ENCH 501 Transport Phenomena University of Calgary Name________________________________

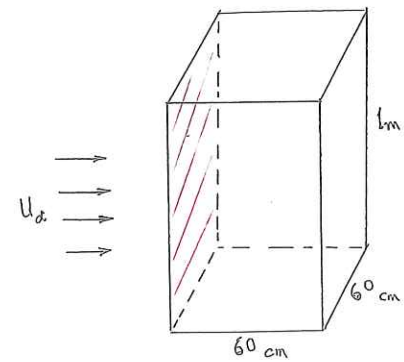
<u>**Quiz #4**</u> / **Time Allowed**: 40 minutes Only a "cheat sheet" is allowed. November 5, 2015 AJ

Rectangular shaped boxes are often mounted on poles or scaffoldings at small malls for advertisement, and as observation posts in the field to monitor forest fires. The boxes are elevated above the ground and are expected to remain in place (not toppled over) when strong wind blows over it. Gusts of wind can exceed 90 km/hr or 10 on the Beaufort scale in Alberta.

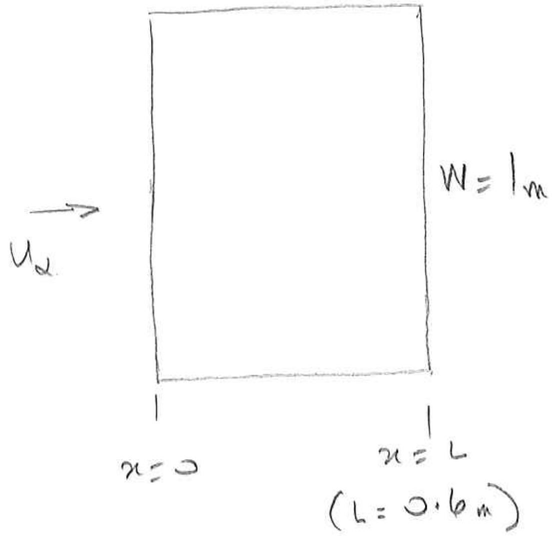
A plexiglas box is 60cm x 60cm x 1m, as shown in the sketch. It contains a camera, a recorder and meteorological equipment, and it is mounted on a tall vertical pole in the forest. On a Fall day, a steady wind of 11.2 m/s was blowing horizontally and normal to one face of the box. The attachment of the box to the (stiff and sturdy) pole is designed to withstand a horizontal force of 46.5N before the box detaches and falls to the ground. The temperature of the air is 15°C, its density is 1.229 kg/m^3 and the dynamic viscosity is 1.73(10^{-5}) Pa.s.

(a) **(7 pts)** Estimate the <u>total</u> force of the wind on the box. Use the **integral method** to estimate the drag component. Will the box remain on top of the pole? Show important steps of your analysis.

(b) **(3 pt)** What are the boundary layer (δ), the displacement (δ_1) and the momentum (δ_2) thicknesses as the rear end of the boundary layer, i.e. at L = 60cm?

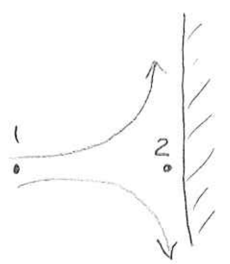


ENCH 501 Solution Quiz 4, Nov. 5, 2015 GJ



The problem has two parts, Pressure due to the velocity head pushes on the <u>front</u> surface, and shear stress acts on the 4 <u>lateral</u> walls. The back wall has no contribution.

(1) The force due to pressure can be estimated using Bernoulli equation. Between points 1 and 2,



$$\frac{P_1}{\rho} + \frac{u_1^2}{2} = \frac{P_2}{\rho} + \frac{u_2^2}{2}$$

where $u_2 = 0$, as the wall re-directs the flow

$$P_2 - P_1 = \frac{\rho u_1^2}{2}$$

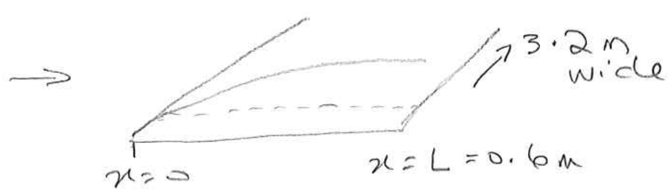
. This is the excess pressure

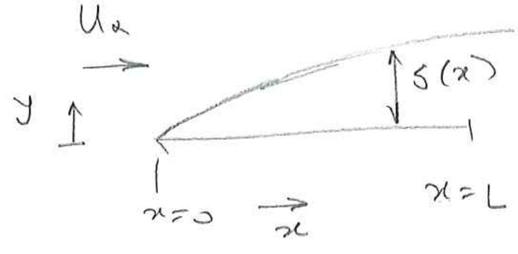
$$= \frac{(1.229)(11.2)^2}{2} = 77.08288 \text{ Pa}$$

The force (Form Drag) $= \Delta P .$ Area

$$= (77.08288)(1 \times 0.6) = 46.2497 \text{ N} \longrightarrow$$

The shear stress is on 4 surfaces equivalent to a wall 3.2m wide and 0.6m long





Check, is the b.l. laminar?

$$Re_x = \frac{U_\alpha \rho L}{\mu} = \frac{11.2(1.229)(0.6)}{1.73(10^{-5})}$$

$$= 4.77(10^5) < 5(10^5)$$

It is laminar.

The integral momentum equation is

$$\mu \left.\frac{du}{dy}\right|_{y=0} = \frac{d}{dx}\left[\int_0^\delta \rho(U_\alpha - u)\,u\,dy\right]$$ From Notes

The conditions are

$y=0$, $u=0$ (no slip), $y=\delta(x)$, $u=U_\alpha=$ const.

$y=\delta(x)$, $\frac{du}{dy}=0$ and $y=0$, $\frac{d^2u}{dy^2}=0$

Assume $u = a + by + cy^2 + dy^3$

Apply conditions: $\frac{u}{U_\alpha} = \frac{3}{2}(y/\delta) - \frac{1}{2}(y/\delta)^3$

Substitute into integral equation to get

$$\frac{3}{2}\frac{\mu U_\alpha}{\delta} = \frac{d}{dx}\left[\rho U_\alpha^2 \delta \cdot \frac{39}{280}\right]$$

Solve with condition $x=0$, $\delta=0$

Boundary layer thickness $\delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}} = \beta x^{1/2}$; $\beta = 4.64\sqrt{\frac{\nu}{U_\alpha}}$

Total drag $D = W\int_0^L \tau|_{y=0}\,dx = \int_0^L \mu \left.\frac{du}{dy}\right|_{y=0} dx$

$$= W\int_0^L \frac{\mu U_\alpha}{\delta}\left(\frac{3}{2}\right)dx$$

$$D = \frac{3}{2}\frac{W\mu U_\alpha}{\beta}\int_0^L x^{-\frac{1}{2}}\,dx$$

$$= 0.6466\left(\rho\mu U_\alpha^3 L\right)^{\frac{1}{2}} \cdot W$$

$$\text{Drag}_s, D = 0.6464\left(1.229 \times 1.73(10^{-5}) \times 11.2^3 \times 0.6\right)^{\frac{1}{2}} \times 3.2$$

$$= 0.277\ \text{N}$$

$$\text{The total force} = 46.25 + 0.277$$

$$= 46.527\ \text{N}$$

Form drag >> Shear forces

This just exceeds the given limit of 46.5 N. The box will fall off!

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(2) Boundary layer thickness $\delta|_L = 4.64\sqrt{\frac{\nu L}{U_\alpha}}$

$$\delta = 4.64\left[\frac{1.73(10^{-5})}{1.229}\frac{(0.6)}{11.2}\right]^{\frac{1}{2}} = 0.004029\ \text{m}$$

or $\simeq$ 4 mm

Displacement thickness, ρ = const

$$\delta_1 = \int_0^\delta\left(1 - \frac{u}{U_\alpha}\right)dy = \frac{3}{8}\delta = 0.001511\ \text{m}$$

or $\simeq$ 1.5 mm

Momentum thickness

$$\delta_2 = \int_0^\delta\left(1 - \frac{u}{U_\alpha}\right)\frac{u}{U_\alpha}\,dy = 0.1392\delta = 0.000561\ \text{m}$$

or $\simeq$ 0.56 mm