CJ Oct 2

University of Calgary - Department of Chemical & Petroleum Engineering Fall 2012

**ENCH 501 Transport Phenomena** **Quiz #3** **Name** ____________________

**Time Allowed:** 45 minutes

**Question #1** (5 points)

The simple shell-and-tube heat exchanger is normally assumed to be arranged as concentric cylinders and to transfer heat between fluids without loss to the surroundings, even when the shell is not insulated. These are often not true.

Consider an exchanger with a tube that has a thin wall, a diameter of 8 cm and a length of 5 m. The tube is located inside a 20 cm diameter shell such that the walls are parallel but separated by a gap of 1 cm at a plane through the centers of the tube and the shell.
If saturated steam at 260 °C is passed at a high rate through the tube, it is a very windy day and the air outside the exposed shell is at -21 °C, estimate the rate of heat loss to the ambient. Assume the space between the tube and the shell is filled with stagnant oil with its density equal 852 kg/m$^3$, heat capacity equals 2.131 kJ/kg K and a thermal conductivity of 0.138 W/m K. Neglect heat losses through the exchanger headers.

**Question #2** (5 points)

Pure solvent $A$ of a fixed volume $\mathbf{V_S}$ is contained in a vessel. Material $B$, provided as **N** uniform sized spheres initially of diameter $\mathbf{d_o}$, is to be dissolved in the solvent. When the spheres are dumped into the solvent, they are kept suspended by strong agitation. As material B dissolves, the concentration of B in the solvent at the sphere-solvent interface is that for a saturated solution, $\mathbf{C_{BS}}$ – a constant in kmols/m$^3$, and the concentration of B in the solvent, $\mathbf{C_{B\alpha}}$ increases with time. The mass transfer coefficient between each sphere and the solvent is $\mathbf{k_B}$, the density of solid B is $\boldsymbol{\rho_B}$ and its molar mass is $\mathbf{M_B}$.
Derive a relationship for $\mathbf{C_{B\alpha}}$ as a function of time from the instant the spheres were introduced into the solvent.
**Hint:** Perform a material balance on B in the solvent. Use the solvent at the control volume. The diameter of each sphere, d, is a function of time. Volume of a sphere is $(4/3)\pi R^3$ and the area is $4\pi R^2$.

**TABLE 4.1** Conduction shape factors and dimensionless conduction heat rates for selected systems.

**(a) Shape factors [$q = Sk(T_1 - T_2)$]**

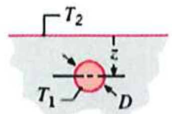
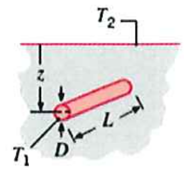
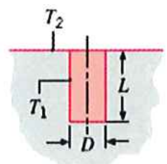
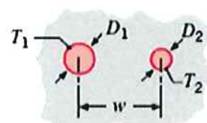
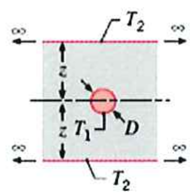
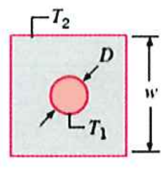
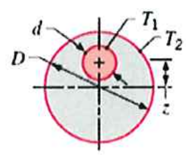
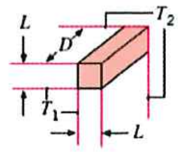
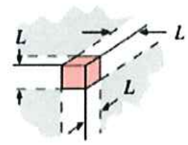
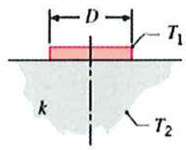
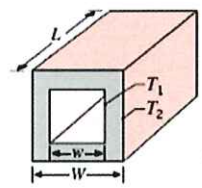
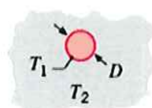
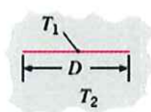
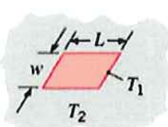
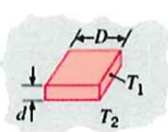
| System | Schematic | Restrictions | Shape Factor |
|---|---|---|---|
| **Case 1**<br>Isothermal sphere buried in a semi-infinite medium |  | $z > D/2$ | $\dfrac{2\pi D}{1 - D/4z}$ |
| **Case 2**<br>Horizontal isothermal cylinder of length $L$ buried in a semi-infinite medium |  | $L \gg D$<br><br>$L \gg D$<br>$z > 3D/2$ | $\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$<br><br>$\dfrac{2\pi L}{\ln(4z/D)}$ |
| **Case 3**<br>Vertical cylinder in a semi-infinite medium |  | $L \gg D$ | $\dfrac{2\pi L}{\ln(4L/D)}$ |
| **Case 4**<br>Conduction between two cylinders of length $L$ in infinite medium |  | $L \gg D_1, D_2$<br>$L \gg w$ | $\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$ |
| **Case 5**<br>Horizontal circular cylinder of length $L$ midway between parallel planes of equal length and infinite width |  | $z \gg D/2$<br>$L \gg z$ | $\dfrac{2\pi L}{\ln(8z/\pi D)}$ |
| **Case 6**<br>Circular cylinder of length $L$ centered in a square solid of equal length |  | $w > D$<br>$L \gg w$ | $\dfrac{2\pi L}{\ln(1.08\, w/D)}$ |
| **Case 7**<br>Eccentric circular cylinder of length $L$ in a cylinder of equal length |  | $D > d$<br>$L \gg D$ | $\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4z^2}{2Dd}\right)}$ |

TABLE 1.1 *Continued*

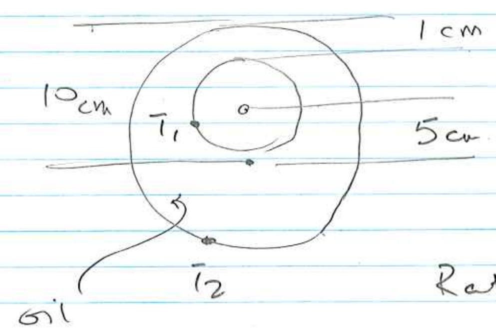
| System | Schematic | Restrictions | Shape Factor |
|---|---|---|---|
| **Case 8**<br>Conduction through the edge of adjoining walls |  | $D > 5L$ | $0.54D$ |
| **Case 9**<br>Conduction through corner of three walls with a temperature difference $\Delta T_{1-2}$ across the walls |  | $L \ll$ length and width of wall | $0.15L$ |
| **Case 10**<br>Disk of diameter $D$ and temperature $T_1$ on a semi-infinite medium of thermal conductivity $k$ and temperature $T_2$ |  | None | $2D$ |
| **Case 11**<br>Square channel of length $L$ |  | $\frac{W}{w} < 1.4$<br>$\frac{W}{w} > 1.4$<br>$L \gg W$ | $\frac{2\pi L}{0.785 \ln (W/w)}$<br>$\frac{2\pi L}{0.930 \ln (W/w) - 0.050}$ |

**(*b*) Dimensionless conduction heat rates [$q = q_{ss}^* k A_s (T_1 - T_2)/L_c$; $L_c \equiv (A_s/4\pi)^{1/2}$]**

| System | Schematic | Active Area, $A_s$ | $q_{ss}^*$ |
|---|---|---|---|
| **Case 12**<br>Isothermal sphere of diameter $D$ and temperature $T_1$ in an infinite medium of temperature $T_2$ |  | $\pi D^2$ | 1 |
| **Case 13**<br>Infinitely thin, isothermal disk of diameter $D$ and temperature $T_1$ in an infinite medium of temperature $T_2$ |  | $\frac{\pi D^2}{2}$ | $\frac{2\sqrt{2}}{\pi} = 0.900$ |
| **Case 14**<br>Infinitely thin rectangle of length $L$, width $w$, and temperature $T_1$ in an infinite medium of temperature $T_2$ |  | $2wL$ | 0.932 |
| **Case 15**<br>Cuboid shape of height $d$ with a square footprint of width $D$ and temperature $T_1$ in an infinite medium of temperature $T_2$ |  | $2D^2 + 4Dd$ | $d/D$ — $q_{ss}^*$<br>0.1 — 0.943<br>1.0 — 0.956<br>2.0 — 0.961<br>10 — 1.111 |

ENCH 501 Quiz #3 Solution Oct. 2, 2012

#1




$T_1 = 260\,°C$

$T_2 = -21\,°C$

Rate of heat loss

$$Q = kS\Delta T$$

from table provided

$$S = \frac{2\pi L}{\cosh^{-1}\left(\frac{D^2 + d^2 - 4z^2}{2Dd}\right)}$$

$L = 5\,m$, $D = 0.2\,m$, $d = 0.08\,m$, $z = 0.05\,m$

$$S = \frac{2\pi(5)}{\cosh^{-1}\left(\frac{(0.2)^2 + (0.08)^2 - 4(0.05)^2}{2(0.2)(0.08)}\right)}$$

$$= \frac{10\pi}{0.518574} = 60.5877\,m$$

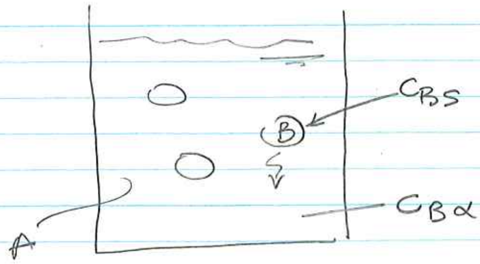
$$\therefore \quad Q = (0.138)(60.5877)(260 - (-21)) \quad W$$

$$= 2,349.468 \quad W$$

$$\text{or} \quad \approx 2.35 \quad kW$$

$\longrightarrow$

\# 2




material balance on B in the solvent.

$$\text{Input} + \underset{\downarrow 0}{\text{Gen}} = \underset{\downarrow 0}{\text{Out}} + \text{Accum.}$$

$$\underset{\text{rate}}{\text{Input}} = \underbrace{N\left(4\pi R^2\right)}_{\text{area}} k_B \left(C_{BS} - C_{B\alpha}\right) \qquad \left(\text{This is similar to } Q = hA\Delta T\right.$$

$$\underset{\text{rate}}{\text{Accum}} = \frac{d}{dt}\left(V_s C_{B\alpha}\right) = V_s \frac{dC_{B\alpha}}{dt} \qquad \left(V_s C_{B\alpha} = \text{moles of B in solvent}\right.$$

i.e. $$N\left(4\pi R^2\right) k_B \left(C_{BS} - C_{B\alpha}\right) = V_s \frac{dC_{B\alpha}}{dt} \qquad (1)$$

In this problem, R (or diameter, d) shrinks with time.

Consider the material transferred from all the spheres into the solvent in time t

$$\underbrace{N \cdot \frac{4}{3}\pi\left(R_o^3 - R^3\right)\rho_B / M_B}_{\text{loss from solids}} = \underbrace{V_s C_{B\alpha}}_{\text{gained by solvent}} \qquad (2)$$

Use equation (2) to remove one of the variables $C_{B\alpha}$ or R in equation (1). We can solve for R and substitute into (2), for example.

Differentiate eq. 2

$$-N\left(\frac{4}{3}\pi\right)\frac{\rho_B}{M_B} 3R^2 \frac{dR}{dt} = V_s \frac{dC_{B\alpha}}{dt} \qquad (3)$$

Equate ③ to ① – since right sides are the same.

$$\cancel{N}\left(4\pi \cancel{R^2}\right) k_B \left(C_{BS} - \alpha\left(R_o^3 - R^3\right)\right) = -\cancel{N}\left(4\pi \cancel{R^2}\right)\frac{\rho_B}{M_B}\cdot\frac{dR}{dt}$$

(from eq. 2)

or

$$-\frac{dR}{dt} = \frac{k_B M_B}{\rho_B}\left(C_{BS} - \alpha R_o^3 + \alpha R^3\right)$$

where $\alpha = \left(N \cdot \frac{4}{3}\pi \frac{\rho_B}{M_B}\frac{1}{V_s}\right)$

$$-\frac{dR}{dt} = \beta\left(C_{BS} - \alpha R_o^3\right) + \alpha\beta R^3$$

where $\beta = \dfrac{k_B M_B}{\rho_B}$

$$\int_{R_o}^{R} \frac{dR}{a + bR^3} = -\int_0^t dt$$

where $a = \beta\left(C_{BS} - \alpha R_o^3\right)$, $b = \alpha\beta$

The solution of this equation relates R to t. The result is substituted into eq. ② to obtain $C_{B\alpha}$ as a function of time.