GJ

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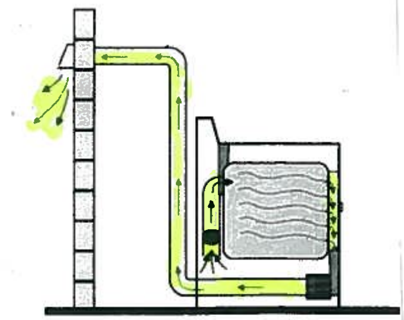
ENCH 501: Transport Phenomena Quiz #4 **October 7, 2008**

Time Allowed: 35 mins. **Name:**

There are many industrial operations in which hot gases are brought in contact with particulate solids to remove liquid films on suspended particles or to promote a chemical reaction. The operation of spray dryers, fluidized catalytic beds, soil remediation systems and cement kilns are examples. A domestic system operating on a similar principle is the clothes dryer. The following problem is on a dryer used to remove wrinkles from clean, dry clothes. The clothes are tossed into the steel tumbler and the dryer is turned on for a few minutes. The tumbler weighs 25 kg. The inlet hot air heats up both the clothes and the tumbler which is thermally isolated from the rest of the equipment.

Wrinkles are to be removed from cotton clothes weighing 5 kg. Air from the room at 20°C is pulled through an electric heater and its temperature raised to 175°C before the air is admitted into the tumbler of the dryer. The air flows through the clothes and then exits the tumbler as shown in the sketch. You may assume that, at any instant, the mass of air in the tumbler is very small compared to the mass of the clothes. The clothes and the tumbler are initially at room temperature and after 4 minutes, the temperature of the gas leaving the tumbler was recorded as 145°C.



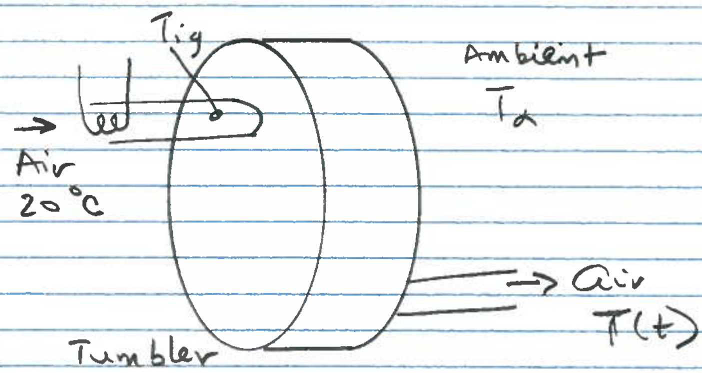
a) Given the data below, estimate the volumetric rate (ft ^{3}/min) at which air is pulled from the room (before being heated). (Note: 1 ft^3/min = 4.7195 (10^{-4}) m^3/s)

b) If the clothes are to be removed from the tumbler when the temperature reaches 162°C, how long should the clothes be in the dryer from the start of the process? Show the equation relating temperature in the tumbler against time elapsed.

Data:

Air: density = 1.205 kg/m^3 at 20°C, specific heat = 1.0057 kJ/kg.K (independent of temperature)
Cotton fibers: density = 400 kg/m^3, specific heat = 1.021 kJ/kg.K
Steel: density = 8700 kg/m^3, specific heat = 0.465 kJ/kg.K

ENCH 501 Quiz #4 Solution Oct. 7, 2008



The hot air leaving the tumbler is assumed to be at the same temperature as the clothes and the tumbler.

Let the tumbler and content be the control volume.

Let mass rate of air $= \dot{m}_1$

the mass of clothes $= m_2$

and the mass of tumbler $= m_3$

Energy Balance on the control volume

$$\text{Input} + \underbrace{\text{Gen}}_{=0} = \text{Output} + \text{Accum.}$$

$$\underbrace{\dot{m}_1 C_{p_1} (T_{ig} - T_\alpha)}_{\text{Heat content of gas in}} = \underbrace{\dot{m}_1 C_{p_1} (T - T_\alpha)}_{\text{Heat content of exit gas}} + \underbrace{\frac{d}{dt}\left\{\left[m_2 C_{p_2} + m_3 C_{p_3}\right](T - T_\alpha)\right\}}_{\text{Rate of heat gain by clothes + tumbler}} \qquad (1)$$

In equation 1, T_α (ambient temp) is the reference temperature for heat content of a body. The mass of air in the tumbler has also been neglected. Otherwise, there would be an extra term in the [ ] of accum term.

Define $\theta = T - T_\alpha$ and $\theta_i = T_{ig} - T_\alpha$

Hence

$$\left(m_2 C_{p_2} + m_3 C_{p_3}\right)\frac{d\theta}{dt} = \dot{m}_1 C_{p_1}\left(\theta_i - \theta\right)$$

or

$$\frac{d\theta}{\theta_i - \theta} = -d\ln(\theta_i - \theta)$$

$$= \left[\frac{\dot{m}_1 C_{p_1}}{m_2 C_{p_2} + m_3 C_{p_3}}\right] dt = \beta\, dt$$

$$\int_0^\theta d\ln(\theta_i - \theta) = -\beta \int_0^t dt \qquad \underline{2}$$

where the initial condition for both the tumbler and clothes $(t = 0,\ T = T_\alpha$ or $\theta = 0)$ has been used

$$\ln(\theta_i - \theta)\Big|_{\theta=0}^{\theta} = -\beta t$$

$$\ln\left(\frac{\theta_i - \theta}{\theta_i}\right) = -\beta t$$

or

$$1 - \frac{\theta}{\theta_i} = e^{-\beta t}$$

Hence

$$\frac{\theta}{\theta_i} = \frac{T - T_\alpha}{T_{ig} - T_\alpha} = 1 - e^{-\beta t} \qquad \underline{3}$$

This equation relates dryer temperature and time.

(a) The volume rate of air can be determined from the mass rate, $\dot{m}_1$

Given $T = 145°C$ when $t = 4$ mins or $240\,s$
$T_\infty = 20°C$,
$T_{ig} = 175°C$

$$\frac{145-20}{175-20} = 0.80645 = 1 - e^{-\beta t}$$

$$e^{-\beta t} = 0.193548$$

$$-\beta t = -1.64228 \quad ; \quad t = 240\,s$$

$$\therefore \quad \beta = 0.006843 \quad s^{-1}$$

Hence

$$\frac{\dot{m}_1 (1005.7)}{5(1021) + 25(465)} = 0.006843$$

$$\dot{m}_1 = \frac{114.477}{1005.7} = 0.113828 \ kg/s$$

Since density of room air $= 1.205\ kg/m^3$,

the intake volume rate for air

$$= \frac{0.113828}{1.205} = 0.094463 \ m^3/s$$

$$\text{or} \quad \frac{0.094463}{4.7195(10^{-4})} = 200.155 \ ft^3/min$$

(b) Use equation 3

$$\frac{162-20}{175-20} = 1 - e^{-(0.006843)t}$$

$t = 362.19\,s$ or a little over 6 minutes