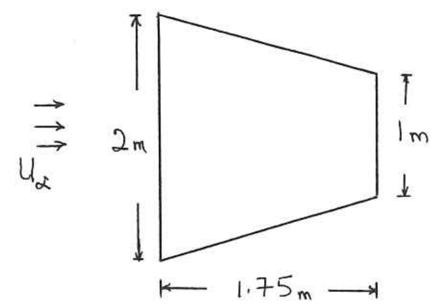
ENCH 501 Transport Phenomena University of Calgary Name________________________________

<u>**Quiz #4**</u> / **Time Allowed**: 45 minutes Only a "cheat sheet" is allowed. November 4, 2014 AJ

At construction sites for tall buildings and sky scrapers, cranes move objects between the ground and the building level where the item is required. Objects moved include power generating equipment, metal plates, concrete slabs, parts of ventilation facilities and buckets of mixed cement to name a few. On windy days, many of the objects lifted up sway as air drags them. Most of the items are hooked securely to the cable and there is very low probability of being dislodged. Awkwardly shaped objects such as plates and slabs, however, are lifted by two or more chain loops. The object sits flat in the loops as harness. A sufficiently strong gust of wind can drag the object out of the harness. The average annual wind speed in Calgary is 14.2 km/hr or 3.94 m/s. (The highest annual wind speed for a Canadian major city is 21.9 km/hr or 6.08 m/s in St. John's, Newfoundland. Winds are classified as strong when speeds exceed 52 km/hr or 14.4 m/s.)

At one site, a flat solar panel is being hoisted to the roof. The shape is as per the sketch. The length L is 1.75 m and the other dimensions are as shown. Air at a steady speed of 4 m/s flows over both sides of the panel, from and normal to the side that is 2m wide. The temperature of the air is 15°C, its density is 1.229 kg/m^3 and the dynamic viscosity is 1.73(10^{-5}) Pa.s.

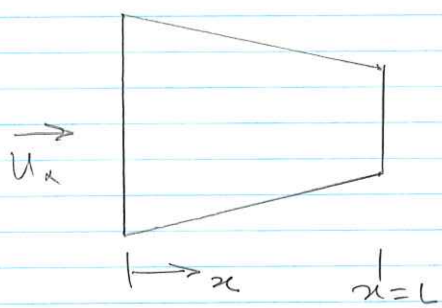


(a) (**7 pts**) Estimate the drag on the panel using the **integral method**. Show the important steps in your analysis.

(b) (**1 pt**) If the flow was from the opposite end, would the drag force be different? Explain.

(c) (**2 pts**) Estimate the boundary layer and displacement thicknesses at distance L from the leading edge.

Quiz 4. Ench 501 Solution Nov. 4, 2014



The integral momentum eq. is

$$\mu \frac{du}{dy}\bigg|_{y=0} = \frac{d}{dx}\left[\int_0^\delta \rho(U_\infty - u)u\,dy\right]$$

since U_∞ and P are assumed constant.

4 boundary conditions are identified in the b.l.

$y = 0$, $u = 0$ (no slip)
$y = \delta(x)$, $u = U_\infty$
$y = \delta(x)$, $\frac{du}{dy} = 0$

the natural b.c.s

and $y = 0$, $\frac{d^2u}{dy^2} = 0$ – from the diff. eq. of b.l.

Assume a polynomial eq. with 4 constants,

$$u = a + by + cy^2 + dy^3$$

Apply b.c.s

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \; ; \text{ where } \delta(x) \text{ unknown}$$

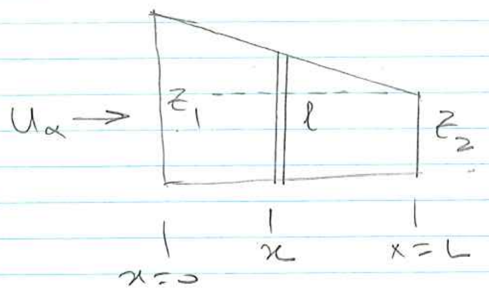
Substitute into the integral equation to get

$$\delta \frac{d\delta}{dx} = \frac{140}{13} \frac{\mu}{\rho U_\infty} \; ; \text{ with } x = 0, \delta = 0$$

solve, $\delta = 4.64\sqrt{\dfrac{\nu x}{U_\infty}}$

This can now be substituted into the vel. profile.

Drag will now be estimated for half of one surface.



The area of differential element at x = $\ell \, dx$

If τ_w is the local wall shear, the force is $\tau_w (\ell \, dx)$

Over the area,

$$D = \int_0^L \ell \, \tau_w \, dx \quad , \quad \text{where } \ell(x)$$

From the geometry, using trigonometry,

$$\ell = z_2 + \frac{z_1 - z_2}{L}(L - x) = z_1 - \left(\frac{z_1 - z_2}{L}\right) x$$

$$= z_1 - \varepsilon x$$

The shear stress at the wall is

$$-\tau_w = \tau\Big|_{y=0} = \mu \frac{du}{dy}\Big|_{y=0} = \frac{\mu U_\infty}{\delta} \frac{d(u/U_\infty)}{d(y/\delta)}\Bigg|_{y=0}$$

From velocity profile,

let $\phi = u/U_\infty$ and $\eta = y/\delta$

$$\phi = \frac{3}{2}\eta - \frac{1}{2}\eta^3 \quad ; \quad d\phi/d\eta = \frac{3}{2} - \frac{3}{2}\eta^2$$

$$\tau\Big|_{y=0} = \frac{\mu U_\infty}{\delta} \frac{d\phi}{d\eta}\Bigg|_{\eta=0} = \frac{3}{2}\left(\frac{\mu U_\infty}{\delta}\right)$$

$$\tau\Big|_{y=0} = \frac{3}{2}\,\frac{\mu U_\alpha}{4.64}\left(\frac{U_\alpha}{\nu}\right)^{\frac{1}{2}} x^{-\frac{1}{2}} = \beta\, x^{-\frac{1}{2}}$$

$$\therefore \; -D = \int_0^L \beta\left(z_1 - \varepsilon x\right) x^{-\frac{1}{2}}\, dx$$

$$= \int_0^L \left(\beta z_1 x^{-\frac{1}{2}} - \beta \varepsilon x^{\frac{1}{2}}\right) dx$$

$$= \left(2\beta z_1 x^{\frac{1}{2}} - \frac{2}{3}\beta\varepsilon x^{\frac{3}{2}}\right)\Big|_0^L$$

$$= 2\beta\left(z_1 - \frac{\varepsilon}{3}x\right)x^{\frac{1}{2}}\Big|_0^L$$

$$= 2\beta\left(z_1 - \frac{\varepsilon}{3}L\right)L^{\frac{1}{2}}$$

Substitute values

$$\beta = \frac{3}{2}\,\frac{(1.73)(10^{-5})(4)}{4.64}\left(\frac{4 \times 1.229}{1.73(10^{-5})}\right)^{\frac{1}{2}}$$

$$= 0.011925$$

$$z_1 = 1\,m$$

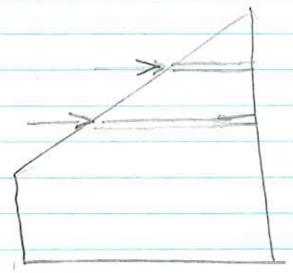
$$\varepsilon = \frac{z_1 - z_2}{L} = \frac{0.5}{1.75} = 0.2857$$

and $L = 1.75\,m$

$$-D = 0.026292 \text{ N}$$

Total drag on plate $= 4(-D) = 0.1052$ N

(b) The drag would be the same if the flow is from the opposite end.
The drag on the rectangular section in the middle will be the same. The drag on the triangular sides will also be the same, only that the leading edge changes as one moves laterally.



(c) The boundary layer thickness at L is given by

$$\delta = 4.64\sqrt{\frac{\mu x}{\rho U_\alpha}}$$

$$= 4.64\sqrt{\frac{1.73(10^{-5})(1.75)}{1.229(4)}}$$

$$= 0.01152\,m \quad \text{or} \quad 11.5\ mm$$

The displacement thickness, δ_1, as shown in Notes, is given by

$$\delta_1 \doteq 3/8\ \delta = 0.00432\,m \quad \text{or} \quad 4.32\ mm$$

* Check that b.l. is always laminar.

$$Re_x = \frac{U_\alpha \rho L}{\mu} = \frac{4(1.229)(1.75)}{1.73(10^{-5})} = 4.97(10^5) < 5(10^5)\ ✓$$