GJ

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ENCH 501: Transport Processes Quiz #4 October 11, 2005

Time Allowed: 45 mins. Name:

Glass is an amorphous solid or very viscous liquid which can be made into intricate shapes by artisans in a series of heating and cooling steps. Expensive Venetian glass art pieces, designed on the Island of Murano, are made from quartz (the polymer $(SiO_2)_n$), 'levant' soda ash (Na_2CO_3) and manganese. The soda ash is a fluxing agent which lowers the melting point of silica, and the manganese removes trace impurities while lengthening the time the glass remains soft and pliable to give time to the glass workers to execute their designs. Lime ($CaCO_3$) is added to other types of glass to increase its hardness and chemical durability.

At a glass workshop, an artist starts with a 300g 'frit" or glass block at room temperature of 25°C. The frit, initially spherical, was placed into a wood-fired furnace at 1400°C. The effective heat transfer coefficient in the furnace is determined to be 15 W/m^{2}K. The glass reaches the 'softening point' (at which the viscosity becomes $10^{6.6}$ Pa.s and the glass will deform under its own weight) at a temperature of 705°C. It attains its 'working point' (at which the viscosity is 10^3 Pa.s and the glass is soft enough to be easily drawn and shaped) at 1140°C. The artisan keeps the glass in the furnace for an extra 2 minutes past the 'working point' and pulls the piece out of the furnace onto a table in the room to quickly work on part of the intended design. The heat transfer coefficient in the room (with a breeze) is 32 W/m^{2}K. The exposed surface area of the glass is increased by 10% at a steady rate in the first 50s after the piece is taken out of the furnace. The rest of the time until the piece is returned into the furnace when it had cooled to 1000°C is for adjustments.

If the artisan repeated the cycles of heating and cooling, and worked on the piece 3 times before completing the design,

a) estimate the time taken to complete the design from the start (with the cold frit) to the finished product cooling to 50°C.

b) What is the highest temperature that the glass will attain in the cycles?

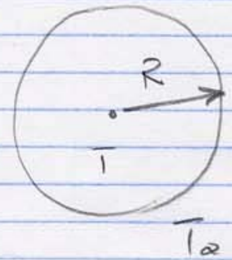
Use the ***Lumped capacity*** method and show all important steps.

Data: Properties of glass (assume constant) :

Density = 2,170 kg/m^3 ; Specific heat = 0.84 kJ/kg K ; Thermal conductivity = 1.69 W/m K

ENCH 501 Quiz 4 Solution Oct. 11, 2005

☑ Cycle 1 – start with a sphere



$$\text{Mass} = 300\,g \quad \text{or} \quad 0.3\,kg$$

$$\text{Volume} = \frac{4}{3}\pi R^3 = \frac{\text{mass}}{\text{density}} \quad \text{or} \quad \frac{0.3}{2170}$$

$$= 1.3825\,(10^{-4})\,m^3$$

$$\therefore R = 3.2088\,(10^{-2})\,m \quad \text{or } 3.2\text{ cm}$$

Area of sphere, $A_o = 4\pi R^2 = 1.2939\,(10^{-2})\,m^2$

☐ Check applicability of lumped capacity method, in furnace

$$\frac{h\,(\forall/A)}{k_{solid}} = \frac{15\,(\forall/A_o)}{1.69} = \frac{15\,(0.0107)}{1.69} = 0.095 < 0.1$$

OK

In air, because $h = 32\ W/m^2K$, the ratio is slightly > 0.1 and solution will be only a good approximation. anyway, problem states that we use the lumped method.

Perform an energy balance on the frit (spherical)

$$-\rho \forall C_p \frac{dT}{dt} = hA\,(T - T_\infty) \quad \text{subject to} \quad \begin{matrix} t = 0 \\ T = T_o \end{matrix}$$

Solution
$$\frac{T - T_\infty}{T_o - T_\infty} = \exp\left[-\frac{hA_o}{\rho \forall C_p} t\right] \quad ; \quad \frac{\forall}{A_o} = 0.0107\,m$$

Find the time to heat block from 25°C to 1140°C

$$\frac{1140 - 1400}{25 - 1400} = \frac{260}{1375} = \exp\left[-\frac{15}{2170(0.0107)840} t\right]$$

$$t = 2165.6\ s \quad (\text{or } 36\text{ min})$$

Give an extra 2 minutes, total time = 2165.6 + 120

or $t = 2285.6$ s

Use this to estimate the temperature at the end of the first heating cycle

$$\frac{T - 1400}{25 - 1400} = \exp\left[-\frac{15}{2170(0.0107)840} \cdot 2285.6\right]$$

$$T = 1162.9\ ^\circ C.$$

This temperature is the start of the cooling part of the first cycle. Cooling is in 2 stages.

0 - 50 s, area is increased steadily - by 10%, That is

$$A = A_0(1 + 0.1\ t/50) \quad ; \quad t \text{ in sec.}$$

The energy balance equation is:

$$-\rho V C_p \frac{dT}{dt} = h A_0 (1 + 0.002t)(T - T_\alpha)$$

or $$\frac{dT}{T - T_\alpha} = -\beta(1 + 0.002t)\,dt \quad ; \quad \beta = \frac{hA_0}{\rho V C_p}$$

Solve

$$\ln\left(\frac{T - T_\alpha}{T_0 - T_\alpha}\right) = -\beta(t + 0.001t^2)\Big|_0^t$$

where $T_0 = 1162.9\ ^\circ C$, $T_\alpha = 25\ ^\circ C$, $t = 50$ s

$$\frac{T - 25}{1162.9 - 25} = \exp\left[-\frac{32}{0.0107(2170)(840)}(50 + 0.001(2500))\right]$$

$$T = 1068.9\ ^\circ C$$

The final area for cycle 1 is $A_1 = 1.2939(10^{-2})(1.1)\ m^2$
$= A_0(1.1)$

The time for adjustments in cycle 1

$$\frac{1000 - 25}{1068.9 - 25} = \exp\left[-\frac{32(1.1)}{0.0107(2170)(840)} t\right]$$

$$t = 37.8 \text{ s}$$

Total time, cycle 1 = 2285.6 + 50 + 37.8 = 2373.4 s →

☑ Cycle 2

Return to furnace

1st stage: Heat back up to working pt.

$$\frac{1140 - 1400}{1000 - 1400} = \exp\left[-\frac{15(1.1)}{2170(0.0107)(840)} t\right]$$

$$t = 509.2 \text{ s}$$

Add 2 mins, $t = 509.2 + 120 = 629.2$ s

Calculate temp. at end of heating

$$\frac{T - 1400}{1000 - 1400} = \exp\left[-\frac{15(1.1)(629.2)}{2170(0.0107)(840)}\right]$$

$$T = 1165.1 \text{ °C}$$

Remove piece from furnace — cooling in 2 stages

Area change, cycle 2 $A = A_1(1 + 0.1\, t/_{50})$; $A_1 = A_0(1.1)$

First 50 s (as for cycle 1)

$$\frac{T - 25}{1165.1 - 25} = \exp\left[-\frac{32(1.1)}{0.0107(2170)(840)} 52.5\right]$$

$$T = 1062 \text{ °C}$$

Area after the 50 s period $A_2 = 1.1 A_1 = 1.21 A_0$ at end of 50 s.

The time for adjustment in cycle 2

$$\frac{1000 - 25}{1062 - 25} = \exp\left[- \frac{32(1.21)}{2170(0.0107)(840)} t \right]$$

$$t = 31.05 \text{ s}$$

Total time for Cycle 2 = 629.2 + 50 + 31.05

= 710.3 s ⟶

☑ Cycle 3

Return to furnace

1st stage

$$\frac{1140 - 1400}{1000 - 1400} = \exp\left[- \frac{15(1.21)}{2170(0.0107)(840)} t \right]$$

$$t = 462.9 \text{ s}$$

Add 2 minutes, t = 462.9 + 120 = 582.9 s.

Calculate temperature at the end of heating

$$\frac{T - 1400}{1000 - 1400} = \exp\left[- \frac{15(1.21)}{2170(0.0107)(840)} 582.9 \right]$$

$$T = 1167.5 \,°C$$ ⟶

Remove from furnace

Area change, cycle 3 $A = A_2 \left(1 + 0.1\, t/50\right)$

First 50 s cooling, with area change with time

$$\frac{T - 25}{1167.5 - 25} = \exp\left[- \frac{32(1.21)}{2170(0.0107)(840)} 52.5 \right]$$

$$T = 1054.4 \,°C$$

Final area, $A_3 = 1.1 A_2 = 1.331 A_o$

The piece is now allowed to cool to 50 °C

$$\frac{50-25}{1054.4-25} = \exp\left[-\frac{32(1.331)}{2170(0.0107)(840)} t\right]$$

$$t = 1702.8 \text{ s}$$

Total time, cycle 3 = 582.9 + 50 + 1702.8

= 2335.7 s

(a) Total time for processing =

2373.4 + 710.3 + 2335.7 = 5419.4 s

or 1.505 hours

⟶

(b) The hottest temperature is 1167.5 °C at the end of the heating phase in cycle 3. For each cycle, the peak temperature increased compared to the previous one.