coefficient at the pool-wood boundary is k_c. The void fraction of the wood (filled with water) is ϕ. Assume that the initial concentration of CCA in the water in the wood pores C_{Ao} is negligible. The CCA reacts with the cellulose at a rate proportional to the local concentration C_A in the pores and the reaction constant is k_1, i.e. per unit volume of wood (cells and void), the rate of CCA removal is $-k_1C_A$.

Use the ***integral method*** to derive an expression for the concentration profile $C_A(z,t)$ in the wood. You may assume that the wood is a semi-infinite domain, $C_A \approx C\, y_A$ and y_A is not small, i.e. not $\ll 1$. Show all your steps.

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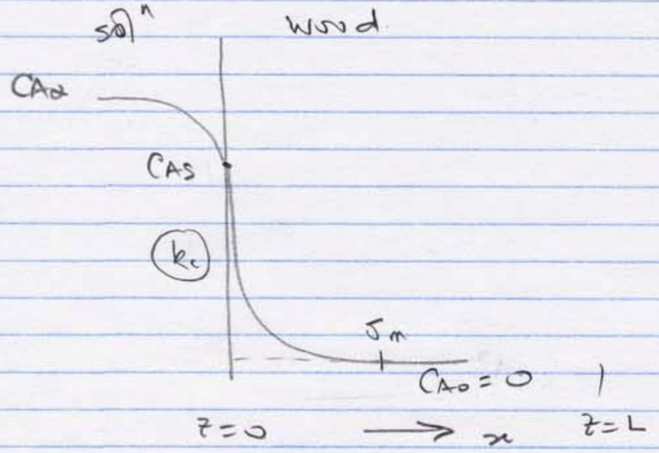

Consider the region $0 \leq z < L$ where $L >> \delta_m$ or the penetration depth of CCA into the wood.

Let CCA be compound A.

The material balance on A is given / unit area of surface as

$$\underset{\text{Input}}{N_A\Big|_{z=0}} - \underset{\text{Rate of loss by reaction}}{\int_0^L k_1 C_A \, dz} = \underset{\text{Rate of Accum.}}{\frac{d}{dt}\left[\int_0^L C_A \phi \, dz\right]} \quad ①$$

The flux is defined as:

$$N_A = -D_{AB}\phi \frac{dC_A}{dz} + y_A(N_A + \underbrace{N_B}_{0})$$

since diffusion occurs only through the pores. $\quad$ water stationary

$$\therefore \quad N_A = -\frac{D_{AB}\phi C}{1-y_A}\frac{dy_A}{dz} \quad ; \text{ where } C_A = y_A C \quad ②$$

This is a convective boundary condition problem.

bc ① $\quad z = 0, \quad N_A\Big|_{z=0} = -\left(\frac{D_{AB}\phi C}{1-y_A}\frac{dy_A}{dz}\right)\Bigg|_{z=0} = k_c\left(C_{A\alpha} - C_{AS}\right)$

bc ② $\quad z = \delta_m, \quad C_A = 0 = y_A$

bc ③ $\quad z = \delta_m \quad \frac{dC_A}{dz} = \frac{dy_A}{dz} = 0$

For a start, use $z=0$, $y_A = y_{AS}$ in place of bc (1).
Assume a concentration profile as follows:

$$y_A = a + bz + cz^2$$

$$\frac{dy_A}{dz} = b + 2cz$$

Use b.c.s, $$\frac{y_A}{y_{AS}} = \left(1 - \frac{z}{\delta_m}\right)^2 \qquad (3)$$

$\therefore$

$$N_A\Big|_{z=0} = -\frac{D_{AB}\phi C}{1-y_{AS}}\left(-\frac{2y_{AS}}{\delta_m}\right) = k_c C\left(y_{Ad} - y_{AS}\right)$$

or $$\frac{(y_{Ad} - y_{AS})(1-y_{AS})}{y_{AS}} = \frac{2D_{AB}\phi}{k_c}\left(\frac{1}{\delta_m}\right) \qquad (4)$$

The material balance equation is (with upper integral limit changed to δ_m)

$$k_c(y_{Ad} - y_{AS}) - \int_0^{\delta_m} k_1 y_{AS}\left(1-\frac{z}{\delta_m}\right)^2 dz = \frac{d}{dt}\left[\int_0^{\delta_m} y_{AS}\left(1-\frac{z}{\delta_m}\right)^2 \phi \, dz\right] \qquad (5)$$

$$k_c(y_{Ad} - y_{AS}) - k_1 y_{AS}\int_0^{\delta_m}\left(1 - \frac{2z}{\delta_m} + \frac{z^2}{\delta_m^2}\right)dz =$$
$$\frac{d}{dt}\left[\phi y_{AS}\int_0^{\delta_m}\left(1 - \frac{2z}{\delta_m} + \frac{z^2}{\delta_m^2}\right)dz\right]$$

$$k_c(y_{Ad} - y_{AS}) - k_1 y_{AS}\left(z - \frac{z^2}{\delta_m} + \frac{1}{3}\frac{z^3}{\delta_m^2}\right)\Bigg|_0^{\delta_m} =$$
$$\frac{d}{dt}\left[\phi y_{AS}\left(1 - \frac{z^2}{\delta_m} + \frac{1}{3}\frac{z^3}{\delta_m^2}\right)\Bigg|_0^{\delta_m}\right]$$

$$k_c\,(y_{A\alpha} - y_{AS}) - k_1\, y_{AS}\left(\frac{\delta_m}{3}\right) = \frac{d}{dt}\left[\phi\, y_{AS}\,\frac{\delta_m}{3}\right]$$

$$\frac{3\,k_c}{\phi}\,(y_{A\alpha} - y_{AS}) - \frac{k_1}{\phi}\, y_{AS}\,\delta_m = \frac{d}{dt}\left[y_{AS}\,\delta_m\right] \qquad (4)$$

From eq. (4)

$$\delta_m = \beta\,\frac{y_{AS}}{(y_{A\alpha} - y_{AS})(1 - y_{AS})} \; ; \quad \beta = \frac{2\,D_{AB}\,\phi}{k_c}$$