The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #6 **November 16, 2004**

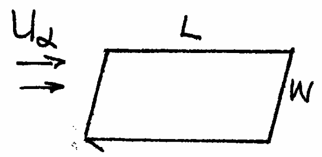
Time Allowed: 50 mins. **Name:**

Hydrocarbon liquids in commerce, such as gasoline and kerosine (aviation fluid), are sometimes spilled on the ground. Similar accidents occur at drilling sites and around wells. Air which flows over the site then carries vapor of the hydrocarbons downstream, often over long distances. The rate of such dispersal is to be studied in a laboratory.

The test table is L long and W wide. The table is covered with a cloth material soaked in and saturated with liquid kerosine. Air free of any hydrocarbons is passed over the surface of the table in a direction normal to the W edge at a free-stream velocity of U_∞. Both the air and the kerosine-soaked cloth are maintained at temperature T. At T, the vapor pressure of kerosine is P_A and its diffusion coefficient in air is D_{AB}. The ambient pressure is P_t.

Derive an expression for the mole fraction profile $y_A(x,y)$ of kerosine in the boundary layer above the plate. Use the ***integral method***. Show all your steps.

Hints and Data:



Assume that the velocity distribution above the table is given by:

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

where $\delta(x)$ is the thickness of the momentum boundary layer, $\boldsymbol{U_\infty}$ is the free-stream velocity and $\boldsymbol{y}$ is distance from the table.

The Schmidt number ($Sc = \nu / D_{AB}$) for the gas phase diffusion is 2.62. Hence the momentum boundary layer is thicker than the species boundary layer. (**Sc** for mass transfer is like **Pr** for heat transfer.)

Assume that the mass flux for the kerosine into air is given by:

$$N_A = -cD_{AB}\frac{\partial y_A}{\partial y}$$

that is, the convective component of the flux is negligible. The term c is the molar concentration of the gas mixture and y_A is the mole fraction.

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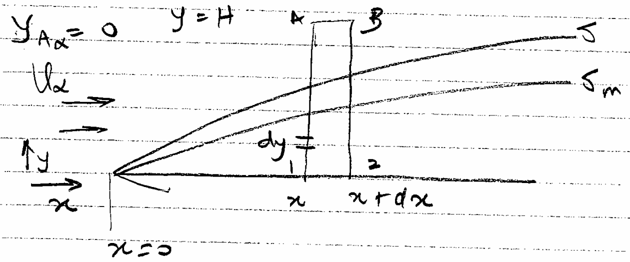

$\delta > \delta_m$

because $Sc = 2.62 > 1$

The velocity distribution is also given as:

$$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Because the gas temp. is constant at 20°C and the total pressure is constant at 1 atm, the molar concentration of the gas mixture,

$$c = \frac{n}{V} = \frac{P}{RT} = \text{constant} \quad (R = \text{universal gas const.})$$

Consider the differential element 1AB2.

Perform a mass balance on the kerosine (species A) in the gas phase.

Take a differential element dy and unit depth, the input of A thro the area is $= c\, y_A\, u\, dy$

Total Input A1 $= \int_0^H c\, y_A\, u\, dy$

Input thro 12 $= N_A\big|_{y=0}\, dx$

Output thro B2 $= \int_0^H c\, y_A\, u\, dy + \frac{d}{dx}\left[\int_0^H c\, y_A\, u\, dy\right] dx$

There is no species A output at AB

Hence overall balance is — The Integral Mass Balance eq:

$$N_A\big|_{y=0} = -cD_{AB}\frac{\partial y_A}{\partial y}\bigg|_{y=0} = \frac{d}{dx}\left[\int_0^H c\, u\, y_A\, dy\right] \qquad (1)$$

To solve this eq., u and y_A are needed as functions of y.

Boundary
Conditions for the problem - concentrations

$$y = 0 \quad y_A = y_{AS} = \frac{P_{vp}^A}{P_{Total}} = \text{a constant at } T$$

$$y = \delta_m \quad y_A = y_{A\alpha} = 0$$

$$y = \delta_m \quad \frac{dy_A}{dy} = 0$$

Assume that $\quad y_A = a + by + cy^2 \quad , \quad \frac{dy_A}{dy} = b + 2cy$

and use the b.c.

at $y = 0 \quad y_{AS} = a$

$$y = \delta_m \quad \frac{dy_A}{dy} = 0 = b + 2c\delta_m$$

$$y = \delta_m \quad 0 = y_{AS} + b\delta_m + c\delta_m^2$$

solve

$$c = \frac{y_{AS}}{\delta_m^2}$$

$$b = -\frac{2y_{AS}}{\delta_m}$$

Substitute

$$y_A = y_{AS} - 2y_{AS}\frac{y}{\delta_m} + y_{AS}\frac{y^2}{\delta_m^2}$$

or

$$\frac{y_A}{y_{AS}} = \left(1 - \frac{y}{\delta_m}\right)^2 \quad \text{where } \delta_m(x) \qquad (2)$$

substitute terms into the Integral equation (1)

$$-c\,D_{AB}\left(-\frac{2y_{AS}}{\delta_m}\right) = \frac{d}{dx}\left[c\,U_\alpha y_{AS}\int_0^{\delta_m}\left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right)\left(1 - \frac{y}{\delta_m}\right)^2 dy\right] \qquad (3)$$

where the limit of the integral has been changed from H to δ_m because $y_A = 0$ for $y \geq \delta_m$

or

$$\frac{2\cancel{c}\,D_{AB}\,\cancel{y_{AS}}}{\delta_m\,\cancel{c}\,U_\alpha\,\cancel{y_{AS}}} = \frac{d}{dx}\left[\int_0^{\delta_m}\left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3\right)\left(1 - \frac{y}{\delta_m}\right)^2 dy\right]$$

$$\frac{2\,D_{AB}}{U_\alpha \, \delta_m} = \frac{d}{dx}\left[\delta \int_0^{\xi} \left(\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \right)\left(1 - \frac{y}{\xi \delta}\right)^2 \frac{dy}{\delta} \right]$$

where a new variable has been defined as

$\xi = \delta_m / \delta$. Let $\eta = y/\delta$

$$\frac{2\,D_{AB}}{U_\alpha \, \xi \delta} = \frac{d}{dx}\left[\delta \int_0^{\xi} \left(\frac{3}{2}\eta - \frac{3\eta^2}{\xi} + \frac{3}{2}\frac{\eta^3}{\xi^2} - \frac{1}{2}\eta^3 + \frac{\eta^4}{\xi} - \frac{1}{2}\frac{\eta^5}{\xi^2} \right) d\eta \right]$$

$$\frac{2\,D_{AB}}{U_\alpha \, \xi \delta} = \frac{d}{dx}\left[\delta \left\{ \frac{1}{8}\xi^2 - \frac{1}{120}\xi^4 \right\} \right]$$

Since $\xi < 1$, neglect 2^{nd} term in bracket of r.h.s.

$$\therefore \quad \frac{16\,D_{AB}}{U_\alpha}\left(\frac{1}{\xi \delta}\right) = \frac{d}{dx}\left(\delta \xi^2\right) = \xi^2 \frac{d\delta}{dx} + \delta \frac{d\xi^2}{dx}$$

$$\frac{16\,D_{AB}}{U_\alpha} = \xi^3 \frac{\delta\, d\delta}{dx} + \delta^2 \xi \frac{d\xi^2}{dx}$$

From the Notes: $\quad \delta\, d\delta = \frac{140}{13}\frac{\nu}{U_\alpha}\, dx \qquad$ eq. 5.18

and $\quad \delta^2 = \frac{280}{13}\frac{\nu x}{U_\alpha} \qquad$ eq. 5.19

Hence

$$\frac{16\,D_{AB}}{U_\alpha} = \xi^3\left(\frac{140}{13}\frac{\nu}{U_\alpha}\right) + \frac{280}{13}\frac{\nu}{U_\alpha} x \, \xi \frac{d\xi^2}{dx} \qquad (4)$$

This is an ordinary differential equation for ξ.

Following the heat transfer example on p. 116 of Notes

$$2x\xi\frac{d\xi^2}{dx} + \xi^3 = \frac{\overset{4}{\cancel{16}}D_{AB}}{\cancel{U_\alpha}}\;\frac{13.\cancel{U_\alpha}}{\underset{35}{\cancel{140}}\,\nu} = \frac{52}{35}\frac{D_{AB}}{\nu}$$

$$4x\xi^2\frac{d\xi}{dx} + \xi^3 = \frac{52}{35}\frac{D_{AB}}{\nu} \qquad (5)$$

This equation is similar to eq. 5.81 (Notes) except the r.h.s.

The solution is

$$\xi^3 = C x^{-3/4} + \frac{52}{35}\frac{D_{AB}}{\nu} \qquad (6)$$

If a section $0 \le x < x_0$ has no kerosine (similar to the unheated leading edge), then

$$\delta_m/\delta = 0 \quad \text{for } x \le x_0$$

$$\therefore \quad \xi = \left[\frac{52}{35}\frac{D_{AB}}{\nu}\right]^{1/3}\left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{1/3}$$

$\therefore$ When $x_0 = 0$

$$\xi = \frac{\delta_m}{\delta} = 1.141\left[\frac{D_{AB}}{\nu}\right]^{1/3} = 1.141\,Sc^{-1/3}$$

$$\therefore \quad \delta_m = 1.141\,Sc^{-1/3}\,\delta \quad , \text{ where } \delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}$$

$$(7)$$

Equation (7) is substituted into eq. (2) to give the concentration profiles required,

i.e. $$\frac{y_A}{y_{As}} = \left(1 - \frac{y}{\delta_m}\right)^2$$

$\longrightarrow$