aJ

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ENCH 501: Transport Phenomena Quiz #3 October 5, 2010

Time Allowed: 45 mins. Name:

In North America, as for most of the world, power plants for generation of electricity are predominantly fueled by coal. For many plants, coal is ground and burnt as powder. The residue of the combustion is fly ash which is primarily silicon, calcium, aluminum and iron oxides, and small quantities of highly toxic materials such as arsenic, barium, boron, lead, thorium, uranium and dioxins. It is legal requirement that fly ash and bottom ash in the burner be recovered and disposed off. One of the methods is to suspend the particles in water and discharge the slurry formed into a pond. There the solids settle and the sludge can be dredged for disposal in a landfill. Occasionally, there is an accident as occurred at the Tennessee Valley Authority Kingston Fossil plant on December 22, 2008. Runoff from heavy rain poured into a pond, churned the slurry and broke the dike. About 4.2 (10^6) m^3 of slurry was released into the surroundings and into the Clinch river. The on-going clean up is very expensive. An engineer has suggested that a way to control the problem of a potential accident is to have a fresh-water pond near and below the slurry-fill pond. If the dike of the slurry pond failed or a rain runoff caused the containment bank to overspill, the slurry flow will be diluted in the fresh-water pond and the discharge into the nearby river will be less catastrophic. You are asked to analyze the proposal by doing some calculations as below.

A slurry pond has a volume of 6(10^6) m^3 or 1.57 billion US gallons and a surface area of 100 acres. It is full to the brim. The suspended solid volume fraction in the slurry at the start was 42%. Below this pond is a fresh water pond, volume 3.2(10^6) m^3 and it is also filled to the brim. The fresh-water pond initially contained no suspended solids, and a river is below it into which its discharge will flow.

A sudden flash flood from the nearby land poured water (assumed free of solids) at a rate of 8(10^5) m^3/hour into the slurry pond. The run-off thoroughly stirred the suspension and a slurry discharge flowed from the slurry pond into the fresh-water pond at the same volumetric rate. This pond was also well stirred and its discharge then flowed into the river at the volume rate above.

a) Derive a relationship for the volume concentration of solids in the fresh water pond as a function of time.
b) At what instant is the maximum amount of solid waste being discharged into the river? What is this rate (volume of solids/hour)?

Please show all your steps.

14.509 $$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

14.510 $$\int xe^{ax}\,dx = \frac{e^{ax}}{a}\left(x - \frac{1}{a}\right)$$

14.511 $$\int x^2e^{ax}\,dx = \frac{e^{ax}}{a}\left(x^2 - \frac{2x}{a} + \frac{2}{a^2}\right)$$

14.512 $$\int x^ne^{ax}\,dx = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$$
$$= \frac{e^{ax}}{a}\left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \cdots \frac{(-1)^n n!}{a^n}\right) \quad \text{if } n = \text{positive integer}$$

14.513 $$\int \frac{e^{ax}}{x}\,dx = \ln x + \frac{ax}{1\cdot 1!} + \frac{(ax)^2}{2\cdot 2!} + \frac{(ax)^3}{3\cdot 3!} + \cdots$$

14.514 $$\int \frac{e^{ax}}{x^n}\,dx = \frac{-e^{ax}}{(n-1)x^{n-1}} + \frac{a}{n-1}\int \frac{e^{ax}}{x^{n-1}}\,dx$$

14.515 $$\int \frac{dx}{p + qe^{ax}} = \frac{x}{p} - \frac{1}{ap}\ln(p + qe^{ax})$$

14.516 $$\int \frac{dx}{(p + qe^{ax})^2} = \frac{x}{p^2} + \frac{1}{ap(p + qe^{ax})} - \frac{1}{ap^2}\ln(p + qe^{ax})$$

14.517 $$\int \frac{dx}{pe^{ax} + qe^{-ax}} = \begin{cases} \dfrac{1}{a\sqrt{pq}}\tan^{-1}\left(\sqrt{\dfrac{p}{q}}\,e^{ax}\right) \\ \dfrac{1}{2a\sqrt{-pq}}\ln\left(\dfrac{e^{ax} - \sqrt{-q/p}}{e^{ax} + \sqrt{-q/p}}\right) \end{cases}$$

14.518 $$\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2 + b^2}$$

14.519 $$\int e^{ax}\cos bx\,dx = \frac{e^{ax}(a\cos bx + b\sin bx)}{a^2 + b^2}$$

14.520 $$\int xe^{ax}\sin bx\,dx = \frac{xe^{ax}(a\sin bx - b\cos bx)}{a^2 + b^2} - \frac{e^{ax}\{(a^2 - b^2)\sin bx - 2ab\cos bx\}}{(a^2 + b^2)^2}$$

14.521 $$\int xe^{ax}\cos bx\,dx = \frac{xe^{ax}(a\cos bx + b\sin bx)}{a^2 + b^2} - \frac{e^{ax}\{(a^2 - b^2)\cos bx + 2ab\sin bx\}}{(a^2 + b^2)^2}$$

14.522 $$\int e^{ax}\ln x\,dx = \frac{e^{ax}\ln x}{a} - \frac{1}{a}\int \frac{e^{ax}}{x}\,dx$$

14.523 $$\int e^{ax}\sin^n bx\,dx = \frac{e^{ax}\sin^{n-1} bx}{a^2 + n^2b^2}(a\sin bx - nb\cos bx) + \frac{n(n-1)b^2}{a^2 + n^2b^2}\int e^{ax}\sin^{n-2} bx\,dx$$

14.524 $$\int e^{ax}\cos^n bx\,dx = \frac{e^{ax}\cos^{n-1} bx}{a^2 + n^2b^2}(a\cos bx + nb\sin bx) + \frac{n(n-1)b^2}{a^2 + n^2b^2}\int e^{ax}\cos^{n-2} bx\,dx$$

DIFFERENTIAL EQUATION	SOLUTION
18.1 Separation of variables	
$f_1(x)\,g_1(y)\,dx + f_2(x)\,g_2(y)\,dy = 0$	$\int \frac{f_1(x)}{f_2(x)}\,dx + \int \frac{g_2(y)}{g_1(y)}\,dy = c$
18.2 Linear first order equation	
$\frac{dy}{dx} + P(x)y = Q(x)$	$y\,e^{\int P\,dx} = \int Q\,e^{\int P\,dx}\,dx + c$
18.3 Bernoulli's equation	
$\frac{dy}{dx} + P(x)y = Q(x)y^n$	$v\,e^{(1-n)\int P\,dx} = (1-n)\int Q\,e^{(1-n)\int P\,dx}\,dx + c$ where $v = y^{1-n}$. If $n = 1$, the solution is $\ln y = \int (Q-P)\,dx + c$
18.4 Exact equation	
$M(x,y)\,dx + N(x,y)\,dy = 0$ where $\partial M/\partial y = \partial N/\partial x$.	$\int M\,\partial x + \int\left(N - \frac{\partial}{\partial y}\int M\,\partial x\right)dy = c$ where ∂x indicates that the integration is to be performed with respect to x keeping y constant.
18.5 Homogeneous equation	
$\frac{dy}{dx} = F\left(\frac{y}{x}\right)$	$\ln x = \int \frac{dv}{F(v) - v} + c$ where $v = y/x$. If $F(v) = v$, the solution is $y = cx$.

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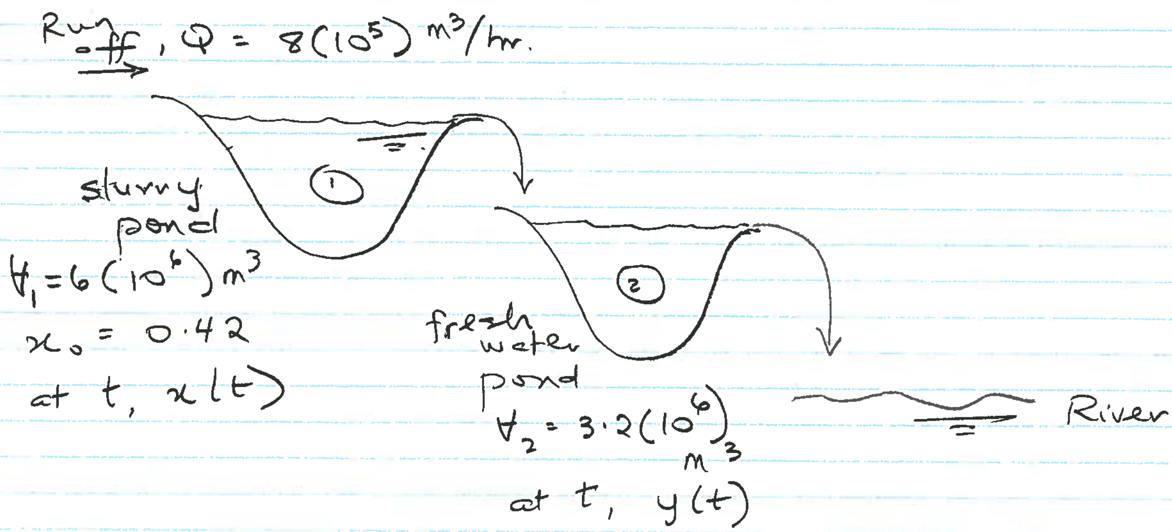

Let the volume fractions of solids in ponds ① and ② be $x(t)$ and $y(t)$ respectively.

Perform material balance on the suspended solids in pond ① — total volume = $\forall_1$

$$\cancel{\text{Input}}^{\,0} + \cancel{\text{Generation}}^{\,0} = \underset{xQ}{\text{Output}} + \underset{\frac{d(\forall_1 x)}{dt}}{\text{Accum}}$$

$$\therefore \quad \frac{d(\forall_1 x)}{dt} = -xQ$$

subject to $t = 0 \quad x = x_0$ (initial condition)

Since $\forall_1$ = constant, $\dfrac{dx}{dt} = -\dfrac{Q}{\forall_1} x = -\beta x$

Re-arrange $\dfrac{dx}{x} = d\ln x = -\beta\, dt$

$\therefore \quad \ln x = -\beta t + \text{constant.}$

Use i.c. $\ln(x/x_0) = -\beta t$ or $x = x_0 e^{-\beta t}$

$\longrightarrow$

This defines the concentration of solids in the flow stream entering pond ②.

Perform a material balance on the suspended solids in pond (2) — total volume $= \forall_2$

Input + Generation = Output + Accum

$$Qx \qquad 0 \qquad Qy \qquad \frac{d}{dt}(\forall_2 y)$$

or $$\frac{d(\forall_2 y)}{dt} = Q(x-y) \quad ; \quad \forall_2 = \text{constant}$$

$$\therefore \frac{dy}{dt} = \frac{Q}{\forall_2}\left[x_o e^{-\beta t} - y\right] = \alpha\left[x_o e^{-\beta t} - y\right]$$

where $\alpha = Q/\forall_2$

This is an ordinary differential equation

$$\frac{dy}{dt} + \alpha y = \alpha x_o e^{-\beta t} \quad \text{with i.c. } t=0,\ y=0$$

Solve using the integration factor method

$$y e^{\int \alpha\, dt} = \int e^{\int \alpha\, dt} \alpha x_o e^{-\beta t} dt + C_1$$

$$y e^{\alpha t} = \alpha x_o \int e^{\alpha t} e^{-\beta t} dt + C_1$$

$$y e^{\alpha t} = \frac{\alpha x_o}{\alpha - \beta} e^{(\alpha-\beta)t} + C_1$$

use i.c. $$C_1 = -\frac{\alpha x_o}{\alpha - \beta}$$

(a) $$\therefore y = \frac{\alpha x_o}{\alpha - \beta}\left(e^{-\beta t} - e^{-\alpha t}\right)$$

but $x_o = 0.42$, $\alpha = 0.25$, $\beta = 0.1333$

$$y = 0.9\left(e^{-0.1333t} - e^{-0.25t}\right); \quad t \text{ hrs}$$

(b) The rate of solid discharge into the river

$$W = Q \cdot y \quad ; \quad Q = \text{constant}.$$

$$\therefore \quad W_{max} = Q \cdot y_{max}$$

at y_{max}, $\dfrac{dy}{dt} = 0 = -0.1333\, e^{-0.1333t} + 0.25\, e^{-0.25t}$

or $\dfrac{0.1333}{0.25} = e^{(-0.25 + 0.1333)t}$

$$\therefore \quad t = 5.3883 \text{ hrs} \longrightarrow$$

~~At~~ this instant,

$$y_{max} = 0.9 \left(e^{-0.1333(5.3883)} - e^{-0.25(5.3883)} \right)$$

$$= 0.9\,(0.4876 - 0.26) = 0.20484$$

$$\therefore \quad W_{max} = 8(10^5)(0.20484) \text{ m}^3/\text{hr}.$$

$$= 1.6387\,(10^5) \text{ m}^3/\text{hr}. \longrightarrow$$

This is the max. solid discharge rate into the river.