ENCH 501 Transport Phenomena University of Calgary Name________________________________

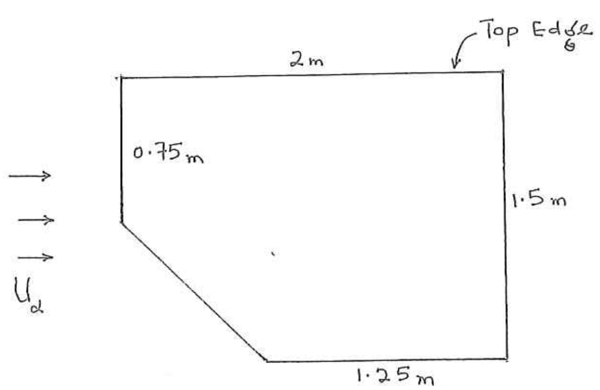
Quiz #4 / Time Allowed: 40 minutes Only a "cheat sheet" is allowed. November 8, 2016 aJ

Large commercial aircrafts have sophisticated landing gears that retract on take-off and are lowered for landing. While the plane is in normal flight, the gears are stored in wheel wells and covered by panels. Opening or closing of the panels and folding or lowering of the gear assemblies are actuated hydraulically or electrically. After full deployment of the system for landing, a panel is seen attached to the gear but a distance away from the wheel parts. Each such panel contributes to a parasitic drag on the plane.

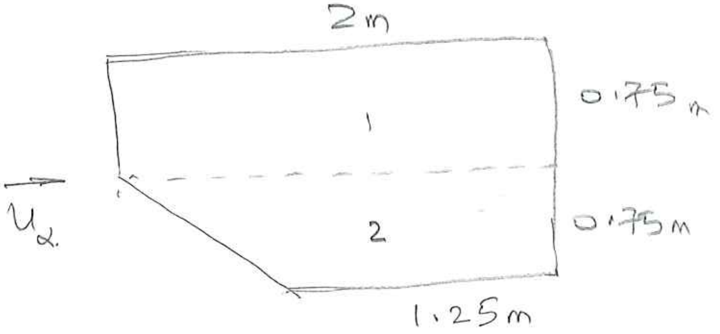
A panel that is attached to a gear assembly of an aircraft has the shape and dimensions as in the sketch below. The plane gears have been deployed in the last 5 minutes of a flight and the plane's current speed is 244.8 km/hr. Ambient air is assumed still and the panel attached to the outside of each wheel assembly is vertically oriented. The properties of the air at the position of the plane (at an elevation of 1,500 m above the runway at sea level) are: density 1.058 kg/m^3 and the dynamic viscosity 0.0174 mPa.s.

(a) **(8 pts)** Estimate the drag on each of the panels. Use the **integral method.** Show the important steps {equations, boundary conditions, chosen u(y), expressions for $\delta(x)$ and $\tau(x)$} of your analysis. For the estimate, assume the entire boundary layer is laminar.

(b) **(2 pts)** If a stowaway passenger such as small reptile (e.g. a gecko) attached itself to the panel at a distance of 0.5 m from the top edge and its force of adhesion to the plate is such that a local shear stress of 0.71 Pa is required to dislodge it (that is shake it free), how far away from the leading edge of the panel must the reptile be to be safe from being sheared off?



ENCH 501 Quiz #4 Solution F16 G. Jeje

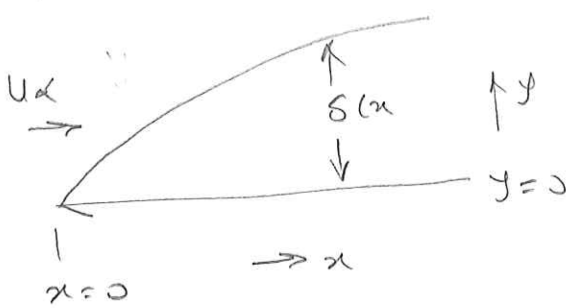


The panel is divided into 2 sections on each side.

The momentum integral equation is

$$\mu \left.\frac{du}{dy}\right|_{y=0} = \frac{d}{dx}\left[\int_0^\delta \rho\,(U_\alpha - u)\,u\,dy\right] \quad \text{eq. 6.18 (Notes)}$$

For flow over a stationary plate (the reverse of problem given – from a different reference frame), the conditions are:



$y = 0 \quad u = 0$

$y = \delta \quad u = U_\alpha$

$y = \delta \quad \dfrac{du}{dy} = 0$

$y = 0 \quad \dfrac{d^2u}{dy^2} = 0$

Assume a velocity profile

$$u = a + by + cy^2 + dy^3$$

Apply conditions to yeild

$$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \quad \text{where } \delta(x) \text{ is unknown}$$

substitute the profile into the integral equation to obtain

$$\delta \frac{d\delta}{dx} = \frac{140}{13}\frac{\nu}{U_\alpha} \; ;$$

subject to the condition $x = 0$, $\delta = 0$ at the leading edge

Solve to get

$$\delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}$$

The drag on the surface is obtained over a strip of width dy from

$$\tau_w = \mu \frac{du}{dy}\bigg|_{y=0} = \frac{3}{2}\mu\frac{U_\alpha}{\delta} = \frac{1.5}{4.64}\left(\mu\rho U_\alpha^3\right)^{\frac{1}{2}} x^{-\frac{1}{2}}$$

$$= \beta x^{-\frac{1}{2}} \quad \text{where } \beta = 0.3233\left(\mu\rho U_\alpha^3\right)^{\frac{1}{2}}$$

Drag over strip of width dy is

$$dD = \int_{x=a(y)}^{x=b(y)} \tau_w \, dx \, dy$$

where a is where the stream engages the solid and b is where the solid ends.

For a series of strips in the y-direction from $y=0$ to $y=W$, the total drag is given by

$$D = \int_0^W \int_{x=a}^{x=b} \tau_w \, dx \, dy$$

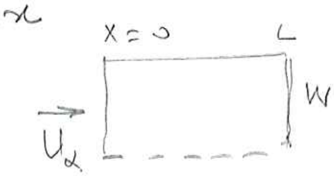
When a and b are constants, the integral can be switched and

$$D = W\int_a^b \tau_w \, dx$$

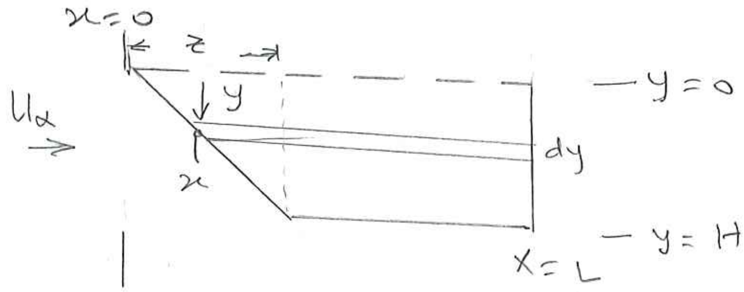
For the panel, consider the rectangular section ①

$$F_1 = W\int_0^L \tau_w \, dx = W\int_0^L \beta x^{-\frac{1}{2}} dx$$

$$= 2W\beta x^{\frac{1}{2}}\Big|_0^L = 2\beta W L^{\frac{1}{2}}$$



For the trapezoidal section ②, the air engages the leading edge of the panel at variable x.



x is related to y by

$$\frac{y}{x} = \frac{H}{z} \quad \text{or} \quad x = \frac{yz}{H}$$

Re-do

$$F_2 = \int_0^H \int_{x=\frac{yz}{H}}^L \tau_w \, dx \, dy = \int_0^H \int_{x=\frac{yz}{H}}^L \beta x^{-\frac{1}{2}} dx\,dy$$

Check:
$\int_0^H \int_0^{z-y} \tau_w \, dx\, dy$
is the correct solution.

$$= 2\beta \int_0^H x^{\frac{1}{2}}\Big|_{\frac{yz}{H}}^L dy = 2\beta \int_0^H \left[L^{\frac{1}{2}} - \left(\frac{z}{H}\right)^{\frac{1}{2}} y^{\frac{1}{2}}\right] dy$$

$$= 2\beta \left[L^{\frac{1}{2}} y - \frac{2}{3}\left(\frac{z}{H}\right)^{\frac{1}{2}} y^{\frac{3}{2}}\right]_0^H$$

$$= 2\beta \left[L^{\frac{1}{2}} H - \frac{2}{3}\left(\frac{z}{H}\right)^{\frac{1}{2}} H^{\frac{3}{2}}\right]$$

$$= 2\beta H \left[L^{\frac{1}{2}} - \frac{2}{3} z^{\frac{1}{2}}\right]$$

The total Force on the two surfaces of the panel, $D = 2(F_1 + F_2)$

$$D = 4\beta\left[W L^{\frac{1}{2}} + H\left(L^{\frac{1}{2}} - \frac{2}{3} z^{\frac{1}{2}}\right)\right]$$

Calculations : $u_\infty = 244.8$ km/hr $= 68$ m/s

$$\beta = 0.3233\left(\mu \rho u_\infty^3\right)^{\frac{1}{2}} = 0.3233\left[1.74(10^{-5}) \times 1.058 \times 68^3\right]^{\frac{1}{2}}$$

$$= 0.7778$$

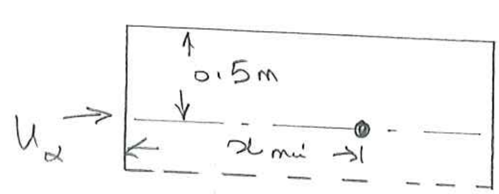
$W = 0.75$ m , $z = 0.75$ m , $H = 0.75$ m , $L = 2$ m

$$D = 4(0.7778)\left[0.75(2^{\frac{1}{2}}) + 0.75\left(2^{\frac{1}{2}} - \frac{2}{3}(0.75)^{\frac{1}{2}}\right)\right]$$

$$= 4(0.7778)\left[1.06066 + 0.62765\right]$$

$$= 5.2523 \text{ N} \longrightarrow$$

(b)



The shear stress at the min. location x_{mi} for the object is

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0} = \beta x^{-\frac{1}{2}} = 0.71 \text{ Pa}$$

That is $0.7778\, x_{mi}^{-\frac{1}{2}} = 0.71$

$$x_{mi} = 1.2 \text{ m} \longrightarrow$$