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ENCH 501: Transport Processes Quiz #4 **October 18, 2002**

Time Allowed: 50 mins. **Name:**

Problem #1 (5 points)

Heat is generated at a constant and uniform rate, g' per unit volume, in a flat plate. The plate has a thickness 2L and it is contacted with a large reservoir of air on both sides at a temperature T_∞. The heat transfer coefficient h is the same on both exposed surfaces.

If the plate was at an initial temperature T_o before the heat generation started, use the Integral Method to derive a function for the temperature in the plate as a function of space and time, i.e. T(x,t). Show all your steps and state your assumptions.

Problem #2 (5 points)

In cold climates, heavy-oil-gathering pipes, pipes in oil refineries or chemical plants exposed to ambient are often "heat traced". A method is to run an insulated electrical cable or ribbon filament along the length of the pipe and pass a current through the wire. The heat generated is transferred to the pipe and it keeps the material flowing within warm. You are instructed by your boss to heat trace a pipe using a thin ribbon coated with a thin plastic layer. The plastic starts to soften and degrade at a temperature of 130°C. So you decided to conduct a test first.

The ribbon available is molybdenum, 1mm thick, and it is wide enough that you can assume it is an infinite plate. The initial temperature of the ribbon is -8°C. You pass a current through so that heat is generated at 39.192 MW/m^3.

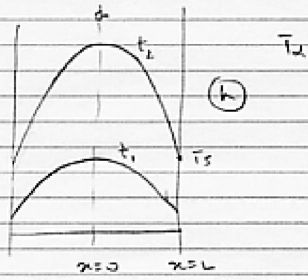
(a) If your test ribbon is uninsulated and exposed to air at -8°C on both sides, what is the value for the heat transfer coefficient (h) below which the ribbon surface temperature will attain or exceed 130°C?

(b) For the condition in part (a), what is the temperature at the midplane of the ribbon 5s after heat generation started?

Data: Properties of Molybdenum
k= 118 W/m K ; C_p= 0.251 kJ/kg K ; ρ= 10,220 kg/m^3

ENCH 501 Solution Q. 4 Oct. 18, 2002

Problem #1



Energy balance over the region

$0 \le x \le L$

$$\frac{d}{dt}\left[\int_0^L \rho C_p (T-T_d)\,dx\right] - g^+ L = -q_x\Big|_{x=L}$$

accum gen output

Subject to

i.c. $t=0$ $T = T_d$

b.c. $x=0$ $\frac{dT}{dx} = 0$ symmetry

Convective $x=L$ $-k\frac{dT}{dx}\Big|_L = h(T_L - T_d)$

Assume $T - T_d = X(x)\,\Gamma(t)$ where $X(x)$ is the steady state solution and $\Gamma(t) \to 1$ as $t \to \infty$

Also $\Gamma(t) = 0$ at $t = 0$

Find first $X(x)$, the steady state solution

Energy balance on shaded element

$$g^+ x = -k\frac{dT}{dx}$$

Integrate

$$-\frac{g^+}{k}\int_x^L x\,dx = \int_T^{T_s} dT$$

or $\frac{1}{2}\left(-g^+/k\right)\left(L^2 - x^2\right) = \left(T_s - T\right)$

But at $x = L$ $-k\frac{dT}{dx} = g^+ L = h(T_s - T_d)$

or $T_s - T_d = \frac{g^+ L}{h}$

$$\therefore \quad -\frac{1}{2}\left(\frac{g^+L^2}{k}\right)\left(1-\frac{x^2}{L^2}\right) = T_a + \frac{g^+L}{h} - T$$

$$\text{or} \quad (T - T_a) = \frac{g^+L}{h} + \frac{1}{2}\left(\frac{g^+L^2}{k}\right)\left(1-\frac{x^2}{L^2}\right)$$

$$= \frac{1}{2}\frac{g^+L^2}{k}\left[\frac{2k}{hL} + 1 - \frac{x^2}{L^2}\right]$$

Hence the steady profile is

$$T(x,\infty) - T_a = \frac{1}{2}\frac{g^+L^2}{k}\left\{1 - \frac{x^2}{L^2} + \frac{2}{Bi}\right\}$$

where Biot #, $Bi = hL/k$.

The transient temperature profile is

$$\frac{T-T_a}{\left(\frac{g^+L^2}{k}\right)} = \frac{1}{2}\left[1 - \frac{x^2}{L^2} + \frac{2k}{hL}\right]\Gamma(t)$$

Substitute into

$$\frac{d}{dt}\left[\int_0^L \rho\, c_p (T-T_a)\,dx\right] - g^+L = k\left.\frac{dT}{dx}\right|_{x=L}$$

to yeild

$$\frac{1}{1-\Gamma}\frac{d\Gamma}{dt} = \left(\frac{Bi}{Bi+3}\right)\frac{3\alpha}{L^2}$$

subject to $t=0 \quad \Gamma = 0$

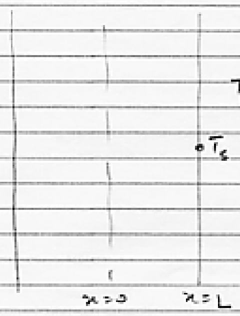
solve : $\Gamma = 1 - \exp\left(-\frac{3\alpha t}{L^2}\frac{Bi}{Bi+3}\right)$

Hence final solution

$$\frac{T-T_a}{\left(\frac{g^+L^2}{k}\right)} = \frac{1}{2}\left[1 - \frac{x^2}{L^2} + \frac{2}{Bi}\right]\left[1 - \exp\left(-\frac{3\alpha t}{L^2}\frac{Bi}{Bi+3}\right)\right]$$

$\longrightarrow$

Problem # 2



(a) At steady state

$$q''' L = h(T_s - T_\alpha)$$

$$T_s = 130\,^\circ C, \quad T_\alpha = -8\,^\circ C$$

$$h = \frac{39.192(10^6)(0.5)(10^{-3})}{138}$$

$$= 142 \ W/m^2K \longrightarrow$$

(b) Use the solution in problem #1

$$Bi = \frac{hL}{k} = \frac{142(0.5)(10^{-3})}{118} = 6.0169(10^{-4})$$

$$\alpha = \frac{k}{\rho c_p} = \frac{118}{10220(251)} = 4.6(10^{-5}) \ m^2/s$$

$$\exp\left[-\frac{3\alpha t}{L^2}\cdot\frac{Bi}{Bi+3}\right] = \exp\left[-0.11069\times 5\right] = 0.575$$

$$T\Big|_{x=0} = -8 + \frac{39.192(10^6)(0.5)^2(10^{-6})}{118}\cdot\frac{1}{2}\left[1+\frac{2}{6.0169(10^{-4})}\right]\left[\ \right]$$

$$= -8 + 138.04\,(1-0.575)$$

$$= 50.67\ ^\circ C \longrightarrow$$