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The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #5 **October 14, 2008**

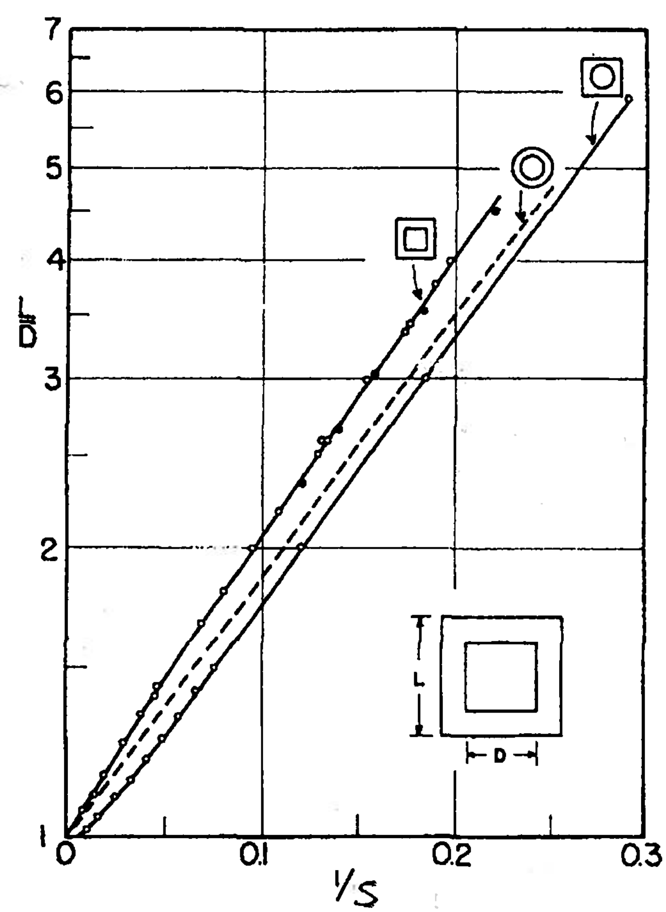
Time Allowed: 45 mins. **Name:**

Waste heat from power plants and industrial facilities are discharged into the atmosphere through chimneys or smoke stacks. The cross-section of the smoke stacks may be square, or circular or a combination such as a round hole in a square wall. You are requested to design a brick chimney which is 12 m tall. Hot combustion gases enter at the base of the chimney at 260°C and a flow rate of 3.9 kg/min. The ambient air is at 10°C and there is a strong wind such that the heat transfer coefficient on the outside wall of the chimney can be assumed to be very large. The heat transfer coefficient inside the chimney is also assumed to be large.

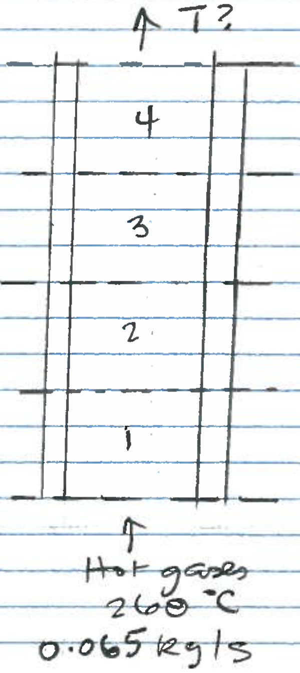
a) If the cross-section of the chimney is square both inside and outside - 0.8 X 0.8 m on the inside and wall is 0.3 m thick, estimate the temperature at which the gas would exit the chimney.
b) If the same exit temperature is to be achieved for a chimney with a circular cross-section (both inside and outside), how thick should the wall be, given that the gas flow rate per unit cross-sectional area of the chimney remains the same?

Data: You are provided a **shape factor** diagram, with S (in m) given per meter height of the chimney.
The heat capacity of the hot gases is 0.78 kJ/kg K.. The thermal conductivity of brick is 0.24 W/mK.

Hint: Treat each 3-meter length of the chimney, from the bottom up, as the control volume. Apply the shape factor method to each segment to estimate the amount of heat loss, assuming the inside and outside wall temperatures are constant at the values at the inlet. Subtract the energy lost that you calculate from the energy content of the flowing gas to get the temperature for the inside wall for the next control volume. For higher accuracy, you may use 1 m length as the control volume.



ENCH 501 Quiz #5 Solution Oct. 14, 2008

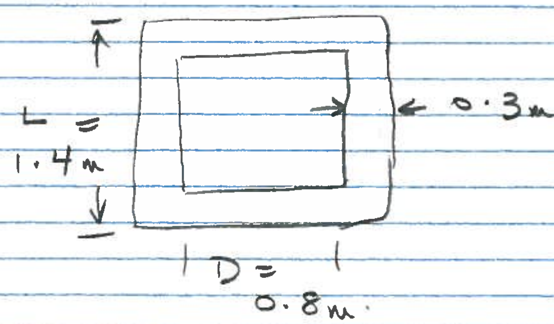


$T_a = 10°C$

$h \to \infty$

If h external to the chimney is very large, the outside wall of the chimney is 10 °C

(a) Consider the square chimney.



$$\therefore \frac{L}{D} = \frac{1.4}{0.8} = 1.75$$

From diagram, $\frac{1}{S} \doteq 0.08$ or $S = 12.5$ m/m height

$\therefore$ For a 3 m segment, $S = 3(12.5)$
$= 37.5$ m

For segment 1

$$Q = k_{wall} S \Delta T = 0.24(37.5)(260-10)$$
$$= 2250 \text{ W}$$

This heat is subtracted from the gas, i.e.

$$2250 = \dot{m} C_p (\Delta T_g) = 0.065(780)(260-T_2)$$

$$T_2 = 260 - 44.38 = 215.62°C$$

This is the approximate temperature inside segment 2.

For segment 2

$$Q = 0.24(37.5)(215.62-10) = 0.065(780)(215.62 - T_3)$$

$$\therefore \quad T_3 = 215.62 - 36.5 = 179.12\ ^\circ C$$

For segment 3

$$Q = 0.24(37.5)(179.12-10) = 0.065(780)(179.12 - T_4)$$

$$\therefore \quad T_4 = 179.12 - 30.02 = 149.1\ ^\circ C$$

For segment 4

$$Q = 0.24(37.5)(149.1-10) = 0.065(780)(149.1 - T_{exit})$$

$$T_{exit} = 149.1 - 24.69 = 124.4\ ^\circ C$$

Exit gas temp. ⟶

(b) If the flux and flow rates are the same for the 2 geometries, then the inside areas are also equal.

$$\therefore \quad \pi R^2 = 0.8(0.8) \quad \text{on the inside}$$

$$R = 0.451\ m \quad \text{or} \quad D = 0.903\ m \longrightarrow$$

For same performance, S must be the same for the 2 systems

$\therefore$ From plot, $S = 12.5$, $\frac{1}{S} = 0.08$

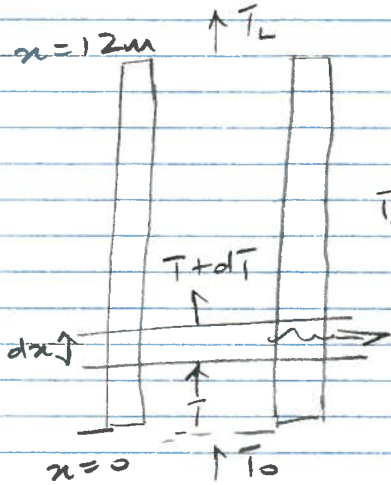
(Log scale), $L/D \doteq 1.6$ $\quad \therefore \quad L = 1.6(0.903)\ m$

or outside diam. $= 1.444\ m$

Wall thickness $= \dfrac{1.444 - 0.903}{2} = 0.271\ m \longrightarrow$

Part (a)

Alternate, more accurate solution — differential



For square wall,

$L/D = 1.75 \quad \therefore \quad \frac{L}{S} = 0.08$

$T_\alpha = 10°C$

or $S = 12.5$ m/m height

$dQ = k\left[S\,dx\right](T - T_\alpha)$

Rate of heat loss from wall to ambient

Rate of energy loss from hot gases in a differential element $= -\dot{m}\,C_p\,dT = k\,S\,(T - T_\alpha)\,dx$

Hence

$$\int_{T_0}^{T_L} \frac{dT}{T - T_\alpha} = -\frac{kS}{\dot{m}\,C_p}\int_0^L dx \quad ; \quad L = 12\,m$$

$$\ln(T - T_\alpha)\Big|_{T_0}^{T_L} = -\frac{kSL}{\dot{m}\,C_p}$$

$$\ln\left(\frac{T_L - T_\alpha}{T_0 - T_\alpha}\right) = -\frac{kSL}{\dot{m}\,C_p}$$

or

$$\frac{T_L - T_\alpha}{T_0 - T_\alpha} = \exp\left[-\frac{kSL}{\dot{m}\,C_p}\right]$$

$$T_L = T_\alpha + (T_0 - T_\alpha)\exp\left[-\frac{kSL}{\dot{m}\,C_p}\right]$$

Substitute values

$$T_L = 10 + (260 - 10)\exp\left[-\frac{(0.24)(12.5)(12)}{(3.9/60)\,780}\right]$$

$$= 132.9\ °C$$

$\longrightarrow$