GJ

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ENCH 501: Transport Phenomena Quiz #4 October 13, 2009

Time Allowed: 45 mins. Name:

Many household appliances now come clad in thin sheets of stainless steel, both inside and outside, for a modern and sophisticated look. These sheets have negligible thermal resistance to heat flow between the inside and the outside of the appliances but they help maintain a uniform temperature at all points on the surface because of rapid lateral heat conduction. Of current interest is a free-standing freezer.

The freezer is 86 cm tall, 104 cm wide and 54 cm deep, all external dimensions. The wall is 6 cm thick and it is essentially made of styrofoam (between the stainless steel sheets). The desired average temperature inside is -15°C. To achieve this, the temperature set points are 3°C above and below this value. Thus when the freezer contents warm up to -12°C, the compressor and the refrigeration unit come on and the freezer is cooled to -18°C before power is turned off. The freezer is located where the air temperature is maintained constant at 17°C. The bottom of the freezer seats of a layer of foam and it can be assumed perfectly insulated. Air circulates over the exposed external surface of the freezer such that the heat transfer coefficient is determined to be 12 W/m^2K.

a) If the freezer compressor comes back on 18 hours after it was last triggered off when the inside temperature was at -18°C, estimate the mass of the items in the freezer.

b) The compressor then runs for the next 6 hours and the freezer compartment temperature drops *linearly* over the period. How much total energy would the refrigeration unit extract as heat from the freezer chamber in one cooling cycle?

Note: Since you are given the temperature of the air in the room, not the temperature of the external surface of the freezer, you may need to obtain a term equivalent to UA in heat exchanger calculations. Assume the temperature in the freezer chamber is always uniform at any instant.

Data:
Thermal conductivity of the styrofoam in the freezer wall is 0.035 W/mK. The average specific heat of the content of the freezer is 3.1 kJ/kg K.

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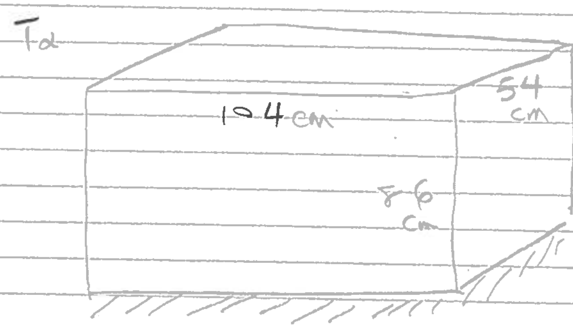

Internal dimensions are 92 x 42 x 74 cm.

The shape factor, S, is given by:

Flat walls $\left[(0.92 \times 0.42) + 2(0.92 \times 0.74) + 2(0.74 \times 0.42)\right] \cdot \frac{1}{0.06}$

+

Edges $\left[4(0.92) + 4(0.74) + 4(0.42)\right] \times 0.54$

+

Corners $\left[8(0.15)(0.06)\right]$

$S = 39.4933 + 4.4928 + 0.072 = 44.0581\ m$

(a) Consider the period when the compressor is off. →

Heat is gained by the freezer content

Let air temp = T_a, freezer external wall temp = T_s; and freezer internal temp = T_i

Energy balance, pseudo-steady state,

$$\dot{Q} = h A_{ext} (T_a - T_s) = k S (T_s - T_i) = m C_p \frac{dT_i}{dt} \Rightarrow \text{where } T_i(t)$$

(Eq. 1) Heat transfer rate from air; Heat gain rate by freezer

But T_s is unknown and can be eliminated as follows:

$$T_\alpha - T_s = \frac{\dot{Q}}{h A_{ext}}$$

$$T_s - T_i = \frac{\dot{Q}}{k S}$$

Add

$$T_\alpha - T_i = \dot{Q}\left[\frac{1}{h A_{ext}} + \frac{1}{kS}\right] = \frac{\dot{Q}}{\beta}$$

$$\therefore \quad \dot{Q} = \beta (T_\alpha - T_i) = m C_p \frac{dT_i}{dt} \qquad \text{Eq. 2}$$

This is an o.d.e. subject to the conditions: $t = 0$, $T_i = -18°C$; $t = 18 \text{ hrs}$, $T_i = -12°C$ and $T_\alpha = 17°C$

$$\frac{dT_i}{T_\alpha - T_i} = \frac{\beta}{m C_p} dt$$

$$-\int_{T_i = -18°C}^{T_i = -12°C} d \ln (T_\alpha - T_i) = \frac{\beta}{m C_p} \int_0^{18(3600)} dt$$

$$\ln \frac{(17 + 18)}{(17 + 12)} = \frac{\beta}{m C_p} (18)(3600)$$

from which m is to be determined.

$$\frac{1}{\beta} = \frac{1}{h A_{ext}} + \frac{1}{kS} \quad ; \quad A_{ext} = 2(1.04)(0.86) + 2(0.86 \times 0.54) + (1.04)(0.54) = 3.2792 \, m^2$$

$$\frac{1}{\beta} = \frac{1}{12(3.2792)} + \frac{1}{(0.035)(44.058)} = 0.6739$$

$\beta = 1.4839 \ W/K$

$\therefore \ m = \dfrac{1.4839 \ (18)(3600)}{\ln\left(\frac{35}{29}\right)(3100)} \ kg$

$= 164.95 \ kg$

→

(b) When the compressor and the refrigeration units are on, both the heat already gained by the freezer content and heat being transferred from the air outside over the 6 hours are to be removed.

The heat removed from content $= m C_p \Delta T$

$(164.95)(3100)(-12-(-18)) = 3.0679 (10^6) \ J$

kg $\quad$ J/kg K $\quad$ K

The rate of heat input from outside

$\dfrac{dQ}{dt} = \beta (T_o - T_i)$

Since T_i decreased linearly, the average rate is given by $\bar{\dot{Q}} = 1.4839 \ (17-(-15)) \ W$

$\therefore$ Total heat input $= \bar{\dot{Q}} \ (6)(3600) \ J$

$= 1.0257 (10^6) \ J$

Total heat removed = the sum $= 4.0936 (10^6) \ J$

→