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ENCH 501: Transport Processes Quiz #5 **November 9, 2005**

Time Allowed: 45 mins. **Name:**

Electricity from nuclear power is generated from uranium U_{238} enriched to 2-3% with the isotope U_{235}. The fuel is made into 1.8cm diameter ceramic cylinders which are stacked inside thin-walled zirconium or stainless steel tubes to form fuel rods. The tubes are 3.66m long and, compared to the diameter, are considered very long. 200 to 300 fuel rods and as many or more rods of graphite or boron (moderators) form the core of the nuclear reactor. The fission rate and, therefore, the rate of energy release as heat by the decay of uranium is controlled (increased or decreased) by raising or lowering the moderator rods in between the fuel rods.

Consider one of the fuel rods. At start-up of the nuclear plant (after every shut down to change fuel rods or for maintenance), the pressurized water in which the rods are immersed (in one reactor type) is pre-heated to 316°C. This is the initial temperature of the rods. At t=0, the moderator rods are pulled up to a certain level so that heat is generated at a constant rate per unit volume of the fuel rods. The water is circulated between the reactor core and a boiler to generate steam to drive a turbine and produce electricity.

For the following problems, use ***the integral method*** and show all your derivations. Neglect the thermal resistance of the thin wall around the fuel.

a) If at steady state, the centreline of the fuel rod is at a temperature of 925°C and the surface is at 400°C, estimate the rate of heat generation per unit volume of the fuel rod.

b) Estimate the heat transfer coefficient (h) around the fuel rod.

c) After how long from the start will the centreline temperature become 815°C? What is the surface temperature of the fuel rod at this instant?

Data: Properties of uranium (assume constant) :

Density = 19,050 kg/m^3 ; Specific heat = 0.1164 kJ/kg K ; Thermal conductivity = 27 W/m K

Melting pt. 1132.2°C ; P_{water} ~ 2000 psia; Rods about 2 yr life

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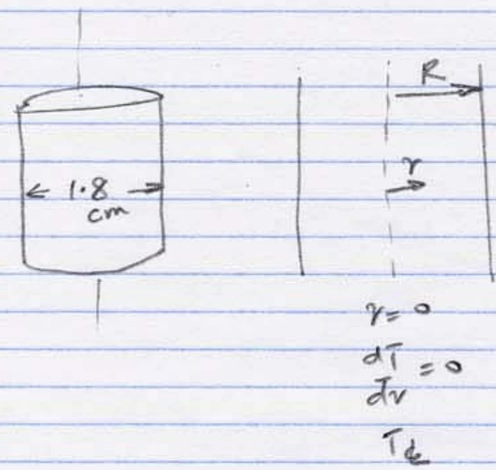
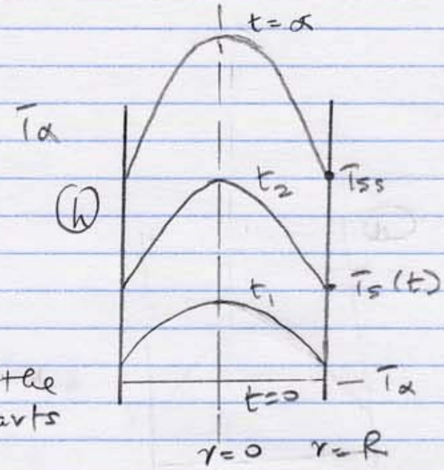
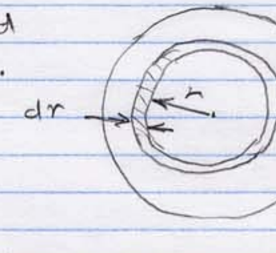


$T_\alpha = 316°C$



The time-dependent temp. pattern in the rod is sketched to the right. The rod starts at a uniform temperature T_α. As heat is generated, some is stored in the rod and the rest is transferred into the pressurized water at T_α.



The energy balance for a length L of rod is given by:

Input + Generation = Output + Accum.

$$\underbrace{\text{Input}}_{0} + g^+(\pi R^2 L) = q_{r=R}(2\pi R L) + \frac{d}{dt}\left[\text{Energy Content}\right]$$

where g^+ is energy generation rate / unit volume

and $$\text{Energy Content} = \int_0^R (T-T_\alpha)\rho C_p 2\pi r\,dr \cdot L$$

$$\therefore \quad g^+(\cancel{\pi} R^2 \cancel{L}) = -k\frac{dT}{dr}\bigg|_{r=R}(2\cancel{\pi} R\cancel{L}) + \frac{d}{dt}\int_0^R \rho C_p (T-T_\alpha) 2\cancel{\pi}\cancel{L}\, r\,dr$$

$$g^+ R^2 = -k\frac{dT}{dr}\bigg|_R (2R) + \frac{d}{dt}\left[2\rho C_p R^2 \int_0^1 (T-T_\alpha)\frac{r}{R}\frac{dr}{R}\right]$$

$\therefore$ The integral energy equation is:

$$\frac{g^+}{2\rho C_p} = -\frac{k}{\rho C_p R}\frac{dT}{dr}\bigg|_R + \frac{d}{dt}\left[\int_0^1 (T-T_\alpha)\frac{r}{R}\frac{dr}{R}\right]$$

$\Longrightarrow$

To solve this equation, an expression is required for T.

Define $\theta = T - T_\alpha$ and assume

$$\theta(r,t) = Y(r)\,\Gamma(t)$$

where $\Gamma(0) = 0$ and $\Gamma(\alpha) = 1$, i.e. $Y(r)$ is the steady state solution.

☐ Let's find the steady state solution, i.e at $t = \alpha$.

Conditions are: $\quad r = 0 \quad \frac{dT}{dr} = 0 \quad$ (symmetry)

$$r = R \qquad -k\left.\frac{dT}{dr}\right|_R (2\pi R \cancel{L}) = g^+(\pi R^2)\cancel{L}$$

and $\quad r = R \quad T = T_{ss} \quad (> T_\alpha)$

(See the sketch on front page.)

Assume $\quad \theta(r,\alpha) = Y(r) = a + br + cr^2$

$$\frac{dY}{dr} = b + 2cr$$

at $r = 0 \quad \frac{dY}{dr} = 0 \quad \Rightarrow \quad b = 0$

at $r = R \quad \frac{dY}{dr} = 2cR = -\frac{g^+R}{2k} \quad$ (from b.c.)

$$c = -g^+/4k$$

and at $r = R$

$$T_{ss} - T_\alpha = a - \frac{g^+R^2}{4k} \quad \text{or} \quad a = T_{ss} - T_\alpha + \frac{g^+R^2}{4k}$$

$$\therefore \quad Y(r) = \theta(r,\alpha) = T_{ss} - T_\alpha + \frac{g^+R^2}{4k} - \frac{g^+r^2}{4k}$$

$$\text{or} \quad T(r,\alpha) - T_\alpha = T_{ss} - T_\alpha + \frac{g^+R^2}{4k}\left(1 - r^2/R^2\right)$$

$$T(r,\alpha) - T_{ss} = \frac{g^+R^2}{4k}\left(1 - r^2/R^2\right)$$

(a) At steady state, when $r=0$, $T(0,\alpha) - T_{ss} = \frac{g^+ R^2}{4k}$

where $T(0,\alpha) = 925°C$, and $T_{ss} = T(R,\alpha) = 400°C$

Also $k = 27\ W/mK$ and $R = 0.009\ m$

$$\therefore \quad 925 - 400 = \frac{g^+(0.009)^2}{4(27)}$$

$$g^+ = 700{,}000{,}000 \quad W/m^3$$

$$\text{or} \quad 700 \quad MW/m^3$$

$\longrightarrow$

(b) The boundary condition for this problem is convective,

i.e. at $r = R$ $\quad -k\frac{dT}{dr}\Big|_R = h(T_{ss} - T_\alpha) = \frac{g^+(\cancel{\pi} R^{\cancel{2}})\cancel{L}}{2\cancel{\pi}\cancel{R}\cancel{L}}$

$$\therefore \quad h = \frac{g^+ R}{2(T_{ss} - T_\alpha)} = \frac{7(10^8)(0.009)}{2(400-316)}$$

$$= 37{,}500 \quad W/m^2K$$

$\longrightarrow$

(c) This section involves the transient part of the problem

$$\theta = T(r,t) - T_\alpha = \left\{(T_{ss} - T_\alpha) + \frac{g^+R^2}{4k}\left(1 - \frac{r^2}{R^2}\right)\right\}\Gamma(t)$$

The function $\Gamma(t)$ is determined by substituting the expression

$$T(r,t) - T_\alpha = \left\{84 + 525\left(1 - \frac{r^2}{R^2}\right)\right\}\Gamma(t)$$

into the integral equation.

solve

$$\int_0^1 \left[84 + 525(1-\eta^2)\right]\eta\, d\eta \qquad \text{where} \quad \eta = r/R$$

$$\int_0^1 (84\eta + 525\eta - 525\eta^3)\, d\eta = \int_0^1 (609\eta - 525\eta^3)\, d\eta$$

$$= \left[\frac{609}{2}\eta^2 - \frac{525}{4}\eta^4\right]_0^1 = \frac{693}{4} = 173.25$$

$\therefore$

$$\frac{g^+}{2\rho C_p} = -\frac{\alpha}{R}\left.\frac{dT}{dr}\right|_R + \frac{d}{dt}\left[173.25\, \Gamma(t)\right]$$

$$\left.\frac{dT}{dr}\right|_R = 525\, \Gamma(t)\left(-\frac{2}{R}\right) = -\frac{1050}{0.009}\Gamma(t)$$

or $$\frac{d\Gamma(t)}{dt} + \frac{\alpha(1050)}{(173.25)(0.009)^2}\Gamma - \frac{7(10^8)}{(173.25)(2)(19050)(116.4)} = 0$$

$$\alpha = k/\rho C_p = \frac{27}{19050(116.4)}$$

$$\frac{d\Gamma(t)}{dt} + 0.9111(\Gamma - 1) = 0 \qquad \Bigg| \quad \text{subject to } t=0,\ \Gamma=0$$

$$\frac{d\Gamma}{\Gamma - 1} = -0.9111\, dt = d\ln(\Gamma - 1)$$

$$1 - \Gamma(t) = \exp(-0.9111\, t)$$

or $$\Gamma(t) = 1 - \exp(-0.9111\, t)$$

The temperature profile is:

$$T(r,t) - T_\alpha = \left\{ 84 + 525\left(1 - \frac{r^2}{R^2}\right)\right\}\left(1 - \exp(-0.9111\, t)\right)$$

At centre-line, $r = 0$, $T(0,t) = 815°C$, $T_\alpha = 316°C$

$$815 - 316 = (84 + 525)\left(1 - \exp(-0.9111\, t)\right)$$

$$t = 1.878 \text{ s} \longrightarrow$$

At this instant

$$T(R,t) - 316 = 84\left(\frac{499}{609}\right)$$

$$T(R,t) = 384.8\ °C \longrightarrow$$