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ENCH 501: Transport Phenomena Quiz #5 **November 10, 2009**

Time Allowed: 40 mins. **Name:**

Cements hydrates and concretes are used for many applications in construction. This includes for home basements and driveways, public buildings, walls of wells in petroleum fields, highways and airfield runways. As the materials cure and harden, there is a risk of cracking due to thermal and other stresses. At a pre-cast plant, a thick slab is heated in different ways to determine an optimal curing rate.

A concrete slab, initially at 8°C, is placed on the floor and radiant heat from lamps is beamed down onto the top surface. For one of the tests, the flux of heat is varied linearly with time as:

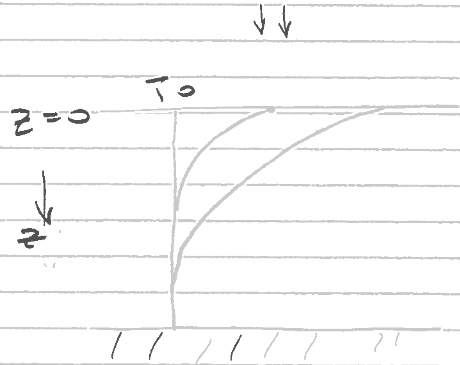
$\mathbf{q} = \mathbf{q_o}(1 + \mathbf{bt})$; where $\mathbf{q_o}$ = 1.2 kW/m^2, b = 1.5(10^{-4}) W/m^2 s, and **t** is time from the start of heating - in seconds.

Use the ***integral method*** to estimate:
a) the times required for the heat to penetrate into the concrete to depths of 10 cm and 25 cm,
b) the temperatures at the surface of the concrete slab at the instants for part a),
c) the total amounts of heat gained by the concrete at the instants of part a).

Assume no heat is lost from the concrete surface by convection into the room.

Data: Properties of the concrete slab
$k = 0.88$ W/m K ; $\rho = 2100$ kg/m^3 ; $C_p = 1.6$ kJ/kg K

ENCH 501 Quiz 5 Solution Nov. 10, 2009



This is a problem involving a semi-infinite domain and prescribed heat flux. This is in the Notes.

The Integral Energy equation is ; basis $1\,m^2$ area

$$q_{z=0} = \frac{d}{dt}\left[\int_0^\delta \rho C_p T\, dz\right] - \rho C_p T_o \frac{d\delta}{dt}$$

p 103, Eq. 5.48

The b.cs. are

$$z = 0, \quad \frac{dT}{dz} = -\frac{F(t)}{k}$$

$$z = \delta \quad \frac{dT}{dz} = 0$$

$$z = \delta \quad T = T_o$$

Assume

$$T = a + bz + cz^2$$ and apply b.c.s

$$T - T_o = \frac{F(t)\delta}{2k}\left(1 - \frac{z}{\delta}\right)^2 \qquad \text{Eq. 5.55.}$$

Substitute into the integral equation + simplify to get

$$\frac{\alpha F(t)}{k} = \frac{d}{dt}\left[\frac{F(t)\delta^2}{6k}\right]$$

Subject to $t = 0, \ \delta = 0$

Solve
$$\delta(t) = \sqrt{6\alpha}\left[\frac{1}{F(t)}\int_0^t F(t)\,dt\right]^{\frac{1}{2}}$$ Eq. 5.57

(a) $F(t) = a + bt$

$$\delta(t) = \sqrt{6\alpha}\left[\frac{1}{a+bt}\cdot at + \frac{b}{2}t^2\right]^{\frac{1}{2}}$$

where $a = 1{,}200 \ W/m^2$

$b = 1.5(10^{-4}) \ W/m^2 s$

(i) $\delta(t) = 0.1 \ m$

$$\alpha = \frac{k}{\rho C_p} = \frac{0.88}{(2100)(1600)} = 2.619(10^{-7}) \ m^2/s$$

$$\therefore \quad 0.1 = \sqrt{6(2.619)(10^{-7})}\left[\frac{12\cancel{00}\left(t + \frac{1.5(10^{-4})}{2}t^2\right)}{12\cancel{00}\left(1 + 1.5(10^{-4})t\right)}\right]^{\frac{1}{2}}$$

$$= 1.2536(10^{-3})\left[\frac{t + 0.75(10^{-4})t^2}{1 + 1.5(10^{-4})t}\right]^{\frac{1}{2}}$$

$$6.3636(10^3) = \frac{t + 0.75(10^{-4})t^2}{1 + 1.5(10^{-4})t}$$

or $t^2 + 606.13t - 8.4848(10^{+7}) = 0$

$$t = \frac{-606.13 \pm \sqrt{(606.13)^2 + 4(8.4848)(10^7)}}{2}$$

$= 8913.2 \ s$ is positive root $\longrightarrow$ for $\delta = 10 \ cm.$

(ii) $\delta(t) = 0.25\,m$

$$3.9771(10^4) = \frac{t + 0.75(10^{-4})t^2}{1 + 1.5(10^{-4})t}$$

or

$$t^2 - 6.6208(10^4)t - 5.3028(10^8) = 0$$

$t = 7.343(10^4)\,s$ is the +ve root.

for $\delta = 25\,cm$

(b) surface temperatures are given as

$$T - T_0 = \frac{F(t)\delta}{2k}\left(1 - \frac{z}{\delta}\right)^2$$

$z \to 0$

(i) $t = 8913.2\,s$, $\delta = 0.1\,m$

$$F(t) = 1.2(10^3)(1 + 1.5(10^{-4})(8913.2))$$

$$= 2.8044(10^3)\ W/m^2$$

$$\therefore T_s = 8 + \frac{2.8044(10^{+3})(0.1)}{2(0.88)}$$

$$= 167.34\ ^\circ C$$

(ii) $t = 7.343(10^4)\,s$, $\delta = 0.25\,m$

$$F(t) = 1.2(10^3)(1 + 1.5(10^{-4})(7.343)(10^4))$$

$$= 1.4417(10^4)\ W/m^2$$

$$T_s = 8 + \frac{1.4417\times(10^4)(0.25)}{2(0.88)}$$

$T_s = 2055.9\ ^\circ C \longrightarrow$

(c) The total amount of heat in slab can be evaluated in two ways —

$$Q = \int_0^t F(t)\,dt \quad \text{or} \quad \int_0^L \rho c_p (T - T_0)\,dx$$

With the first,

(i) $Q = \int_0^t (a + bt)\,dt = at + \frac{b}{2}t^2$

$t = 8913.2\ s$

$$Q = 1.2(10^3)\left(t + 0.75(10^{-4})\,t^2\right)$$

$$= 1.7846(10^7)\ J/m^2$$

(ii) $t = 7.343(10^4)\ s$

$$Q = 5.7339(10^8)\ J/m^2$$

$\longrightarrow$

Notes:

				Ratio
δ	=	$0.1\ m$	$0.25\ m$	2.5
t	=	$8.9132(10^3)\ s$	$7.343(10^4)\ s$	8.24
Q	=	$1.7846(10^7)\ W/m^2$	$5.7339(10^8)\ J/m^2$	32.13