GJ

The University of Calgary
Department of Chemical & Petroleum Engineering

ENCH 501: Transport Processes Quiz #3 **October 3, 2006**

Time Allowed: 35 mins. **Name:**

---

1) (6 pts) The law of radioactive decay of a substance is that the instantaneous rate of disintegration is directly proportional to the mass of the substance remaining. If 0.5% of a batch of radium disappears in 12 years, a) what % would disappear in 1000 years? b) If the half life is the time it takes for ½ of the substance to disappear, what is the half life of the batch of radium?

2) (4 pts) A viscometer consists of two long, concentric, rotating cylinders. The space between them is filled with an incompressible fluid. The inner cylinder has a radius κR and it is rotated at a rate Ω_i rpm. The outer cylinder has a radius R and it is rotated at Ω_o. If the resulting flow is laminar, the velocity distribution for the tangential direction u(r) is given as:

$$u = [R^2(1-\kappa^2)]^{-1}\{r[\Omega_o R^2 - \Omega_i \kappa^2 R^2] - [(\kappa^2 R^4)/r](\Omega_o - \Omega_i)\}$$

Define the rate of angular distortion in the flow. Obtain an expression for the radial position at which the rate of angular distortion equals zero?

ENCH 501 Quiz 3 Solution Oct. 3, 2006

1. Let the quantity of radium = A at time t.

$\therefore \frac{dA}{dt} \propto A$ or $\frac{dA}{dt} = -\beta A$ where $\beta > 0$

The negative sign is because A is decreasing as time increases.

If the mass of radium = A_0 at $t = 0$, the equation can be solved

$$\frac{dA}{A} = -\beta dt \quad \text{or} \quad \int_{A_0}^{A} d \ln A = \int_0^t -\beta dt$$

$$\ln A - \ln A_0 = \ln \frac{A}{A_0} = -\beta t$$

Given $t = 12$ yrs, $\frac{A}{A_0} = \frac{1 - 0.005}{1} = 0.995$

$$\ln(0.995) = -\beta(12)$$

or $\beta = 4.177(10^{-4})$ yr^{-1}

$\therefore$ The general equation for A remaining is $A = A_0 \exp(-\beta t)$

(a) $t = 1000$ yrs $\frac{A}{A_0} = \exp(-0.4177)$

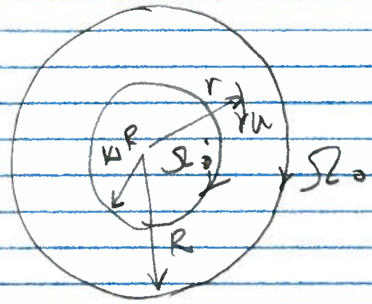
$\frac{A}{A_0} = 0.6586$ (fraction remaining)

$\therefore$ Radium fraction disappeared = 0.3414 or 34.14%

(b) Half-life $\Rightarrow A/A_0 = 0.5 = \exp(-\beta t)$

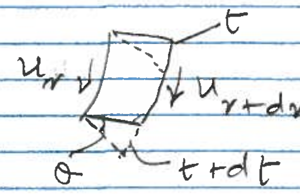
$t = 1659.4$ years.

2.



$$u(r) = \frac{1}{R^2(1-k^2)} \left\{ r(\Omega_o R^2 - \Omega_i k^2 R^2) - \frac{k^2 R^4}{r}(\Omega_o - \Omega_i) \right\}$$

Consider a small element in the annular space



The shapes at t and t+dt are shown.
The distortion angle is θ.

Rate of angular deformation

$$\frac{d\theta}{dt} = \frac{\partial u}{\partial r}$$

$$\tan\theta \simeq \theta \simeq \frac{\frac{\partial u}{\partial r}\,dr\,dt}{dr}$$

Rate of distortion =

$\frac{1}{2}$ (rate of angular deformation)

$$\dot{\varepsilon}_{r\theta} = \frac{1}{2}\frac{\partial u}{\partial r}$$

substitute.

From eq. given

$$\frac{\partial u}{\partial r} = \frac{1}{R^2(1-k^2)} \left\{ (\Omega_o R^2 - \Omega_i k^2 R^2) + \frac{k^2 R^4}{r^2}(\Omega_o - \Omega_i) \right\}$$

$$\dot{\varepsilon}_{r\theta} = 0 \Rightarrow \Omega_o R^2 - \Omega_i k^2 R^2 + \frac{k^2 R^4}{r^2}(\Omega_o - \Omega_i) = 0$$

$$\therefore r = \sqrt{-\frac{\Omega_o R^2 - \Omega_i k^2 R^2}{k^2 R^4(\Omega_o - \Omega_i)}}$$

Possible if $\Omega_i > \Omega_o$ and $\Omega_i k^2 < \Omega_o$ or reverse