The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501 : Mathematical Methods in Chemical Engineering
Quiz #6

Time Allowed: 50 min. **November 20, 2001** *AJ*

A barge containing 0.2 million barrels of crude oil was involved in an accident in the middle of a large calm lake and the vessel broke into two parts, spilling its entire cargo. The oil formed a circular patch of a thickness of 1.6 cm initially on the lake. Then it started to spread. Given the assumptions and data below,

i) Plot the radius versus time pattern for the oil layer and explain your method and reasoning;

ii) Estimate how long the oil slick will take for its thickness to decrease to 1 mm.

Assumptions and Data:

1) There are three regimes of spreading and the transitions between the regimes occur at oil layer thicknesses of 0.65 and 0.2 cm.
2) One barrel of oil is equivalent to 159 litres.
3) Densities of water and oil are 998 and 952 kg/m^3 respectively.
4) The interfacial tension between oil and water is 0.025 N/m.
5) Viscosity of water is 1.6 mPa.s.
6) The oil is more viscous than water; it does not vaporize, expand or contract over the period of interest. The oil neither absorbs substances from water nor releases compounds into the water. There is also no biological activity.
7) There are no currents in the lake and the air is still.

ENCH 501 Quiz #6 Sol'n Nov. 20, 2001

Volume spilled = 0.2 m barrels or $\forall = 31{,}800\ m^3$

Given the patch is a disc, $\pi R^2 h = \forall$

$h = 1.6$ cm or 0.016 m

At $t = 0$, $R = 795.39$ m

When $h = 1$ mm or 0.001 m, $R = 3181.55$ m.

$h = 6.5$ mm or 0.0065 m $R = 1247.9$ m

and $h = 2$ mm $R = 2249.7$ m

$$\Delta = \frac{998 - 952}{998} \simeq 0.046$$

$\mu = 1.6$ mPa·s or $1.6(10^{-3})$ mPa·s.

(a) First stage of spreading — Dominant forces — gravity + Inertia

By order of magnitude, $R \sim (g \Delta \forall)^{1/4} t^{1/2}$ (from Notes) (1)

$2^{1/4} \sim 1.19$

For second stage, Dominant forces — gravity + viscous

and $R \sim \nu^{-1/12} (g\Delta)^{1/6} \forall^{1/3} t^{1/4}$ (2)

and for 3rd stage, Dominant forces — surface tension + viscous

$R \sim \sigma^{1/2} (\rho\mu)^{-1/4} t^{3/4}$ (3).

Time t has entered into all these relationships (as shown in the Notes) from velocity, acceleration or rate of change. It is not absolute time. Only for the first one is time relative to the start of spreading!

If a log-log plot is used, eq. ① becomes

$$\ln\left(\frac{R}{\beta}\right) \sim \frac{1}{2}\ln t \quad ; \quad \beta = (g\Delta\forall)^{1/4}$$

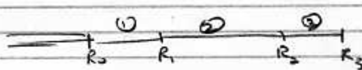
$$= 10.9449 \ \frac{m}{s^{1/2}}$$

and both sides are dimensionally consistent.

When $R = 795.39$ m, $t = 5281$ s

Use log-log plot + locate point.
Draw line slope $\frac{1}{2}$ to $R = 1248$ m.
In $1248 < R < 2250$ m, draw slope $1/4$
for $R > 2250$ m, draw slope $3/4$. ⟶

ⓑ Final time $\sim 246{,}000$ s for $h = 1$ mm
Time difference $= (246{,}000 - 5281) = 66.87$ hrs ⟶
< 2.8 days.

Alternate. 

for the different regimes.

$$R_1 - R_0 = (g\Delta\forall)^{1/4}(t^{1/2}) \qquad t_1 \sim 1708 \text{ s}$$

$$R_2 - R_1 = \nu^{-1/12}(g\Delta)^{1/6}\forall^{1/3}\left[t_2^{1/4} - t_1^{1/4}\right] \qquad t_2 \sim 1.12(10^5)\text{ s}$$

$$R_3 - R_2 = \sigma^{1/2}(\rho\mu)^{-1/4}\left[t_3^{3/4} - t_2^{3/4}\right] \qquad t_3 \sim 2.968(10^5)\text{ s}$$

$\therefore$ time ~ 82.46 hrs.

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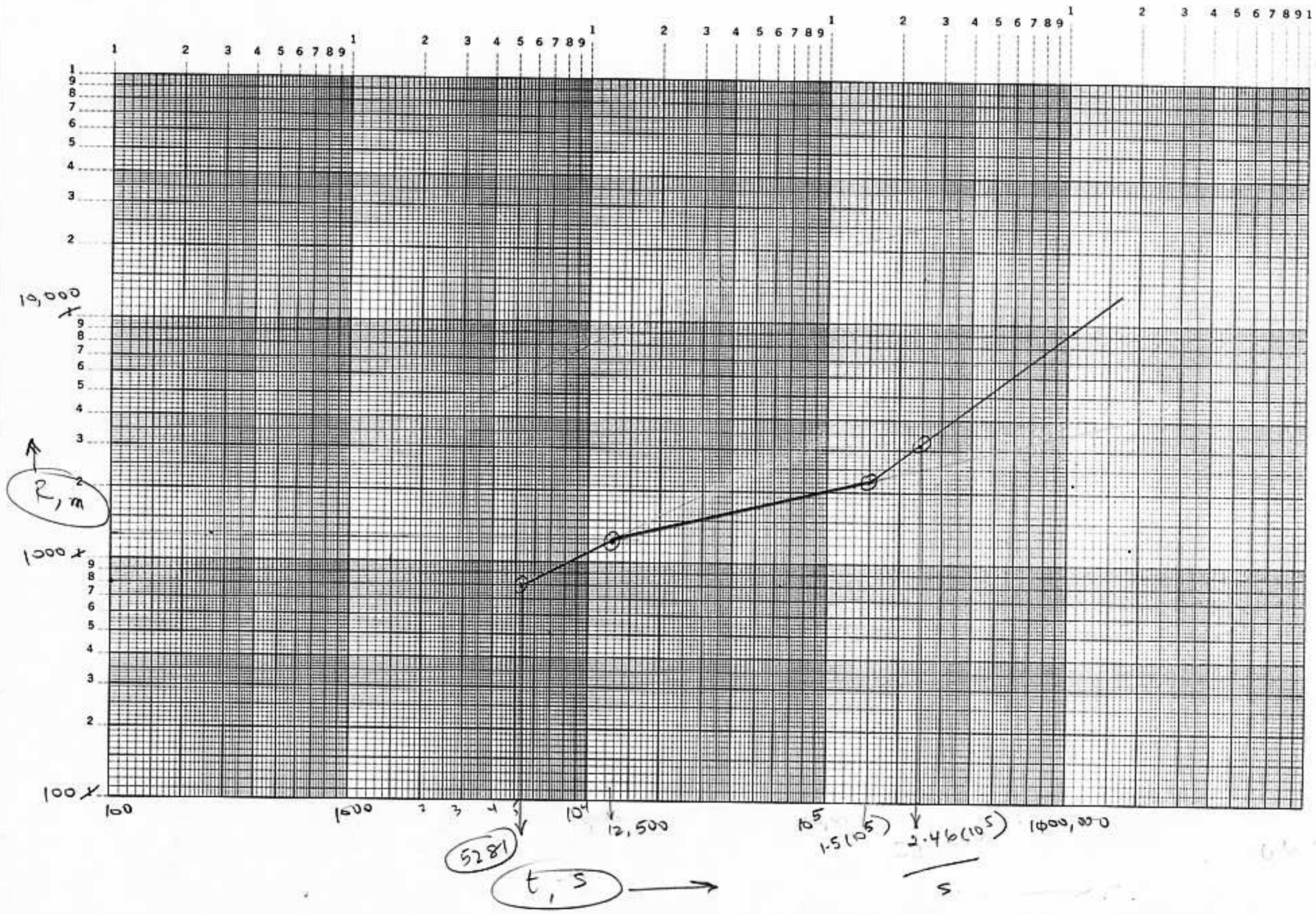


□ Oil Spreading on Calm Water

4 basic forces – cause / retard

$\rho_o = \rho_w (1-\Delta)$; $\Delta \ll 1$

oil in hydrostatic eqm in vertical direction

$\rho_o h = \rho_w y$

$y = (1-\Delta) h$, i.e. Δh is above mean water surface.

$\leftarrow \frac{L(t)}{\text{radius}} \rightarrow$

$\leftarrow P = \rho g \Delta h$

more accurately, force on submerged surface $= \frac{1}{2}\rho_w g h(1-\Delta)\Delta \times$ area

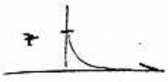
pressure $F \sim (\rho g \Delta h)(Lh)$. (2/11)

inertia mass × accel^n $\sim \rho h L^2 \left(\frac{L}{t^2}\right)$

viscous $\mu \frac{du}{dz} \cdot 2\pi r dr \sim \mu \left(\frac{L}{t}\right) z^{-1} \cdot L^2$

$z \sim \sqrt{\nu t}$

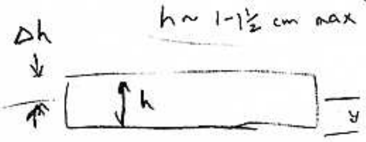
interfacial σl



stoke's first problem

From Notes

$h \sim 1$-$1\frac{1}{2}$ cm max



pressure (gravity) – cause spreading

inertia – retard

viscous – retard

interfacial tension – const.

neglect – evaporation of lighter compon's
– dissolution in sea water
– biological degradation

$t \to 0$

gravity > surface tension

$h > \left[\frac{\sigma}{\rho g \Delta}\right]^{\frac{1}{2}} \sim 1$ cm. ; $V \sim h l^2$

① long time

$h \to 0$

viscous / surface tension

$l = 1.33\, \sigma^{\frac{1}{2}} \left(\frac{\rho}{\mu}\right)^{\frac{1}{4}} t^{\frac{3}{4}}$

for volume released

② short time

pressure / inertia.

gravity / inertia (spreading)

$l \sim [g\Delta V]^{\frac{1}{4}} t^{\frac{1}{2}}$

$\frac{dl}{dt} \sim t^{-\frac{1}{2}}$

gravity / viscous (viscous regime)

$l \sim \nu^{-\frac{1}{12}} (g\Delta)^{\frac{1}{6}} V^{\frac{1}{3}} t^{\frac{1}{4}}$

$\frac{dl}{dt} \sim t^{-\frac{3}{4}}$

surface tension / viscous

$l \sim \sigma^{\frac{1}{2}} \left[\frac{\rho}{\mu}\right]^{\frac{1}{4}} t^{\frac{3}{4}}$

$\frac{dl}{dt} \sim t^{-\frac{1}{4}}$