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ENCH 501: Transport Processes Quiz #4 **October 12, 2004**

Time Allowed: 50 mins. **Name:**

The microwave oven is commonly used to boil water (for coffee or tea) in a mug or used to warm food on a plate. When the mug or plate is removed from the oven, it frequently feels warm or hot to touch. The container thus loses heat to the ambient during the heating process. Since heat is generated by the microwave radiation vibrating primarily the molecules of water in the mug or in the food (with no effects on ceramic or porcelain), the heat acquired by the container has been transferred by its content. Similar heat losses to the ambient via container walls are observed for many industrial systems such as evaporators with immersed steam chests or heating coils.

A cylindrical ceramic mug (8cm o.d., 0.4cm wall thickness and 10cm tall) weighs 301g empty, and 584.3g when filled with water. The initial temperature of both the water and mug was 24°C. The mug was placed in a microwave oven (at a constant power level of 1,688.3 W/kg of water) for 90s. The temperature of the water rose to 51°C. A fan circulates the air in the oven and exchanges it with outside air. Neglect evaporation of water.

(10 pts.) If it is assumed that the ceramic mug temperature is exactly the same as for the water at any instant and the ambient temperature is constant at 24°C in the oven, estimate the heat transfer coefficient between the mug and the air while heating. Assume that both the top and base of the mug are insulated, the water is well mixed and that the lumped heat capacity method is valid.

Bonus (2 pts.) If the mug and the water are not at the same temperature (except at t=0) and the heat transfer coefficient between the water and the mug is $\boldsymbol{h_i}$ (while the heat transfer coefficient between the mug and air is $\boldsymbol{h_o}$), derive an expression that, when solved, allows the determination of the water temperature as a function of time while the oven is operating. Do not solve.

Data:

Ceramic ρ = 1748.9 kg/m^3 ; C_p = 0.96 kJ/kg K

Water ρ = 999.8 kg/m^3 ; C_p = 4.22 kJ/kg K

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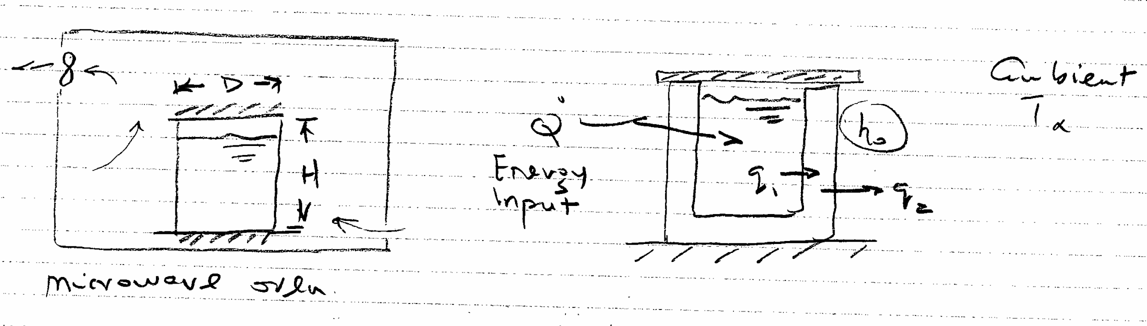


For the main problem, it is given that the temperatures of the water and the ceramic were always the same. $\therefore$ Energy Balance on water + mug —

Input + Generation = Output + Accum.
(Generation = 0)

ϕ Input is from the microwave oven, and it is based only on water.

Mass of water = 584.3 − 301 = 283.3 g

or 0.2833 kg

Energy input rate $\dot{Q}$ = (0.2833)(1688.3) = 478.3 W

ϕ Output = $h_o A_o (T - T_\alpha)$; $A_o = \pi D H$

ϕ Accumulation = $\left(m_w C_{p_w} + m_c C_{p_c}\right)\dfrac{dT}{dt}$, composite system

$$\therefore \quad \dot{Q} = h_o A_o (T - T_\alpha) + \left(m_w C_{p_w} + m_c C_{p_c}\right)\frac{dT}{dt}$$

Let $\alpha = \left(m_w C_{p_w} + m_c C_{p_c}\right)/(h_o A_o)$ = constant

$\beta = \dot{Q}/h_o A_o$ = constant

Then $$\alpha \frac{dT}{dt} + (T - T_\alpha) - \beta = 0$$

Let $$\theta = T - (T_\alpha + \beta)$$

$$\alpha \frac{d\theta}{dt} + \theta = 0$$; Initial condition $t = 0, \ \theta = \theta_0$

Solve.

$$\ln \frac{\theta}{\theta_0} = -\frac{t}{\alpha}$$

$$\therefore \quad \frac{T - (T_\alpha + \beta)}{T_0 - (T_\alpha + \beta)} = \exp\left[-\frac{t}{\alpha}\right]$$

Note that $T_0 = T_\alpha = 24°C$, $t = 90s$, $T = 51°C$

$$\beta = \frac{478.3}{h_0(\pi \times 0.08 \times 0.1)} = \frac{19030.77}{h_0}$$

$$\alpha = \frac{(0.2833)(4220) + (0.301)(960)}{h_0(\pi \times 0.08 \times 0.1)}$$

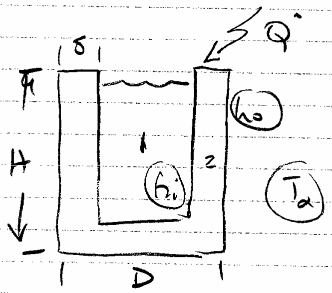
substitute and simplify

$$\frac{27}{19030.77} \times h_0 - 1 = \exp\left[-1.693(10^{-5}) \times 90 \times h_0\right]$$

Solve for h_0. $\quad h_0 \doteq 887.4 \ W/m^2K.$

Bonus Question

For a typical system, the ceramic mug temperature will lag behind the temperature of the water.



Let $1 \equiv$ water
$2 \equiv$ ceramic mug

Heat input by oven $= \dot{Q}$

Energy balance on water

$$\dot{Q} = h_i A_i (T_1 - T_2) + m_1 C_{p_1} \frac{dT_1}{dt} \quad (1)$$

Energy balance on ceramic

$$h_i A_i (T_1 - T_2) = h_o A_o (T_2 - T_\alpha) + m_2 C_{p_2} \frac{dT_2}{dt} \quad (2)$$

These are 2 equations in 2 unknowns, T_1 and T_2.

From eq. (1)

$$T_2 = T_1 - \frac{1}{h_i A_i}\left(\dot{Q} - m_1 C_{p_1} \frac{dT_1}{dt}\right) \quad (3)$$

Differentiate w.r.t. time

$$\frac{dT_2}{dt} = \frac{dT_1}{dt} + \frac{m_1 C_{p_1}}{h_i A_i} \frac{d^2 T_1}{dt^2} \quad (4)$$

Combine equations (1) and (2), eliminating $h_i A_i (T_1 - T_2)$ and substituting (3) and (4)

$$\dot{Q} - m_1 C_{p_1} \frac{dT_1}{dt} = h_o A_o \left\{ T_1 - \frac{1}{h_i A_i}\left(\dot{Q} - m_1 C_{p_1} \frac{dT_1}{dt}\right) - T_\alpha \right\}$$

$$+ m_2 C_{p_2} \left\{ \frac{dT_1}{dt} + \frac{m_1 C_{p_1}}{h_i A_i} \frac{d^2 T_1}{dt^2} \right\} \quad (5)$$

This equation involves only T_1 as the dependent variable and it can be simplified as

$$\alpha \frac{d^2T_1}{dt^2} + \beta \frac{dT_1}{dt} + \gamma (T_1 - T_a) = \delta \text{, a constant.}$$

This requires 2 conditions

$t = 0 \qquad T_1 = T_2 = T_a$

$t = t_1 \qquad T_1 = T_{t_1}$

Where

$$\alpha = \frac{(m_1 C_{p_1})(m_2 C_{p_2})}{h_i A_i}$$

$$\beta = m_1 C_{p_1} \left(1 + \frac{h_o A_o}{h_i A_i} \right) + m_2 C_{p_2}$$

$$\gamma = h_o A_o$$

$$\delta = \dot{Q} \left(1 + \frac{h_o A_o}{h_i A_i} \right)$$

$\longrightarrow$