aJ

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ENCH 501: Transport Processes Quiz #5 **November 2, 2004**

Time Allowed: 50 mins. **Name:**

The regular incandescent bulb consists of a filament of fine tungsten wire (double-coiled) that is suspended in an inert gas such as nitrogen, argon or xenon enclosed in a bulb. The "halogen bulb" is similar except that trace amounts of a halogen (most likely iodine) is added to the inert gas (most likely xenon). The halide reacts with tungsten vapor released from the heating element and deposited on the cooler bulb wall. When the volatile compound molecules (formed by the reaction) next approach the hot filament, they decompose into the elements and the tungsten is deposited back on the filament. This recycling prolongs the life of the bulb.

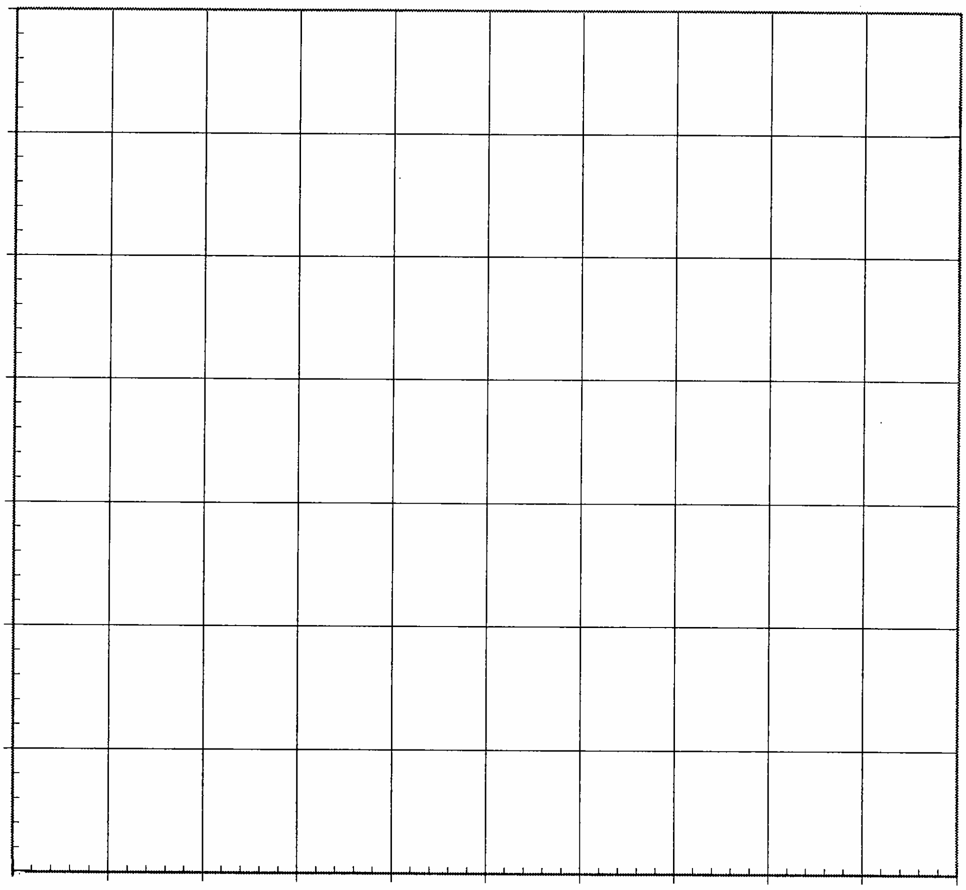
A special light bulb used in the movie industry is made with a ribbon (rather than wire) filament of tungsten. Because the thickness of the ribbon is so much smaller than the width, the ribbon may be treated as an infinite "flat plate" with temperature variations only through the thin section. The thickness of the ribbon is 420μm. When the bulb is turned on, the temperature of the inert gas surrounding the filament very rapidly rises to and remains constant at 256°C. This temperature may be assumed as the initial temperature of the filament, i.e. at t = 0. When the bulb is "on", at steady state, the temperature of the surface of the ribbon (measured with a pyrometer) was recorded as 2552°C. The rate of heat generation by the current passed through the filament is constant at $5.92(10^8)\text{W/m}^3$.

(a) Derive, using the integral method, an expression for the temperature across the thickness of the filament as a function of position and time from the instant the switch is turned on. Treat the heat loss from the filament as if it is entirely by convection. Show **all** important steps.

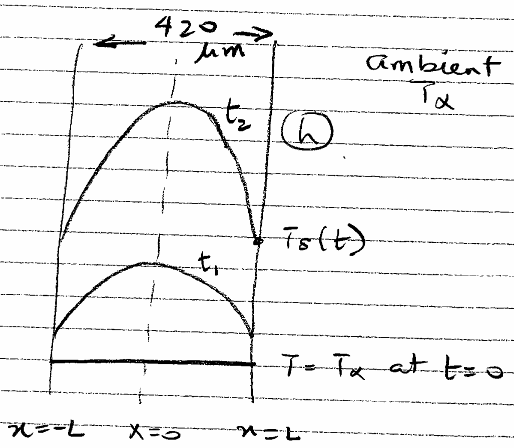
(b) Estimate the effective heat transfer coefficient ***h*** (assumed constant) at the boundary of the filament and the inert gas in the bulb.

(c) Calculate and plot the temperatures at the mid-plane of the filament versus time from t = 0. You need at least 4 properly spaced points to make a good plot.

Data:
Properties of Tungsten
$k = 167.42$ W/mK ; $C_p = 0.134$ kJ/kgK ; $\rho = 19{,}300$ kg/m^3



ENCH 501 Quiz #5 Solution. Nov. 2, 2004



ambient T_α

The filament continuously loses heat to the surrounding inert gas at T_α = constant

Both convective and radiative heat losses are combined using an effective heat transfer coefficient $\underline{h}$.

Overall energy balance per unit area of filament surface, from $0 \le x \le L$ is

Integral Energy Eq. From Notes:

$$\frac{d}{dt}\left[\int_0^L \rho C_p (T - T_\alpha)\, dx\right] - g^+ L + q_x\Big|_L = 0$$

g^+ ↑ generation/volume

Applicable conditions are:

b.c.
$$x = 0, \quad \frac{dT}{dx} = 0 \quad \text{(symmetry)}$$
$$x = L, \quad -k\frac{dT}{dx} = h(T - T_\alpha) \quad \text{(convective)}$$

i.c. $\quad t = 0 \quad T = T_\alpha$

Define $\theta = T - T_\alpha$ and $\theta_s = T_s - T_\alpha$

where $T_s(t)$ is the ribbon surface temp.

Let $\theta(x,t) = X(x)\,\Gamma(t)$ where $X(x)$ is the steady-state solution and $\Gamma(0) = 0$.

The steady state solution may be obtained from:

Conditions
$$x = 0 \quad \frac{dT}{dx} = 0$$
$$x = L \quad T = T_{sf} \quad \text{(final surface temp.)}$$
$$x = L \quad -k\frac{dT}{dx} = g^+ L \quad \text{(all new heat generated goes out.)}$$

Assume $T = a + bx + cx^2$ & apply conditions

$$b = 0 \, , \quad c = -g^+/2k \, , \quad a = T_{sf} + \frac{g^+L^2}{2k}$$

substitute

$$T(x) - T_{sf} = \frac{g^+L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) = \lambda\left(1 - \frac{x^2}{L^2}\right) \quad (1)$$

$$\frac{dT}{dx} = -\frac{2\lambda x}{L^2}$$

$\therefore$ From the boundary condition at steady state

$$\frac{k\,2\lambda}{L} = h\left(T_{sf} - T_\alpha\right) \quad \text{or} \quad h = \frac{2\lambda k}{L\left(T_{sf} - T_\alpha\right)} \quad (2)$$

$\longrightarrow$

L.H.S. of eq. 1 can be re-written as

$$T(x) - T_{sf} = \left(T(x) - T_\alpha\right) - \left(T_{sf} - T_\alpha\right) \quad (3)$$

where $T_{sf} - T_\alpha$ can be evaluated from eq. (2)

i.e. $T(x) - T_\alpha - \dfrac{2\lambda k}{Lh} = \lambda\left(1 - x^2/L^2\right)$

or

$$T(x) - T_\alpha = \lambda\left(1 - x^2/L^2 + \frac{2k}{Lh}\right) \quad (4)$$

Define Biot number, $Bi = Lh/k$

$\therefore$ Steady temperature profile in filament is

$$X(x) = T(x) - T_\alpha = \lambda\left(1 - x^2/L^2 + 2/Bi\right)$$

☐ Unsteady problem, as given

$$T(x,t) - T_\alpha = \theta(x,t) = \lambda\left[1 - x^2/L^2 + 2/Bi\right]\Gamma(t) \quad \text{where} \quad (5)$$

$\Gamma(t)$ is unknown.

Substitute into the integral eq.

$$\rho C_p \lambda \frac{d}{dt}\left[\Gamma(t)\int_0^L \left(1 - x^2/L^2 + 2/Bi\right)dx\right] - g^+L - k\left.\frac{dT}{dx}\right|_{x=L} = 0$$

$$2\rho C_p \lambda L \left(\frac{1}{3} + \frac{1}{Bi}\right)\frac{d\Gamma}{dt} - g^+ L + \left(\frac{2k\lambda}{L}\right)\Gamma = 0 \qquad (6)$$

and use the condition

$t = 0$, $\Gamma = 0$, solve and substitute to get

(a)

$$\frac{T(x,t) - T_a}{\left(\frac{g^+ L^2}{k}\right)} = \frac{1}{2}\left[1 - \frac{x^2}{L^2} + \frac{2}{Bi}\right]\left[1 - \exp\left(-\frac{3\alpha t}{L^2}\frac{Bi}{Bi+3}\right)\right]$$

$\longrightarrow$

(b) From equation (2)

$$h = \frac{\cancel{2k}}{\cancel{k}(T_{sf} - T_a)} \cdot \frac{g^+ L^{\cancel{2}}}{\cancel{2k}} = \frac{g^+ L}{T_{sf} - T_a}$$

$$= \frac{5.92(10^{+8})(210)(10^{-6})}{(2552 - 256)} = 54.15 \ W/m^2K.$$

$\longrightarrow$

(c) At mid-plane of filament , $x = 0$

$$\therefore \quad T(0,t) = T_a + \frac{g^+ L^2}{2k}\left[1 + \frac{2}{Bi}\right]\left[1 - \exp\left(-\frac{3\alpha t}{L^2}\frac{Bi}{Bi+3}\right)\right]$$

$$Bi = \frac{Lh}{k} = \frac{210(10^{-6})(54.15)}{167.42} = 6.7917(10^{-5})$$

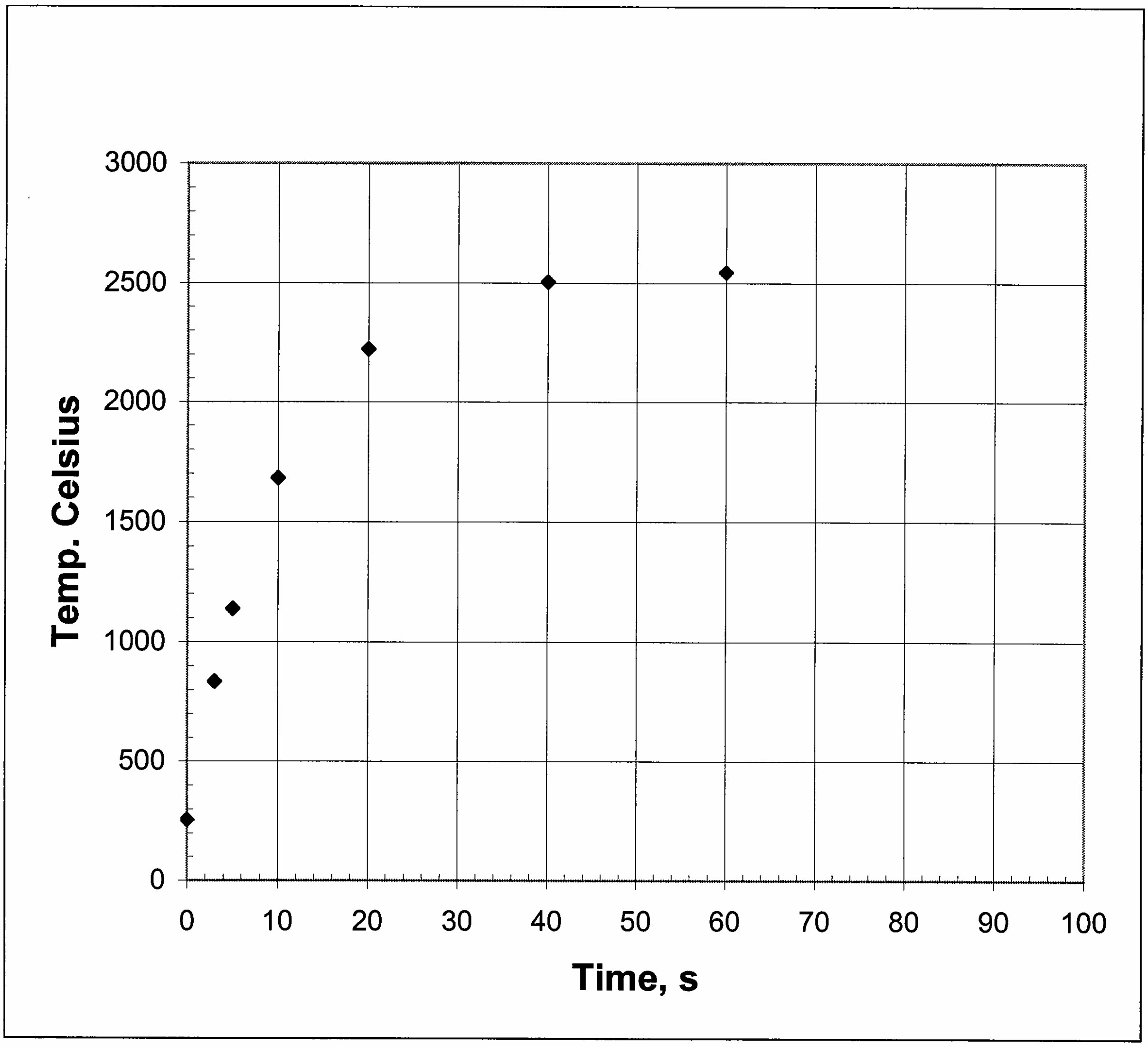
$$\frac{g^+ L^2}{2k} = \frac{5.92(10^8)(210)^2(10^{-12})}{2(167.42)} = 7.7969(10^{-2})$$

$$\alpha = k/\rho c_p = \frac{167.42}{19{,}300(134)} = 6.4736(10^{-5}) \ m^2/s$$

$$T(0,t) = 256 + 2.2961(10^3)\left[1 - \exp(-0.09698\,t)\right]$$

t, s	$T(0,t)$, °C
0	256
3	835.6
5	1138.3
10	1681.5
20	2222.01
40	2504.65
60	2545.3
∞	2552.1

(not much higher than the surface temperature!)



3000
2500
2000
1500
1000
500
0
Temp. Celsius
0
10
20
30
40
50
60
70
80
90
100
Time, s