ENCH 501 Transport Phenomena University of Calgary Name________________________________

Quiz #3 / **Time Allowed**: 45 minutes Only a "cheat sheet" can be consulted. October 7, 2014 AJ

(1) (10 points) A cylindrical glass vessel (vertical and open at the top) has a radius of 10 cm and it contained a salt solution to a level of 25 cm above the base at t = 0. The concentration of salt was 0.5 kg/m^3 in the tank at t = 0. At the bottom of the vessel, a 0.5 m long, 1.2 cm i.d. straight tube is attached but it is plugged at the end open to air. At t = 0, 2(10^{-3}) m^3/s of a solution containing 3 kg/m^3 of salt was fed into the top of the vessel, and simultaneously, the plug in the tube was removed so that solution from the tank can flow out. The vessel is well stirred so that the salt concentration in the vessel can be assumed uniform always. Because the solutions are dilute, the density of the solution (irrespective of its salt content) is assume constant at 1,000 kg/m^3. The viscosity of the solution is 1 mPa.s. If the flow within the tube is assumed laminar and the Hagen-Poiseuille equation is assumed valid,

(a) After how long will the volumetric flow rate out of the vessel be 1% different from (just below or above) the flow rate into the vessel? Show all important steps.
(b) How much salt (in kg) will be in the vessel at this instant?
(c) Is the assumption of laminar flow in the tube valid?

(2) (Bonus: 2 points) A liquid hydrocarbon mixture has 4 components – benzene (1), toluene(2), ethylbenzene (3) and cyclooctane (4). At 20°C, a liquid mixture with the mole fractions given below has a density of 861.8 kg/m^3 (**H. El-Sayed, 2009, Ph.D. thesis, University of Windsor, Canada**).

Estimate the mass concentrations of the components.

Data:

	1	2	3	4
Molar mass, g/mol	78.11	92.13	106.16	112.21
Mole fraction	0.2558	0.2331	0.1754	

Mass fraction, $\omega_i = \frac{\rho_i}{\rho}$ and Mole fraction, $x_i = \frac{\rho_i/M_i}{c}$

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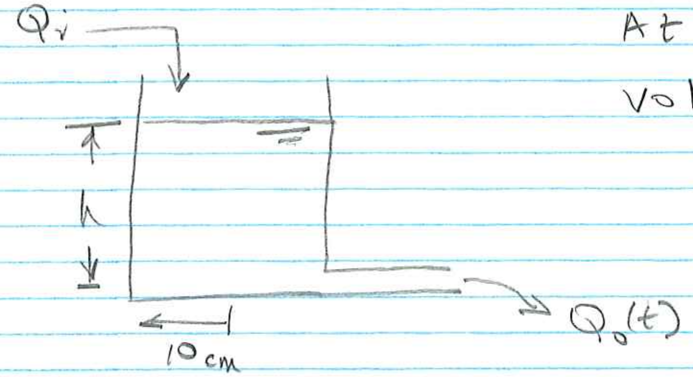

At $t = 0$ $h = 25$ cm

volume of solution

$$= \pi (0.1)^2 (0.25) = 0.007854 \ m^3$$

$\therefore$ amount of salt in vessel at $t = 0$ is

$$(0.007854)(0.5) = 3.927(10^{-3}) \ kg/s$$

The volume rate of feed, $Q_i = 2(10^{-3}) \ m^3/s$

At $t = 0$, the effluent flow Q_o can be estimated from the Hagen-Poiseuille equation

$$Q_o = \frac{\pi (\Delta P) R^4}{8 \mu L} \quad \text{where } \Delta P = h \rho g$$

$$Q_o = \left(\frac{\pi (\rho g) R^4}{8 \mu L} \right) h_o = \beta h_o \quad \text{with } h_o = 0.25 \ m$$

$$Q_o = \frac{\pi (10^3)(9.81)(6)^4 (10^{-12})}{8(10^{-3})(0.5)} \cdot 0.25$$

$$Q_o \Big|_{t=0} = (0.9985)(10^{-2})(0.25) = 0.002496 \ m^3/s$$

The outflow rate is greater than the feed rate, hence the level of the solution will drop with time.

The outflow rate at 1% above inflow is

$$(1.01)(2)(10^{-3}) = 2.02(10^{-3}) \ m^3/s \text{ at } t.$$

The corresponding height of liquid at t can be estimated from the Hagen-Poiseuille eq.

$$Q_t = (0.9985)(10^{-2})\, h_t = 2.02(10^{-3})$$

$$h_t = 0.2023 \text{ m} \quad \text{or} \quad 20.23 \text{ cm}.$$

(a) Balance on liquid in vessel as control volume

$$\text{Input} + \underset{0}{\cancel{\text{Gen}}} = \text{Output} + \text{Accum}$$

$$Q_i \rho \qquad\qquad Q_o \rho + \frac{d(\forall \rho)}{dt}$$

where $\forall$ is volume of solution in the vessel.

Since ρ = constant and $\forall = Ah$ (A constant)

$$Q_i = \beta h + A\frac{dh}{dt} \quad ; \quad A = \pi(0.1)^2 \text{ m}^2$$

$$\int_{h_0}^{h_t} \frac{dh}{Q_i - \beta h} = \int_0^t \frac{dt}{A}$$

Integrate

$$\ln\left[\frac{Q_i - \beta h_t}{Q_i - \beta h_o}\right] = -\frac{\beta}{A} t$$

substitute

$Q_i = 2(10^{-3})$ m^3/s, $\beta = 0.009985$, $h_o = 0.25$ m,

$h_t = 0.2023$ m, $A = \pi(0.01)$ m^2

$$t = 10.1 \text{ s} \longrightarrow$$

(b) Balance on salt in the tank – rates

Input + Gen = Output + Accum
(Gen → 0)

$$2(10^{-3})(3) \qquad Q_o\Big|_t \frac{x}{Ah} + \frac{dx}{dt}$$

where x is mass of salt in the tank

$$6(10^{-3}) = \beta h \frac{x}{Ah} + \frac{dx}{dt}$$

$$\int_{3.927(10^{-3})}^{x} \frac{dx}{6(10^{-3}) - \frac{\beta}{A}x} = \int_0^{10.1s} dt$$

Integrate

$$\ln\left[\frac{0.006 - \beta/_A\, x}{0.006 - \beta/_A (3.927)(10^{-3})}\right] = -\frac{\beta}{A}(10.1)$$

$$\beta/_A = \frac{0.009985}{\pi(0.01)} = 0.317832$$

$$0.006 - 0.317832x = 0.000192$$

$$x = 0.018275 \text{ kg/s} \quad \text{or} \quad 18.3 \text{ g/s}$$

(The vessel contained 3.93 g/s at t = 0.)

(c) The highest velocity in the tube occurs at the very start

$$Q_o\Big|_{t=0} = 2.496\,(10^{-3}) = \bar{u}(\pi)(6)^2(10^{-6})$$

$$\therefore \quad \bar{u} = 22.07 \text{ m/s}$$

$$Re = \frac{D\bar{u}\rho}{\mu} = \frac{1.2(10^{-2})(22.07)\,10^3}{10^{-3}}$$

$$\doteq 2.65\,(10^5) \gg 2100$$

flow expected to be turbulent

2 Bonus Question

Use the following relationships

$$w_i = \rho_i/\rho \quad \text{and} \quad x_i = \frac{\rho_i/M_i}{c}$$

$$\rho_i = c M_i x_i$$

$$= \frac{c M_i x_i}{\sum_{j=1}^{4} c M_j x_j}$$

$$= \frac{x_i M_i}{\sum_{j=1}^{4} x_j M_j}$$

From the data, it is obvious that $x_4 = 0.3357$

Species	$x_i M_i$	w_i	ρ_i kg i/m³ sol^n
Benzene 1	19.9805	0.2044	176.1519
Toluene 2	21.4755	0.2197	189.3375
Ethylbenzene 3	18.6205	0.1905	164.1729
Cyclooctane 4	37.6689	0.3854	332.1377
Σ	97.7454	1.0000	

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