The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Mathematical Methods in Chemical Engineering
Quiz #6

Time Allowed: 45 mins. **November 21, 2000**

Student's Name: ______________________

(a) Show that equation 6.36 in your Notes can be obtained from equation 6.47 when $U_o = 0$.

(b) An atomizer consists of 2 concentric tubes with a liquid flowing in the inner tube and nitrogen in the annular space. The tubes are stainless steel. The outside tube has an external diameter of 5 mm and a wall thickness of 0.5 mm. The inner tube has an i.d. of 2 mm and a wall thickness of 0.4 mm.

(i) If the flow of nitrogen ($\rho = 1.12$ kg/m^3 ; $\mu = 1.79\ (10^{-2})$ mPa•s) is just laminar, i.e. at the transition of being turbulent, what is the maximum velocity in the annulus?

(ii) What is the ratio of the shear forces on the inner and outer tubes (per unit length) due to the nitrogen flow?

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(a) Equation 6.47 in Notes – Couette flow in annular space

$$u = -\beta\left[1-\frac{r^2}{R^2}\right] + \frac{1}{\ln\lambda}\left\{U_0 + \beta(1-\lambda^2)\right\}\ln\frac{r}{R}$$

where λR is radius of external surface of inner tube, R is radius of inner surface of outer tube.

In equation 6.36, κ is equivalent to the above λ. Hence eq. 6.47 is

$$u = -\beta\left[1-r^2/R^2\right] + \frac{1}{\ln\kappa}\left\{U_0 + \beta(1-\kappa^2)\right\}\ln\frac{r}{R}$$

When $U_0 = 0$

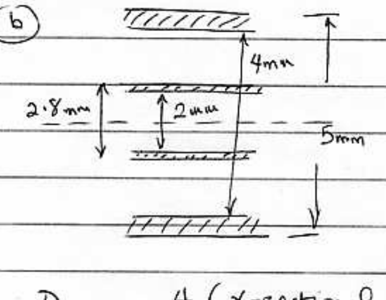
$$u = -\beta\left\{1 - r^2/R^2 - \frac{1-\kappa^2}{\ln\kappa}\ln\frac{r}{R}\right\}$$

$$= -\beta\left\{1 - r^2/R^2 + \frac{1-\kappa^2}{\ln 1/\kappa}\ln r/R\right\}$$

where $\beta = \frac{dP}{dz}\frac{R^2}{4\mu}$

This is the same equation as 6.36 for flow in a horizontal annular space.

(b)



Let $R = 2\,mm$

$\kappa R = 1.4\,mm$

$\kappa = 0.7$

$$Re = \frac{D_H \bar{u} \rho}{\mu} = 2100 \text{ at transition}$$

$$D_H = \frac{4\,(\text{X-sectional area})}{\text{wetted Perimeter}}$$

$$= \frac{4^2 \pi R^2 (1-\kappa^2)}{2\pi R(1+\kappa)} = 2R(1-\kappa)$$

$$D_H = 2(2)(10^{-3})(1-0.7) = 1.2(10^{-3})\ m$$

$$\therefore\ 2100 = \frac{1.2(10^{-3})\,\bar{u}\,(1.12)}{1.79(10^{-5})};\quad \bar{u} = 27.97\ m/s$$

Eq. 6.38

$$\bar{u} = \frac{\xi R^2}{8\mu L}\left[\frac{1-\kappa^4}{1-\kappa^2} - \frac{1-\kappa^2}{\ln\frac{1}{\kappa}}\right] = \frac{\xi}{2}\left[0.06013\right]$$

(ii) Eq. 6.37

$$U_{max} = \xi\left[1 - \left\{\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}}\right\}\left\{1 - \ln\left\{\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}}\right\}\right\}\right]$$

$$\therefore\ \xi = 930.328\ m/s$$

and $U_{max} = 42.01\ m/s$ →

(ii) Ratio of shear forces — inner : outer

$$\Lambda = \left| \frac{-\tau_{rz}|_{\kappa R}\, \cancel{2\pi} \kappa \cancel{R} \cancel{L}}{\tau_{rz}|_{R}\, \cancel{2\pi} \cancel{R} \cancel{L}} \right|$$

$$= \frac{\tau_{rz}|_{\kappa R}\, \kappa}{\tau_{rz}|_{R}}.$$

Using eq. 6.35

$$\Lambda = \frac{\kappa \cdot \cancel{\frac{\Delta \mathcal{P} R}{2L}} \left[\kappa - \frac{1}{2\kappa} \frac{1-\kappa^2}{\ln \frac{1}{\kappa}} \right]}{\cancel{\frac{\Delta \mathcal{P} R}{2L}} \left[1 - \frac{1}{2} \frac{1-\kappa^2}{\ln \frac{1}{\kappa}} \right]}$$

$$= \frac{\kappa^2 - \frac{1}{2} \frac{1-\kappa^2}{\ln(1/\kappa)}}{1 - \frac{1}{2} \frac{1-\kappa^2}{\ln(1/\kappa)}} = \left| \frac{-0.225}{0.285} \right|$$

$$= 0.789 \longrightarrow$$