The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Phenomena Quiz #4 October 12, 2010

Time Allowed: 35 mins. Name:

An automobile radiator is a device usually located in front of the engine compartment to dissipate heat generated by the internal-combustion process. Thermal energy is carried by a mixture of glycol and water that leaves the engine at a temperature of 262°C into the radiator where the liquid is cooled by a cross-flow of air to 186°C for a particular car. The liquid is then re-circulated back into the engine in a closed-loop. The device is a series of vertical tubes joined to horizontal fins for enhanced heat loss. For most cars, the cross-section of the tubes are circular. For some very expensive vehicles, the tubes are elliptical. You are asked to determine whether elliptical tubes are better at dissipating heat.

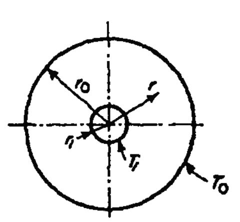
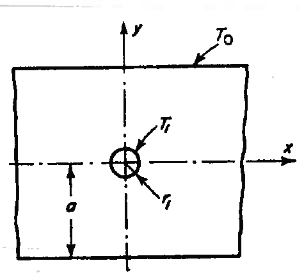
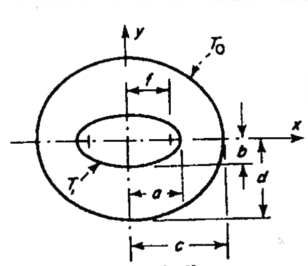
For your test you choose two identical 4 cm, inside radius, nickel steel tubes of circular cross-section. The wall thickness was 4.091 mm. You laid one of the tubes horizontally in a press and deformed the cross-section into an oval. The cross-section of both the inside and outside walls were confocal ellipses. The inside major and minor semi-axes were 5 cm and 3.2 cm, that is, the cross-sectional area of the hole in the tube was not changed. Assume the cross-sectional area of the wall material was also not changed, the wall thickness was measured as δ mm. (Determine δ.) Into both tubes, hot oil at 260°C was passed at the same volume rate. Over the external surfaces of both tubes, air at 12°C was passed at a very high rate that the outside wall temperatures of the tubes may be assume to be the same as the temperature of the air.

a) What length of the elliptical tube is required to cool the oil to 180°C if the circular tube required 56 cm? Show all your steps and assumptions.

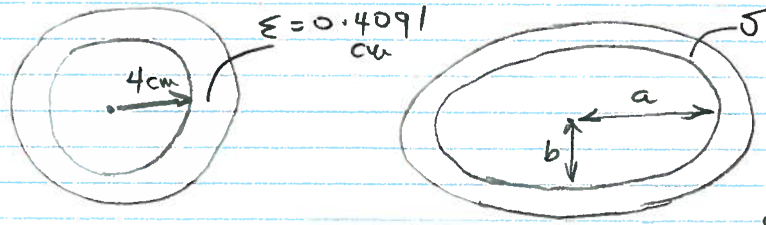
b) What is the volume rate of the oil in the tubes, assumed to be the same for both?

Data: Thermal conductivity of steel = 19 W/m K; heat capacity of oil = 2.307 kJ/kg K; density of oil = 829 kg/m³. Area of an ellipse = π(**ab**), where **a** and **b** are major and minor semi-axes.

Below are some conduction shape factors. L is tube length.

Shape and notation	Shape factor, S $\frac{Q}{L} = kS(T_i - T_0)$
 Concentric circles	$\frac{2\pi}{\ln \frac{r_0}{r_i}}$
 Infinite strip	$\frac{2\pi}{\ln \frac{4a}{\pi r_i}}$
 Confocal ellipses	$\frac{2\pi}{\ln \frac{c+d}{a+b}}$

ENCH 501 Quiz #4 Solution Oct. 12, 2010



$a = 5$ cm

$b = 3.2$ cm

area $= \pi a b$

X-section area of hole $= \pi r^2$
$= 16\pi \text{ cm}^2$

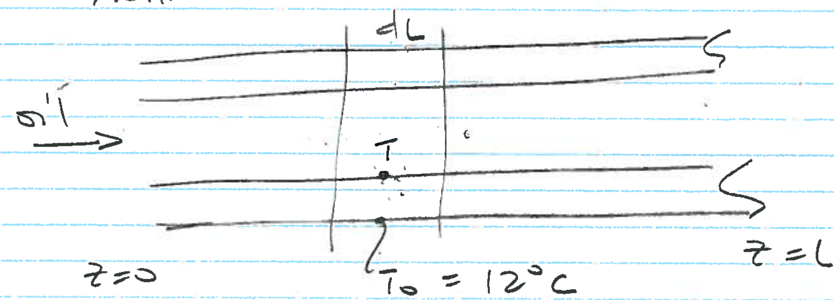
X-section of hole in pipe $= \pi(3.2)(5)$
$= 16\pi \text{ cm}^2$

If area of wall is to remain the same, then

$$\pi(4+\varepsilon)^2 - \pi(4)^2 = \pi(5+\delta)(3.2+\delta) - \pi(5)(3.2)$$

Given $\varepsilon = 0.4091$ cm, substitute, $\delta = 0.4$ cm →

This is a problem with temp. of oil dropping with distance from inlet.



Assume flow turbulent + local fluid temp. = local wall temp.

Choose differential element dL

Shape factor method $dQ = k(S\,dL)(T - T_0)$

The oil loses heat in differential element at rate

$$dQ = -\dot{m}\,C_p\,dT$$

i.e. $$-\frac{dT}{dL} = \frac{kS(T-T_0)}{\dot{m}\,C_p} \quad \text{or} \quad \frac{dT}{T-T_0} = -\frac{kS}{\dot{m}\,C_p}\,dL$$

$$d\ln(T-T_0) = -\beta\,dL \quad ; \quad \beta = \frac{kS}{\dot{m}\,C_p}$$

Integrate :

$$\int_{T_{inlet}}^{T} d\ln(T-T_o) = -\beta\int_0^L dL \quad \text{or} \quad \ln\left(\frac{T-T_o}{T_{inlet}-T_o}\right) = -\beta L$$

This gives

$$T - T_o = (T_{inlet} - T_o)\,e^{-\beta L} \qquad (1)$$

(c) Compare the 2 tubes. T_{exit} are to be the same and T_{inlet} is also the same

$$\therefore \quad \underset{\text{circular}}{\beta_1 L_1} = \underset{\text{elliptical}}{\beta_2 L_2}$$

This is equivalent to $S_1 L_1 = S_2 L_2$

$$\Rightarrow \quad \frac{2\pi}{\ln(r_o/r_i)} L_1 = \frac{2\pi}{\ln\left(\frac{c+d}{a+b}\right)} L_2 \qquad (2)$$

Substitute values

$$\frac{L_1}{\ln\left(\frac{4.4091}{4.0}\right)} = \frac{L_2}{\ln\left(\frac{5.4+3.6}{5.0+3.2}\right)} \quad \text{or} \quad \frac{L_2}{L_1} = 0.956$$

That is, for the same heat transfer, the elliptical tube would be shorter by 4.4 %.

Given $L_1 = 0.56\ m$, $L_2 = 0.535\ m$

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(b) For the circular pipe, $T_{exit} = 180°C$, $T_o = 12°C$, $T_{inlet} = 260°C$. Substitute into (1)

$$(180-12) = (260-12)\,e^{-\beta L} \Rightarrow \beta L = 0.3895$$

$$\text{But } \beta L = \frac{(19)\left(2\pi / \ln\left(\frac{4.4091}{4.0}\right)\right)}{\dot m\,(2307)}(0.56)$$

$$\therefore \quad \dot m = 0.7641\ kg/s$$

Given $\rho_L = 829\ kg/m^3$

Volume rate of oil $\dot{V}_L = 9.22(10^{-4})\ m^3/s$

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