ENCH 501 Transport Phenomena University of Calgary

Names________________________________ ____

Quiz #5 / Take home. Work in pairs and submit one solution by 9am Nov. 23. November 22, 2016 *aJ*

General scope – This problem is on the new solar roofs recently announced by Tesla and SolarCity. The roofs are made of tiles arranged in overlapping patterns (see figure below) like clay tiles on high-end homes.

The solar tiles are photovoltaic panels arranged on a roof just as regular tiles. Embedded in a layer just below the exposed surface of the glass are tiny cells that convert part of the sun's electromagnetic radiation (spectra in visible light and infra-red) incident on each tile into electricity. The tiles, for the purposes of this analysis, are assume to be 1 cm thick, smooth-surfaced rectangular plate (20 x 30 cm) insulated on the back surface and the narrow sides. The sun's radiation at ground level is, for the period of interest, constant at 560 W/m^2 and the beam is oriented normal to the surface. It is given that 7% of the beam's energy is reflected back into the ambient. The rest is absorbed by the tiny photovoltaic cells at the top surface of each tile. The cells convert 21% of the energy into electricity and release the balance as heat. Since temperature is also rising at the surface of and within the tile, heat is transferred by convection from the tile to ambient air at a constant temperature of 15°C. The convective heat transfer coefficient between the exposed tile surface and ambient air is given as constant at 125 W/m^2 K. The tile was initially at ambient temperature before it was suddenly exposed to the sun's radiation. The thermal conductivity, the density and the specific heat for the tile are respectively 0.8 W/m K, 2210 kg/m^3 and 0.752 kJ/kg K. The dimensionless quantity, Biot number (Bi), equals hL/k.

a) *(5pts)* Obtain an expression for the temperature profile T(x, t) within the tile, in the direction of the radiation. Use the ***integral method***. Neglect any heat generated by the flow of current. Show your steps.
b) *(3 pts)* Plot the temperature profiles at 20s, 60s and at steady state (at ~ 10 minutes). Use Excel. What patterns do you observe?
c) *(2 pts)* Plot temperature profiles at 60s for h values of 5, 25 and 125 W/m^2 K. What patterns do you see?

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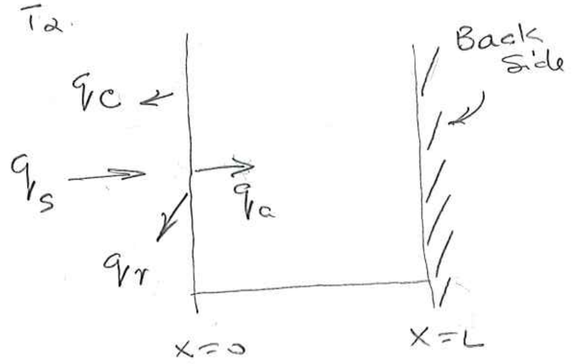
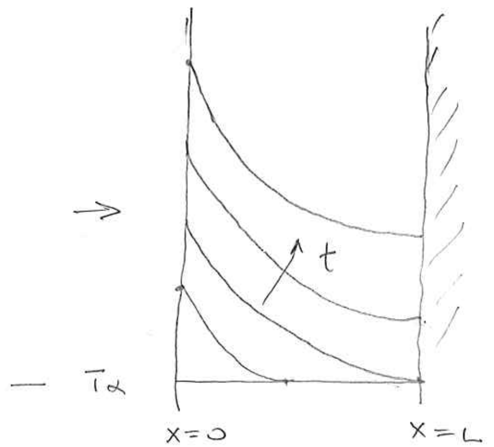


q_s = constant energy flux from the sun

q_r = reflected radiation, $0.07\, q_s$

q_c = convective heat transfer from tile surface

The profiles expected for temperature in the tile are as follows:



In the tile, the rates of absorption q_a and convection q_c will change with time.

The control volume is the space $0 \le x \le L$

Energy balance equation is

$$\text{Input} + \text{Gen} = \text{Output} + \text{Accum}$$

Let the fraction of radiation reflected from incident beam $= \varepsilon$

$$\text{Energy Input} = (1-\varepsilon)\, q_s = q_a$$

Generation is through conversion of radiation into electricity. Given that efficiency equals λ, this energy is withdrawn

$$-\lambda\, q_a$$

Output $\quad q_c = h\left(T\Big|_{x=0} - T_\alpha\right)$

Accumulation

$$\frac{d}{dt}\left[\int_0^L \rho\, c_p (T - T_\alpha)\, dx\right]$$

The heat balance equation is

$$-k \frac{dT}{dx}\Big|_{x=0} = h\left(T\Big|_{x=0} - T_\alpha\right) + \frac{d}{dt}\left[\int_0^L \rho\, c_p (T - T_\alpha)\, dx\right]$$

with the conditions

① $\quad -k \dfrac{\partial T}{\partial x}\Big|_{x=0} = (1-\lambda)\, q_a = (1-\lambda)(1-\varepsilon)\, q_s$

② $\quad \dfrac{\partial T}{\partial x}\Big|_{x=L} = 0 \quad$ (insulated)

③ $\quad t = 0, \quad T = T_\alpha$

Let $\theta(x,t) = T(x,t) - T_\alpha = X(x)\Gamma(t)$

At $t=0$ $\quad \theta(x,t) = 0 \quad \therefore \Gamma(0) = 0$

As $t \to \alpha$, profile is steady. $\Rightarrow \Gamma(\alpha) = 1$

At steady state, the conditions are

(i) $x=0$, $-k\left.\frac{dT}{dx}\right|_{x=0} = (1-\lambda)q_a$

(ii) $x=0$, $h\left(T|_{x=0} - T_\alpha\right) = (1-\lambda)q_a$ heat input = heat removed by convection

(iii) $x=L$ $\frac{dT}{dx} = 0$

Assume a temperature profile (at steady state)

$$T = a + bx + cx^2$$

$$\frac{dT}{dx} = b + 2cx$$

From (iii) $0 = b + 2cL$ $\therefore c = -\frac{b}{2L}$

Using (i) $-\frac{(1-\lambda)}{k}q_a = b$

From (ii) $T|_{x=0} = a = T_\alpha + \frac{(1-\lambda)q_a}{h}$

$$\therefore T = T_\alpha + \frac{(1-\lambda)q_a}{h} - \frac{(1-\lambda)q_a x}{k} + \frac{(1-\lambda)q_a}{2Lk}x^2$$

$$T - T_\alpha = (1-\lambda)q_a\left[\frac{1}{h} - \frac{x}{k} + \frac{x^2}{2Lk}\right]$$

at st. st.

$$T(x,\infty) - T_\infty = (1-\lambda) q_a L \left[\frac{1}{hL} - \frac{1}{k}\frac{x}{L} + \frac{1}{2k}\frac{x^2}{L^2} \right]$$

or

$$X(x) = \frac{(1-\lambda) q_a L}{2k} \left[2\frac{k}{hL} - 2\frac{x}{L} + \frac{x^2}{L^2} \right].$$

$$= \beta \left[\frac{2}{B_i} - 2\eta + \eta^2 \right] \rightarrow \left\{ \begin{array}{l} \beta = \frac{(1-\lambda) L q_a}{2k} \\ Bi = hL/k \\ \eta = x/L \end{array} \right.$$

The general solution, at any time, is

$$\theta(x,t) = \beta \left[\frac{2}{B_i} - 2\eta + \eta^2 \right] \Gamma(t)$$

where $\Gamma(t)$ is an unknown function. Substitute into the integral solution

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=0} = -\frac{k}{L} \left.\frac{\partial \theta}{\partial \eta}\right|_{\eta=0} = (1-\lambda) q_a$$

$$h \left(T|_{x=0} - T_\infty \right) = h\,\theta|_{\eta=0} = h \frac{2\beta}{B_i} \Gamma(t)$$

and

$$\int_0^L \rho c_p (T - T_\infty)\, dx = \rho c_p L \int_0^1 \theta \, d\eta$$

$$= \Gamma(t) . \rho c_p L \beta \int_0^1 \left(\frac{2}{B_i} - 2\eta + \eta^2 \right) d\eta$$

$$= 2\rho C_p L \beta \left(\frac{1}{\beta_i} - \frac{1}{3}\right) \Gamma(t)$$

The energy equation becomes

$$(1-\lambda) q_a = h \frac{2\beta}{Bi} \Gamma(t) + 2\rho C_p L \beta \left(\frac{1}{Bi} - \frac{1}{3}\right) \frac{d\Gamma(t)}{dt}$$

$$\frac{L}{k}\frac{(1-\lambda)}{2\beta} q_a = \Gamma(t) + \frac{L^2}{\alpha}\left(\frac{1}{Bi} - \frac{1}{3}\right)\frac{d\Gamma(t)}{dt}$$

$$-(\Gamma - a) = b\frac{d\Gamma}{dt}$$

where

$$a = \frac{L(1-\lambda)}{2k\beta} q_a$$

$$b = \frac{L^2}{\alpha}\left(\frac{1}{Bi} - \frac{1}{3}\right)$$

$$-\frac{1}{b}\int_0^t dt = \int \frac{d\Gamma}{\Gamma - a} = \int_0^\Gamma d\ln(\Gamma - a)$$

$$\ln\left(\frac{a-\Gamma}{a}\right) = -\frac{t}{b}$$

$$a - \Gamma = a e^{-t/b} \quad \text{or} \quad \Gamma = a\left(1 - e^{-t/b}\right)$$

$$\therefore \theta(x,t) = a\beta\left[\frac{2}{Bi} - 2\eta + \eta^2\right]\left(1 - e^{-t/b}\right)$$

The unsteady temperature profile is

$$T(x,t) - T_\alpha = \frac{L(1-\lambda) q_a}{2k}\left[\frac{2}{Bi} - \frac{2x}{L} + \frac{x^2}{L^2}\right]\left[1 - \exp\left(-\frac{\alpha t}{L^2}\frac{3Bi}{(3-Bi)}\right)\right]$$

b $\quad q_a = 0.93\, q_s = 0.93(540) = 502.2 \ W/m^2$

$$\frac{L(1-\lambda)\, q_a}{2k} = \frac{0.01(1-0.21)(502.2)}{2(0.8)} = 2.4796$$

$$Bi = \frac{hL}{k} = \frac{0.01(125)}{0.8} = 1.5625$$

$$\alpha = k/\rho c_p = \frac{0.8}{2210(752)} = 4.8137(10^{-7}) \ m^2/s$$

Use values in temp. profile and plot.

Results – temperature decreased from surface inwards

– Temperature at $x=0$ rose with time.

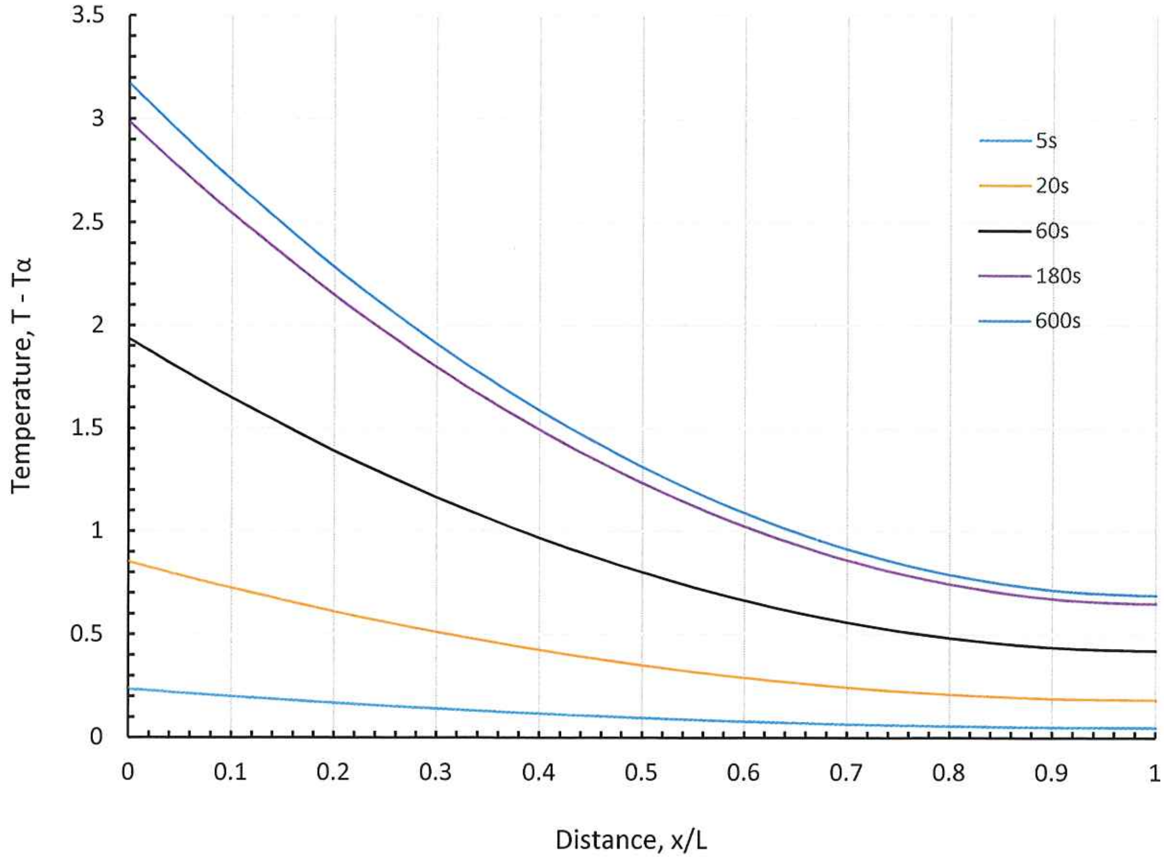
c. With h changing, Bi changes

$h = 5 \ W/m^2K$, $Bi = 0.0625$

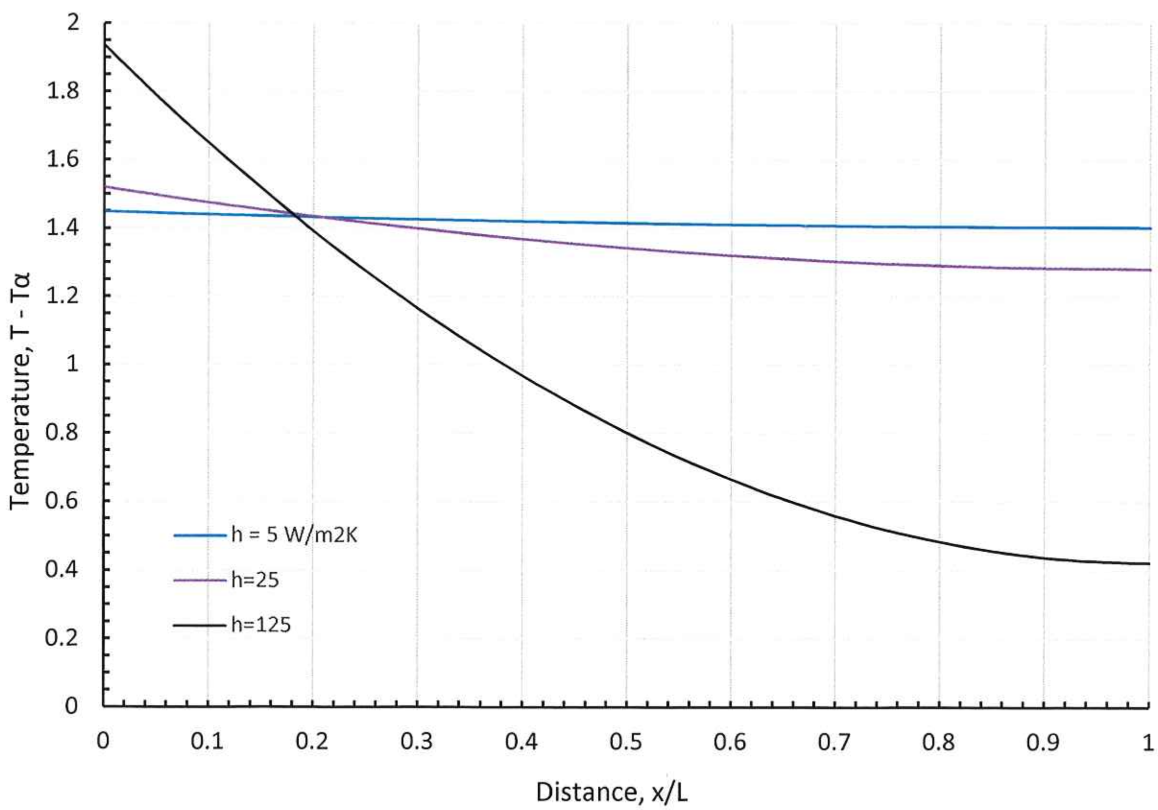
$h = 25 \ W/m^2K$, $Bi = 0.3125$

The plots at 60 s show the following:

- As $h \downarrow$, temperature gradient within the tile decreases – i.e. temp is approaching uniformity
- Temp at $x=0$ $\uparrow$ as $h \uparrow$.
- The curves cross.



3.5
3
2.5
2
1.5
1
0.5
0
Temperature, T - Tα
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Distance, x/L
5s
20s
60s
180s
600s



2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Temperature, T - Tα
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Distance, x/L
h = 5 W/m2K
h=25
h=125

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h = 125 W/m2 K

	t=5	t=20	t=60	t=180	t=600	t=6000
0	0.239576	0.855156	1.936322	2.98573	3.17363	3.173888
0.02	0.232164	0.8287	1.876417	2.893359	3.075446	3.075696
0.05	0.221327	0.790017	1.788829	2.758302	2.931889	2.932127
0.1	0.204014	0.728219	1.648899	2.542536	2.702544	2.702764
0.2	0.172195	0.614644	1.391732	2.145994	2.281047	2.281232
0.3	0.14412	0.51443	1.164819	1.796103	1.909137	1.909292
0.4	0.119788	0.427578	0.968161	1.492865	1.586815	1.586944
0.5	0.099199	0.354088	0.801758	1.236279	1.314081	1.314188
0.6	0.082354	0.29396	0.665611	1.026345	1.090935	1.091024
0.7	0.069252	0.247194	0.559718	0.863063	0.917377	0.917452
0.8	0.059894	0.213789	0.484081	0.746433	0.793408	0.793472
0.9	0.054279	0.193746	0.438698	0.676455	0.719026	0.719084
0.98	0.052482	0.187333	0.424176	0.654062	0.695223	0.69528
1	0.052407	0.187065	0.42357	0.653129	0.694232	0.694288

t=60s

	h = 5	h = 25
0	1.4494	1.520969
0.02	1.447606	1.511558
0.05	1.444984	1.497798
0.1	1.440794	1.475815
0.2	1.433094	1.435414
0.3	1.4263	1.399767
0.4	1.420412	1.368872
0.5	1.41543	1.34273
0.6	1.411353	1.321342
0.7	1.408183	1.304706
0.8	1.405918	1.292824
0.9	1.404559	1.285694
0.98	1.404124	1.283413
1	1.404106	1.283318

Explain what happens at h > 200 W/m2K at t = 60s.