ENCH 501 Transport Phenomena University of Calgary Name_______________________________

Quiz #5 / Time Allowed: 45 minutes You may use class Notes. November 18, 2014 aj

A thick block of cement is to be *cured* by irradiation with heat lamps. The block is initially at 15°C. The heat lamps, directed at the surface of the cement, deliver 170 W/m^2 to the surface. The top 10 cm of the cement is considered *cured* and ready to accept a load of equipment if the temperature at all points within the 10cm top layer has attained a minimum of 45°C and has been maintained at or above this temperature for 90 minutes.

At the instant when the first 10 cm of the layer is *cured,*

(a) How much total energy would have been absorbed by the cement? Use **the integral method** and show the important steps of your analysis.

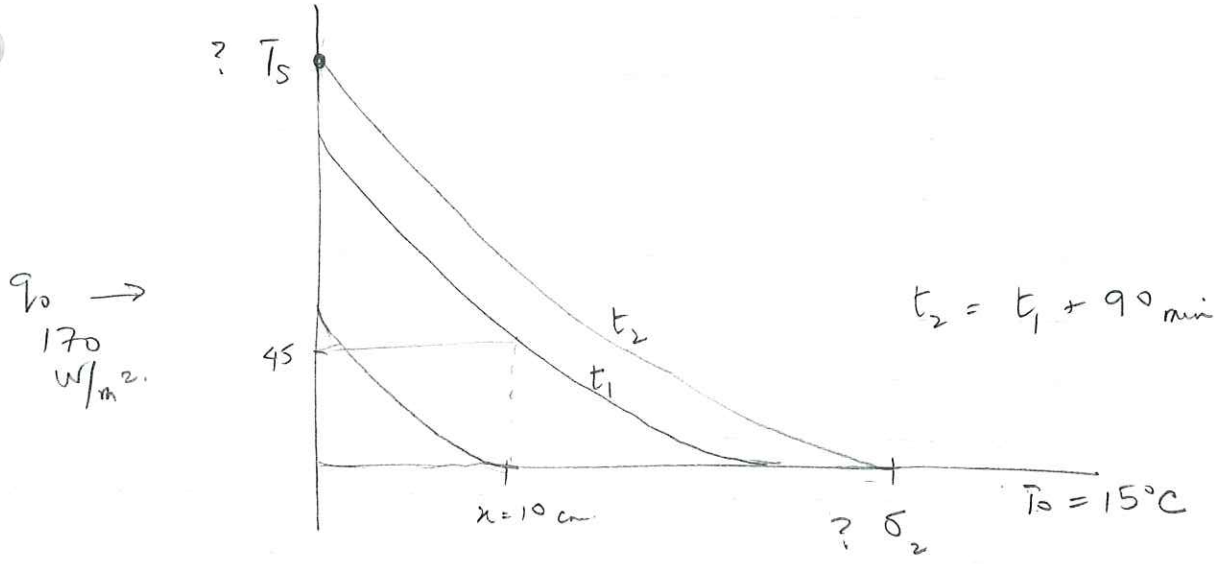
(b) To what depth would heat have penetrated into the cement?

(c) What would the temperature of the exposed surface of the block be at the instant of curing?

Data: Properties of cement – k = 2.9 W/m K, ρ = 1200 kg/m^3, C_p = 0.42 kJ/kg K

ENCH 501 Quiz #5 Sol^n

Nov. 18, 2014



Required to find δ_2, $\int_0^{\delta_2} \rho C_p (T - T_0)\, dx$ or $q_0 \cdot t_2$ and T_s.

$$\alpha = \frac{k}{\rho C_p} = \frac{2.9}{1200(420)} = 5.754 (10^{-6}) \text{ m}^2/\text{s}$$

Use integral method - Notes: Eq. 5.59

$$T - T_0 = \sqrt{\frac{3}{2}} \frac{q_0}{k} \sqrt{\alpha t} \left[1 - 0.4082 \frac{x}{\sqrt{\alpha t}}\right]^2$$

Given that

at t_1 and $x = 0.1$ m, $T = 45$ °C

$$\therefore 45 - 15 = \sqrt{\frac{3}{2}} \frac{(170)}{2.9} \sqrt{\alpha t} \left[1 - 0.4082 \frac{0.1}{\sqrt{\alpha t}}\right]^2$$

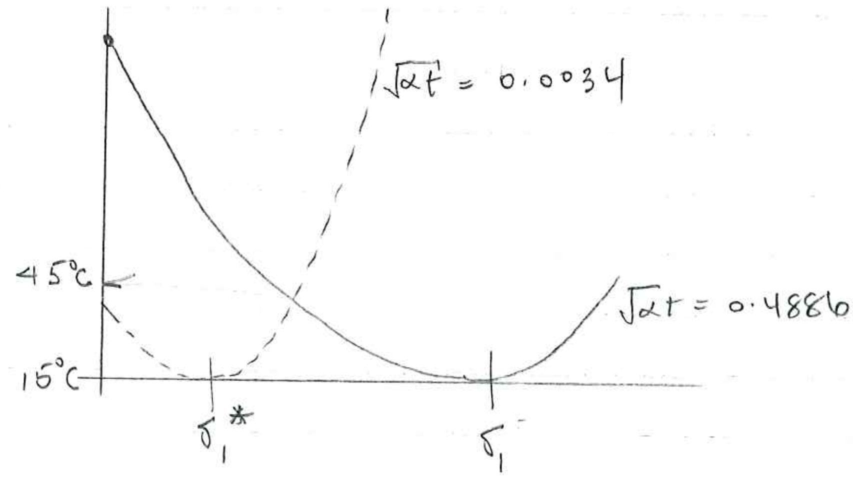
$$30 = 71.795 (\sqrt{\alpha t}) \left[1 - \frac{0.04082}{\sqrt{\alpha t}}\right]^2$$

See attached

$$\frac{0.041}{\sqrt{\alpha t}} \simeq 0.0839 \text{ and } 11.9161$$

{ Exact values - 0.082276 and 12.154292

The correct solution is the first answer since the approximate solution is parabolic.



Note, past δ_1^* & δ_1 solution is not valid!

$\sqrt{\alpha t_1} = 0.4886 \quad \therefore \quad t_1 = 41,482.5\,s$

or 11.523 hours

$t_2 = t_1 + 90_{min} = 46,882.5\,s$

(~ 13 hours) →

$\sqrt{\alpha t_2} = 0.5195$

$\sqrt{6\alpha t} = \delta_2 = 2.4492\sqrt{\alpha t_2} = 1.272\,m$ →

Also $T_s - T_0 = \sqrt{\frac{3}{2}}\,\frac{q_0}{k}\sqrt{\alpha t_2} = \sqrt{\frac{3}{2}}\,\frac{170}{2.9}\sqrt{\alpha t_2}$

$T_s - 15 = 71.795\,(0.5195)$

$T_s = 52.3\,°C$ →

Total Energy absorbed $= q_0 \cdot t_2 = 7.97\,(10^6)\ J/m^2$

→

Let $y = \frac{0.04082}{\sqrt{\alpha t}}$

$$30 = 71.795 \frac{\sqrt{\alpha t}}{0.04082} \cdot 0.04082 \left(1 - \frac{0.04082}{\sqrt{\alpha t}}\right)^2$$

$$= \frac{2.93067}{y}(1-y)^2$$

$$10.236567\, y = 1 - 2y + y^2$$

$$0 = 1 - 12.236567\, y + y^2$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -12.234..$
$c = 1$

y has 2 roots

$y = 0.082276$ and 12.154292.