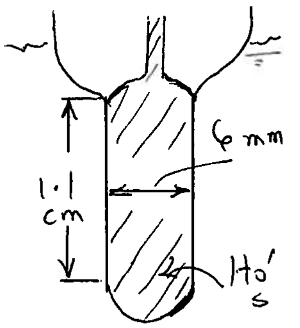
G. J.

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #4 October 9, 2007

Time Allowed: 45 mins. Name:

One of your classmates claims, in a discussion, that temperature measurements using mercury thermometers are often inaccurate. He notes that most people are too impatient and take readings after immersing the thermometer in a fluid or under the tongue for 15s or less.



To prove his point, he suggests you check out a laboratory thermometer illustrated in the sketch. The bulb is a mercury-filled, thin-walled glass "thimble" or a cylinder capped by a hemisphere - with dimensions as shown in the sketch. You may ignore the thin glass wall in your analysis. The upper part of the thermometer is thick-walled glass tube, with a narrow capillary through its axis. Mercury fills the bulb and part of the capillary. You may assume that when the thermometer is immersed in a liquid, heat exchange occurs between the liquid and the mercury only through the bulb. You may also assume that, because the thermal conductivity of mercury is high, all the mercury (in the bulb and the capillary) will be at the same temperature at any instant.

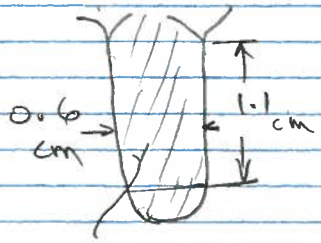
(a) Given the data below, if the thermometer initially at 18°C is inserted into a stirred pool of water at a constant temperature of 96°C, what would the scale read after 15s of immersion?

(b) From part (a), after how long will the scale reading correspond to 99.9% approach to the ultimate reading?

(c) From part (a), at what instant is the rate of temperature rise as indicated by the thermometer equal to 1/4 of the maximum rate?

Data:
Mass on mercury in thermometer = 5g ; Specific heat of mercury = 0.138 kJ/kg K ; Density of mercury = 13,546 kg/m^3 ; Convective heat transfer coefficient around the thermometer bulb, h = 682 W/m^2 K

ENCH 501 Quiz #4 Solution Oct 9, 2007



$T_\alpha = 96°C$

ⓐ Perform an energy balance on the mercury as control volume.

$T(t)$
~~$(t=0,\ T = T_o = 18°C)$~~

$$\text{Input} + \underset{\downarrow\,0}{\text{Gen}} = \underset{\downarrow\,0}{\text{Output}} + \text{Accum}$$

$$\text{Input} = hA(T_\alpha - T)$$, where A = area of bulb

$$\text{Accum} = \frac{d}{dt}\left[m_o C_p (T - T_o)\right]$$

where m_o is mass of mercury and T_o is initial temp.

$$\therefore \quad hA(T_\alpha - T) = m_o C_p \frac{d(T - T_o)}{dt}$$

Define $\theta = T - T_o$

$\theta_\alpha = T_\alpha - T_o$ and $\beta = \dfrac{hA}{m_o C_p}$

$$\beta\left[(T_\alpha - T_o) - (T - T_o)\right] = \frac{d(T - T_o)}{dt}$$

$$\beta(\theta_\alpha - \theta) = \frac{d\theta}{dt}$$

This is a first order o.d.e. subject to the condition $t = 0$, $T = T_o$ or $\theta = 0$

Integrate

$$\int_0^t \beta\, dt = \int_0^\theta \frac{d\theta}{\theta_\alpha - \theta}$$

$$\beta t = -\ln(\theta_\alpha - \theta)\Big|_0^\theta = \ln \frac{\theta_\alpha}{\theta_\alpha - \theta}$$

or $$1 - \frac{\theta}{\theta_\alpha} = e^{-\beta t}$$

or $$\frac{\theta}{\theta_\alpha} = 1 - e^{-\beta t} = \frac{T - T_0}{T_\alpha - T_0}$$

This is the relationship between T and t.

$\longrightarrow$

Given $h = 682 \ W/m^2 K$

$m_0 = 5(10^{-3}) \ kg$

$C_p = 138 \ J/kg/K$

and $A = \pi D z + 2\pi r^2$; $r = 3(10^{-3})m$, $D = 2r$, $z = 1.1(10^{-2})m$

$= \pi(6)(10^{-3})(1.1)(10^{-2}) + 2\pi(9)(10^{-6})$

$= 2.638939(10^{-4}) m^2$

$$\therefore \beta = \frac{682(2.638939)(10^{-4})}{5(10^{-3})(138)} = 0.260834$$

When $t = 15 s$

$$\frac{T - 18}{96 - 18} = 1 - e^{-0.260834(15)} = 0.98001$$

$T = 94.4°C$

Hence, there is an under reading of $\sim 1.6°C$

$\longrightarrow$

(b) A 99.9% approach to the ultimate readings means

$$\frac{T - T_0}{T_a - T_0} = 0.999 = 1 - e^{-\beta t}$$

$$e^{-0.260834\,t} = 0.001$$

$$-0.260834\,t = -6.9078$$

$$t = 26.48 \text{ s} \longrightarrow$$

That is, about 12s more will give higher accuracy.

(c) The rate of temperature rise,

$$\frac{dT}{dt} = \beta(T_a - T)$$

$\left.\frac{dT}{dt}\right|_{\text{maximum}}$ ~~$= \beta(T_a - T_0)$~~ ~~since~~ $(T_a - T_0)$ is largest ΔT

From the solution

$\frac{T - T_0}{T_a - T_0}$ ~~$= 1 - e^{-\beta t}$~~ , $\frac{dT}{dt} = (T_a - T_0)\beta e^{-\beta t}$

Now $$\frac{dT}{dt} = \frac{1}{4}\left.\frac{dT}{dt}\right|_{\max}$$

$$e^{-\beta t} = 0.25$$

$$t = 5.315 \text{ s} \longrightarrow$$