GJ

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ENCH 501: Transport Processes Quiz #5 **November 7, 2006**

Time Allowed: 50 mins. **Name:**

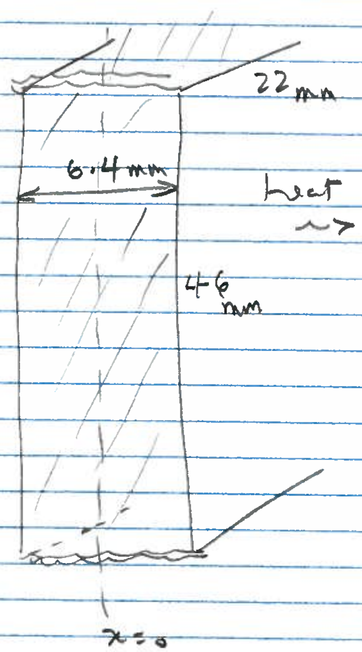
Lithium-Ion batteries have been in the news lately because some have ignited or exploded inside laptops, cell phones and other consumer electronics. This has occurred because of heat build up inside the device. The lithium-ion battery has several advantages over other types of batteries for electronics and electric vehicles, in particularly because it has a high energy storage density, but it self-heats (ohmic) on either charging or discharging (or if short-circuited by contamination). If the temperature exceeds 150°C within the body, it may experience 'thermal runaway' leading to conflagration.

In an electronic device, a lithium-ion battery, 46mm x 22mm x 6.4mm, is held in a bracket along its narrow sides. Felt material acted as both insulator and cushion between the bracket and the battery so that only the two largest areas of the surfaces are exposed to air being moved by a small fan. You may assume that the convective coefficient of heat transfer ***h*** is the same on both sides of the battery. During electrical energy discharge, heat is generated in the device at a rate of 4.5 (10^4) W/m^3 at every location. The initial temperature of the battery is the same as the ambient, constant at 20°C.

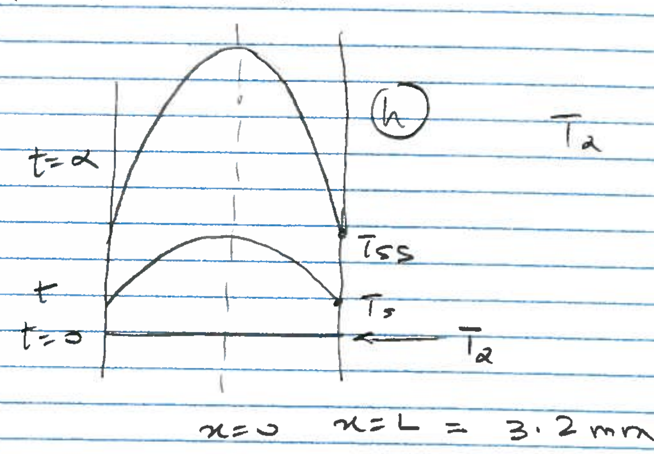
a) *4pts* . Use ***the integral method*** to derive a function for the temperature profiles **T(x,t)** across the thickness of the battery. Show the key steps.
b) *3pts* At what limiting value of ***h*** will the battery be at a risk of a thermal runaway, and what would the surface temperature of the battery be at this condition?
c) *3pts* After how long will the temperature at a distance 2mm into one of the exposed surfaces attain 50°C, and how much heat has been stored in the battery by this time? Assume ***h*** is the same as for part b).

Data: Effective thermophysical properties of Lithium-Ion battery components (with electrolytes)
ρ = 2,680 kg/m^3; C_p = 1.28 kJ/kg K; k = 3.4 W/mK

ENCH 501 Quiz 5 Solution Nov. 7, 2006



The heat transfer is 1-dimensional and unsteady in a finite domain.



The integral energy balance equation in $0 \le x \le L$

Input + Generation = output + Accum.

$$0 \qquad g^{+}AL = q\big|_{x=L} A + \frac{d}{dt}\left[\int_{x=0}^{L} A\rho c_p (T - T_\infty)\, dx\right]$$

Conditions are

b.c. $x = 0$ $\quad \dfrac{dT}{dx} = 0$, symmetry

$x = L$ $\quad -k\dfrac{dT}{dx} = h(T_s - T_\infty)$, convective

Note that T_s is a function of time.

i.c. $t = 0$ $\quad T = T_\infty$

Define $\theta = T - T_\infty$ (local rise in temperature)

and $\theta_s = T_s(t) - T_\infty$

Let $\theta(x,t) = X(x)\,\Gamma(t)$ where $X(x)$ is the steady state solution

The steady-state solution is given by the conditions:

$$x = 0 \qquad \frac{dT}{dx} = 0$$

$$x = L \qquad g^+ L = -k \left.\frac{dT}{dx}\right|_L$$

$$x = L \qquad T = T_{ss}, \text{ constant}$$

assume $T = a + bx + cx^2$, we obtain

$$T(x,\alpha) - T_{ss} = \frac{g^+ L^2}{2k}\left(1 - \frac{x^2}{L^2}\right); \quad \text{Let } \lambda = \frac{g^+ L^2}{2k}$$

$$\frac{dT}{dx} = -\frac{2\lambda}{L^2} x$$

But at steady state

$$g^+ L = -k \left.\frac{dT}{dx}\right|_{x=L} = h\left(T_{ss} - T_\alpha\right)$$

$$\therefore \quad T_{ss} - T_\alpha = +\frac{k}{h}\left(\frac{2\lambda}{L}\right)$$

from above

$$T(x,\alpha) - T_{ss} = \left(T(x,\alpha) - T_\alpha\right) - \left(T_{ss} - T_\alpha\right)$$

$$\lambda\left(1 - \frac{x^2}{L^2}\right) = T(x,\alpha) - T_\alpha\big) - \left(\frac{2k}{hL}\right)\lambda$$

or $$T(x,\alpha) - T_\alpha = \lambda\left(1 - \frac{x^2}{L^2} + \frac{2k}{hL}\right)$$

Define Biot #, $Bi = \frac{hL}{k}$

$$T(x,\alpha) - T_\alpha = \lambda\left(1 - \frac{x^2}{L^2} + \frac{2}{Bi}\right)$$ steady state sol^n.

Unsteady profiles given by

$$\left(T(x,t) - T_\alpha\right) = \theta(x,t) = \lambda\left(1 - \frac{x^2}{L^2} + \frac{2}{Bi}\right)\Gamma(t)$$

substitute this into the integral equation to get

$$\rho c_p \lambda \frac{d}{dt}\left[\Gamma(t)\int_0^L \left(1 - \frac{x^2}{L^2} + \frac{2}{B_i}\right)dx\right] - g^+ L - k\left.\frac{dT}{dx}\right|_{x=L} = 0$$

or

$$2\rho C_p \lambda L\left(\frac{1}{3} + \frac{1}{B_i}\right)\frac{d\Gamma}{dt} - g^+ L + \left(\frac{2k\lambda}{L}\right)\Gamma = 0$$

This is an o.d.e. with condition $t = 0$, $\Gamma = 0$

Solution

(a)
$$\frac{T(x,t) - T_\alpha}{\left(\frac{g^+ L^2}{k}\right)} = \frac{1}{2}\left[1 - \frac{x^2}{L^2} + \frac{2}{B_i}\right]\left[1 - \exp\left(-\frac{3\alpha t}{L^2}\cdot\frac{B_i}{B_i+3}\right)\right]$$

$\longrightarrow$

(b) The limiting temperature $= 150°C$ and this will be attained first at $x = 0$. This solution should also be the steady state profile.

$$\therefore \frac{T(x,\alpha) - T_\alpha}{\left(\frac{g^+ L^2}{k}\right)} = \frac{1}{2}\left[1 + \frac{2}{B_i}\right]$$

$$\frac{g^+ L^2}{k} = \frac{4.5(10^4)(3.2)^2(10^{-6})}{3.4} = 0.1355 \text{ K}$$

$$\frac{150 - 20}{0.1355} = \frac{1}{2}\left[1 + \frac{2}{B_i}\right]$$

$$B_i = \frac{hL}{k} = 1.0431(10^{-3})$$

$$h = \frac{1.0431(10^{-3})(3.4)}{(3.2)(10^{-3})} = 1.108 \ W/m^2 K$$

This is a low value which can be easily obtained.

At this condition, the surface temperature T_{ss} is given by

$$T_{ss} = T(L, \alpha) = T_\alpha + \frac{g^+ L^2}{2k}\left[\cancel{1} - \cancel{\frac{L^2}{L^2}} + \frac{2}{Bi}\right]$$

$$= 20 + \frac{0.1355}{1.0431(10^{-3})} = 149.9 \ °C$$

Since the centre line is at 150°C, temperature gradients across the battery are inst substantial.

(c) $T(x, t) = 50°C$, $x = (3.2 - 2)(10^{-3})\ m$
$= 1.2(10^{-3})\ m$

$T_\alpha = 20°C$

$$\frac{50 - 20}{0.1355} = \frac{1}{2}\left[1 - \left[\frac{1.2(10^{-3})}{3.2(10^{-3})}\right]^2 + \frac{2}{1.0431(10^{-3})}\right] \cdot \left[1 - \exp\left(-\frac{3\alpha t}{(3.2)^2(10^{-6})} \cdot \frac{1.0431(10^{-3})}{3}\right)\right]$$

Thermal diffusivity

$$\alpha = \frac{k}{\rho C_p} = \frac{3.4}{2680(1280)} = 9.9114(10^{-7})\ m^2/s$$

$t = 2{,}599.6\text{s}$ or 0.722 hrs

The heat stored in the battery is obtained by integrating $T(x,t)$ over the space, i.e.

$$Q = 2\int_0^L \rho C_p (T - T_a) A \, dx$$

where 2 is because the integral is over the half space.

$$A = 46(10^{-3}) \times 22(10^{-3}) = 1.012(10^{-3}) \ m^2$$

$$Q = 2\rho C_p A \int_0^L (T(x,t) - T_a) dx$$

$$= 2\rho C_p A \cdot \frac{g^+ L^2}{2k}\left(1 - \exp\left(-\frac{3\alpha t}{L^2} \cdot \frac{B_i}{B_i + 3}\right)\right)\int_0^L \left(1 - \frac{x^2}{L^2} + \frac{2}{B_i}\right) dx$$

$$= 470.4\left(1 - \exp\left(-0.26246\right)\right)\left\{\left(1 + \frac{2}{B_i}\right)L - \frac{L}{3}\right\}$$

$$= 108.589\left\{\frac{1}{3} + \frac{1}{B_i}\right\} 2L \quad \text{Joules}$$

$$= 666.48 \ J \longrightarrow$$

If it is assumed (as suggested earlier) that the temp. gradient is small, then

$$Q \simeq 2LA\rho C_p (T - T_a)$$

$$2(3.2)(10^{-3})(1.012)(10^{-3})(2680)(1280)(50-20)$$

$$\simeq 666.54 \ J \longrightarrow$$

Temperature gradients may be neglected for this problem.