

Quiz #3 / Time Allowed: 40 minutes Only a "cheat sheet" is allowed.

October 13, 2015 AJ

Q. 1 (5 points)

Identify the types of motion prevalent in the following:

1 – Translation, 2 – Linear Deformation, 3 – Angular deformation, 4 – Rotation

Check all the appropriate boxes (There are no part marks for each one.)

	1	2	3	4
1. Blowing wind	✓		✓	✓
2. Gas flow at high rates through a uniform-diameter, straight pipe	✓	✓	✓	✓
3. Water flowing at a slow rate in a straight uniform-diameter tube	✓		✓	
4. Water draining from a full kitchen sink suddenly unplugged	✓			
5. Air flow during inhalation (as you breathe)	✓		✓	✓
6. A drop of water falling through the air	✓			
7. Waves crashing on the seashore	✓		✓	✓
8. Mushroom cloud from detonating an explosion from blasting rock	✓	✓	✓	✓
9. Flow out of the exhaust pipe of a running vehicle	✓	✓	✓	✓
10. Water in a beaker placed at the centre of a slowly rotating turn-table	✓			

eddies

no rotation,
pure translation
(path curved)
- spiralcooling +
shrinkingsolid
body rotation

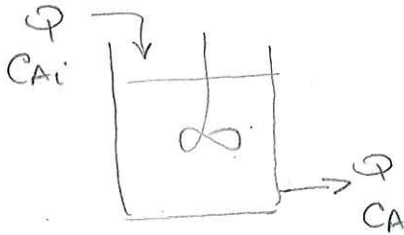
Q. 2 (5 pts)

A batch reactor is a cylindrical tank with a diameter of D m. It is filled to a volume V . The level of the liquid above the bottom of the tank is maintained constant. At time $t = 0$, a solution containing substance A at a concentration C_{Ai} kg/m^3 flows at a rate Q m^3/s into the reactor from the top of the tank. There was no substance A initially in the tank that is always well stirred. Substance A reacts in the tank and it is removed at a rate proportional to its concentration C_A within the tank at any time (a first-order reaction). Per unit volume, the rate of disappearance of A is $k_1 C_A$. A solution flows out of the tank at the same volume rate as the feed from time $t = 0$.

Derive an expression for the mass of A in the tank as a function of time, t .

Show all important steps.

Q. 2.



At $t=0$ $C_A = 0$ in tank.

Control volume - liquid in tank.

Balance - mass of A in tank $\rightarrow A$.

Input + Gen = Output + Accum.

$$Q C_{Ai} - k_1 C_A V = Q C_A + \frac{d(V C_A)}{dt} ; V = \text{const.}$$

$$\frac{d C_A}{dt} = -\left(k_1 + \frac{Q}{V}\right) C_A + \frac{Q}{V} C_{Ai}$$

Group constants, \therefore

$$\frac{d C_A}{dt} = -\alpha C_A + \beta \quad \text{with } t=0, C_A=0$$

$$\text{Let } Y = \beta - \alpha C_A, \quad dY = -\alpha dC_A$$

$$-\frac{1}{\alpha} \frac{dY}{dt} = Y \quad \text{or} \quad \frac{dY}{Y} = -\alpha dt$$

Integrate $t=0, C_A=0$ or $Y=\beta$

$$\int_{\beta}^{\beta - \alpha C_A} \frac{dY}{Y} = -\alpha \int_0^t dt \quad \text{or} \quad \ln Y \Big|_{\beta}^{\beta - \alpha C_A} = -\alpha t$$

$$\ln\left(\frac{\beta - \alpha C_A}{\beta}\right) = -\alpha t \quad \text{or} \quad 1 - \frac{\alpha}{\beta} C_A = e^{-\alpha t}$$

$$\therefore C_A = \frac{\beta}{\alpha} (1 - e^{-\alpha t})$$

$$\text{Mass of A, } A = C_A V = \frac{\beta V}{\alpha} (1 - e^{-\alpha t})$$

$$\text{Total A in tank} \quad A = \left(\frac{Q C_{Ai} V}{k_1 V + Q} \right) \left(1 - e^{-(k_1 + \frac{Q}{V})t} \right) \text{ kg}$$