The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #6 **November 21, 2002**

Time Allowed: 50 mins. **Name:**

Lubricating oils are used in bearings to reduce friction between moving parts. This is common in both domestic and industrial devices and equipment, from the blender, the wheels of a car to rotating heads on drilling rigs.

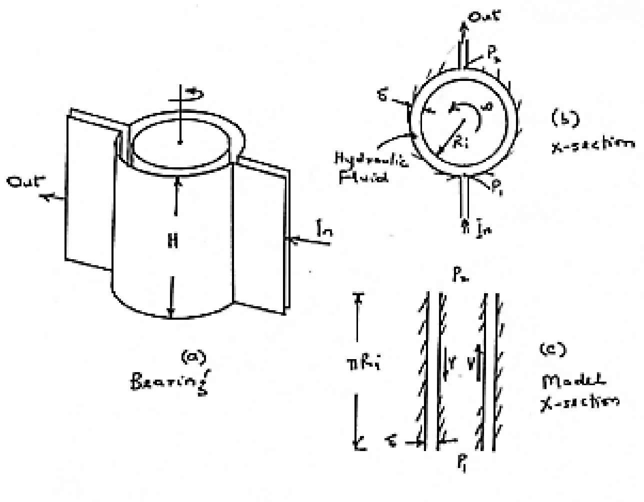
A bearing consists of two parts as illustrated below in fig. (a). The inner part is a cylinder, 12 cm diameter, rotating at 45 rpm. Hydraulic fluid, density 858 kg/m^3 and viscosity 19.2 mPa.s, is between the rotating inner cylinder and the stationary outside cylinder. The outer cylinder is made of two halves, with two slits on opposite ends to admit and flow out the hydraulic fluid. The gap space between the cylinders (δ) is 2 mm. A cross-section of the assembly is shown in fig. (b). The pressures at the slit junctions are p_1 and p_2. It is desired to analyze this system.

Assume that the bearing can be approximated by two flat halves as show in fig. (c). Each side has two flat surfaces one of which moves at velocity V equal the tangential velocity of the inner cylinder and the other surface is stationary. The gap width is δ. For the left half, the pressure opposes the flow of hydraulic fluid. For the right side, the flow is assisted by pressure. The pressures at the ends of the plates are p_1 and p_2.

(a) If the net flow of hydraulic fluid is zero for the left side, estimate the pressure drop across the bearing.

(b) If the height of the bearing is 4 cm, what is the flow rate of the hydraulic fluid through the bearing under the condition for part (a)?

(c) What is the torque (force x radius) that must be applied to turn the inner cylinder for the conditions above?



ENCH 501 Quiz #6 Solution. Nov. 21, 2002

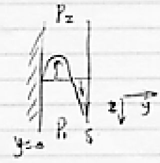
Flow between gap is essentially approximated as flow between flat plates. The velocity distribution across the gap is given by (Notes, p 146 ; Couette flow)

$$\phi = \frac{u}{V} = \frac{y}{\delta} + \Gamma \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) \quad \text{where } \Gamma = + \frac{\delta^2}{2\mu} \frac{\alpha}{L} \frac{1}{V}$$

Let $\eta = y/\delta$

(a) for <u>left</u> side, no net flow $\Rightarrow \int_0^\delta u \, dy = 0$

This is equivalent to



$$\int_0^1 \phi \, d\eta = 0 = \int_0^1 \{\eta + \Gamma(\eta - \eta^2)\} d\eta$$

$$\frac{1}{2}\eta^2 \Big|_0^1 + \Gamma\left(\frac{1}{2}\eta^2 - \frac{1}{3}\eta^3\right)\Big|_0^1 = \frac{1}{2} + \Gamma\left(\frac{1}{2} - \frac{1}{3}\right) \Rightarrow \Gamma = -3$$

$V = R_i\omega \qquad \omega = 2\pi n \; ; \; n = 45 \text{ rpm or } 0.75 \text{ s}^{-1}$

$R_i = 0.06 \text{ m}$

$V = 0.75(2\pi)(0.06) = 0.28274 \text{ m/s}$

$\delta = 2(10^{-3}) \text{ m} \qquad ; \; L = \pi R_i = \pi(0.06) = 0.1885 \text{m}$

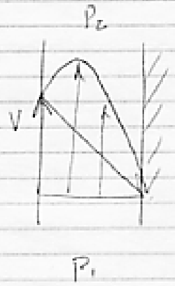
$\mu = 19.2(10^{-3}) \text{ Pa.s} \qquad \text{and} \quad \alpha = P_2 - P_1$

substitute $\Gamma = -3 = -\frac{\delta^2}{2\mu} \frac{P_1 - P_2}{L} \frac{1}{V}$

$\therefore \; P_1 - P_2 = 1534.92 \text{ Pa} \longrightarrow$

(b) There is no net flow in left side, so all the hydraulic fluid flows through the right side with ΔP as for part (a).

For the right side, $\Gamma = 3$ because only the sign of ΔP is reversed



i.e. $\frac{u}{V} = \eta + 3(\eta - \eta^2) = \phi$

The volume rate

$$Q = H\int_0^\delta u\,dy = HV\delta \int_0^1 \phi\,d\eta$$

$$= HV\delta \int_0^1 \left[\eta + 3(\eta - \eta^2)\right] d\eta$$

$$= HV\delta\left[\frac{1}{2} + \frac{3}{6}\right] = HV\delta$$

$$\therefore\ Q = (0.04)(0.28274)(0.002)\ m^3/s$$

$$= 2.26\,(10^{-5})\ m^3/s \longrightarrow$$

(c) The force on the cylinder consists of 2 parts.

For left side $\quad \frac{u}{V} = \phi = \eta - 3(\eta - \eta^2)$

$$\tau_w = +\mu \frac{\partial u}{\partial y}\Big|_{y=\delta} = +\frac{\mu V}{\delta}\frac{\partial \phi}{\partial \eta}\Big|_{\eta=1} \quad ; \quad \frac{\partial \phi}{\partial \eta} = 1 - 3 + 6\eta$$

$$\tau_{w_L} = +\frac{4\mu V}{\delta}, \quad \text{Force} = \tau_w \cdot \text{Area} = \tau_{w_L} \cdot H(\pi R_i)$$

For right side $\quad \frac{u}{V} = \phi = \eta + 3(\eta - \eta^2)$

$$\frac{\partial \phi}{\partial \eta}\Big|_1 = 1 + 3 - 6\eta\Big|_{\eta=1} = -2$$

$$\tau_{w_R} = -\frac{2\mu V}{\delta} \quad \text{and} \quad \text{Force} = \tau_w \cdot H(\pi R_i)$$

$$\text{Net Force} = \left(+\frac{4\mu V}{\delta} - \frac{2\mu V}{\delta}\right) H\pi R_i = +\frac{2\mu V}{\delta} \cdot (H\pi R_i)$$

$$\text{Torque} = +\frac{2\mu V}{\delta} \cdot R_i(H\pi R_i) = +\frac{2(19.2)(10^{-3})(0.28274)}{2(10^{-3})} \cdot 0.06\,(H\pi R_i)$$

$$= 2.456(10^{-3})\ N \cdot m \longrightarrow \quad \text{clockwise}$$