Oct. 23

GJ

ENCH 501 Transport Phenomena **Quiz #4** **Name** ______________________________

Time Allowed: 45 minutes

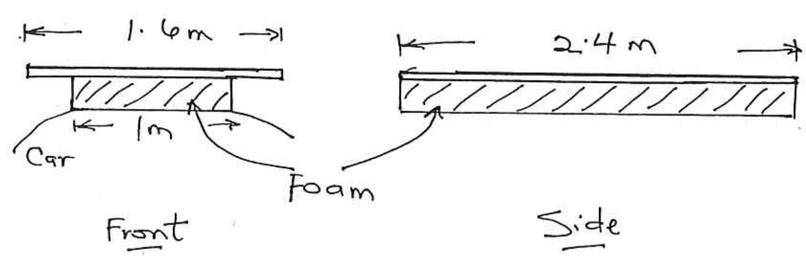
Many people transport objects on the roofs of their vehicles. These include mattresses, pieces of plywood, bicycles, skis, canoes and storage boxes. These items must be held down securely such that they are not dislodged when the vehicle is in motion.

A car has a rectangular piece of plywood tied to the roof rack when the ambient temperature is 22 °C. The piece is 1.6 m wide by 2.4 m long. It is tied with the 1.6 m side normal to the direction of air flow over the wood as the vehicle moves. The wood is placed on a piece of foam plastic that is 1 m wide so that 0.3 m of the wood hangs over the sides of the roof rack, as shown in the sketch below (front view).

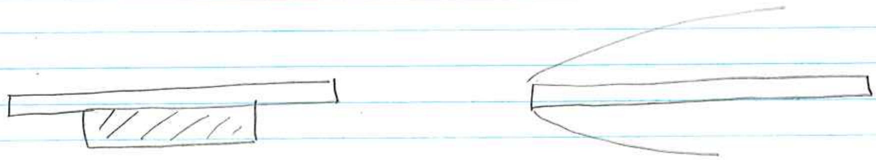
a) (8 pts.) If the force holding the wood down to the roof is given as 0.295 N, estimate the maximum speed (km/hr) that the vehicle can travel without the plywood being dislodged. Use the Integral method and show only important steps in the analysis.

b) (2 pts.) If the temperature of the air is 0 °C, at what speed would the wood be dislodged?

Data: Properties of air at 22 °C – density = 1.18 kg/m^3, viscosity = 1.84 (10^{-5}) Pa s ; at 0 °C, the values are 1.29 kg/m^3 and 1.71 (10^{-5}) Pa s respectively.



ENCH 501 Quiz #4 Solution Oct. 23, 2012



With the vehicle in motion, there will be drag on the top surface and under the surface area not covered by foam.
The effective width of the board is hence $1.6 + 0.3 + 0.3 = 2.2\ m = W$.

By the integral method:
The momentum integral equation is

$$\mu \frac{du}{dy}\Big|_{y=0} = \frac{d}{dx}\left[\int_0^\delta \rho\,(U_\infty - u)\,u\,dy\right]$$

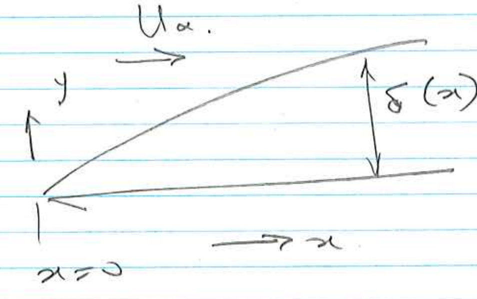
treating the air as incompressible.
The boundary conditions are:

$y = 0 \quad u = 0$

$y = \delta \quad \frac{du}{dy} = 0$

$y = \delta \quad u = U_\infty$

$y = 0 \quad \frac{d^2u}{dy^2} = 0$



Assume a velocity profile

$$u = a + by + cy^2 + dy^3$$

Apply b.c.s

$$\frac{u}{U_\alpha} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Substitute into momentum integral eq. and solve subject to $x = 0$, $\delta = 0$

$$\delta = 4.64\sqrt{\frac{\nu x}{U_\alpha}}$$

Drag is given by

$$F = \int_0^L W \tau\Big|_{y=0} dx \quad ; \quad \tau\Big|_{y=0} = \mu \frac{du}{dy}\Big|_{y=0}$$

$$= \frac{\mu U_\alpha}{\delta} \cdot \frac{3}{2}$$

$$\therefore F = \frac{W \mu U_\alpha}{\beta} \cdot \frac{3}{2} \int_0^L x^{-\frac{1}{2}} dx \quad ; \quad \beta = 4.64\sqrt{\frac{\nu}{U_\alpha}}$$

$$= \frac{3}{\cancel{2}} \cancel{2} \frac{W \mu U_\alpha^{3/2} L^{\frac{1}{2}}}{4.64\sqrt{\nu}} = \frac{3}{4.64} \frac{W \mu U_\alpha^{\frac{3}{2}} L^{\frac{1}{2}}}{\nu^{\frac{1}{2}}}$$

Substitute at 22°C

$$0.295 = \frac{3}{4.64} \frac{(2.2)(1.84)(10^{-5}) U_\alpha^{\frac{3}{2}} (2.4)^{\frac{1}{2}}}{\left[1.5593(10^{-5})\right]^{\frac{1}{2}}}$$

$$U_\alpha = 9.3805 \ m/s$$

$$= \frac{9.3805\ (3600)}{1000} = 33.77 \ km/hr.$$

At $0^\circ C$

$$0.295 = \frac{3}{4.64} \frac{(2.2)(1.71)(10^{-5})\, U_\alpha^{3/2}\, (2.4)^{1/2}}{\left[1.3256\,(10^{-5})\right]^{1/2}}$$

$$U_\alpha = 9.3311 \ m/s$$

$$\text{or} \quad 33.592 \ km/hr.$$

The speeds are low and not much different at the 2 temperatures!

II Check whether b.l. laminar $\longrightarrow$

$$Re_x = \frac{9.3805\,(2.4)}{1.5593\,(10^{-5})} = 1.4438\,(10^{6})$$

This is $> 5(10^5)$

Hence a portion of the boundary layer is turbulent.

Calculations assumed all the b.l. is laminar.

$\longrightarrow$